Bayesian Inference of multi-exponential decays in low resolution NMR

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Outline

Introduction

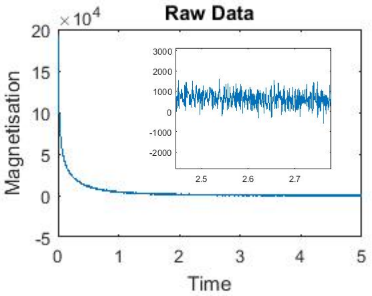
What have we done this week?

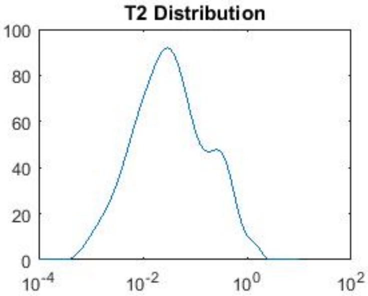
Going forward

Summary

Nuclear Magnetic Resonance

- NMR performed on rocks
- Measure magnetisation over time
- Want to infer distribution of pore sizes $P(T_1, T_2)$ based relaxation times T_1 and T_2





The Problem

Current method - Tikhonov Regularisation

Problems

- Physical properties such as $T_1 \geq T_2$ not satisfied
- No uncertainty estimates

Faster regularisation

Non-Negative Flexible Conjugate Gradient Least Squares (NNFCGLS)

$$\min_{\mathbf{x}\geq 0} ||\mathbf{b} - \mathbf{A}\mathbf{x}||_2^2$$

$$\min_{\mathbf{x}\geq 0} \left\| \begin{bmatrix} \mathbf{A} \\ \lambda \mathbf{I} \end{bmatrix} \mathbf{x} - \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \right\|_2^2 \text{with regularisation}$$

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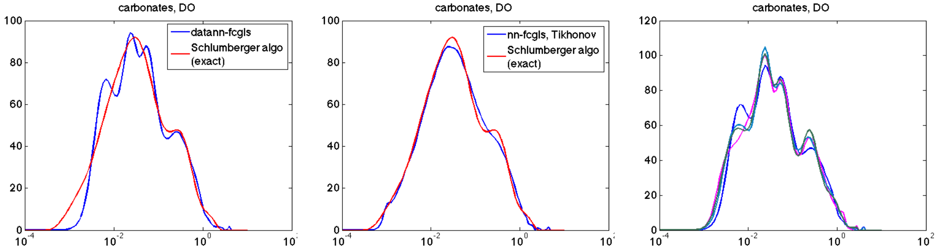

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- Faster in high dimension
- Uncertainty estimates still missing ...

New method

Proposed - Bayesian inference

- Use BayeSys/Massinf
- MCMC with atomic prior

BayeSys Experiments

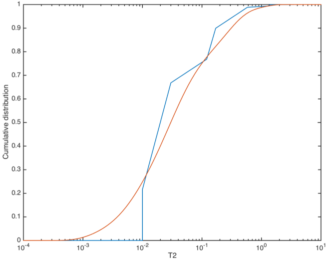
- Implemented simplified version of model in BayeSys

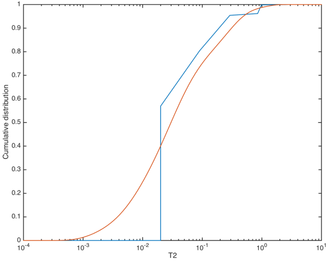
The model

$$M(\tau_2^{(j)}) = \int_0^{T_{2,B}} P(T_2) \exp\left\{-\frac{\tau_2^{(j)}}{T_2}\right\} dT_2 + e(\tau_2^{(j)}) \quad e \sim N(0, \sigma\mathbf{I})$$

$$P(T_2) = \sum_j^r \omega_j \delta_{\tau_2^{(j)}}(T_2)$$

Prior $\quad r \sim Geom(n) \qquad \tau_2 \quad \sim U(0,5) \qquad \exp^{-\frac{\omega}{10^4}} \sim U(0,1)$


Cumulative distribution
T2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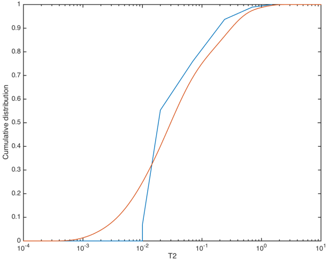
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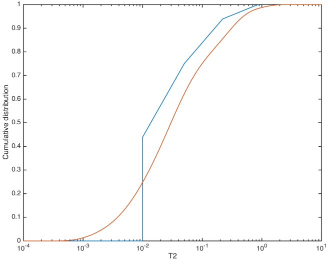
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Developments - Coding

- Improve efficiency

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- Extend to 2D (τ_1 and τ_2)

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- Build MatLab interface

Developments - Model Comparison

- Bayes Factors

$$\frac{\Pr(\mathbf{y}|M_1)}{\Pr(\mathbf{y}|M_2)}$$

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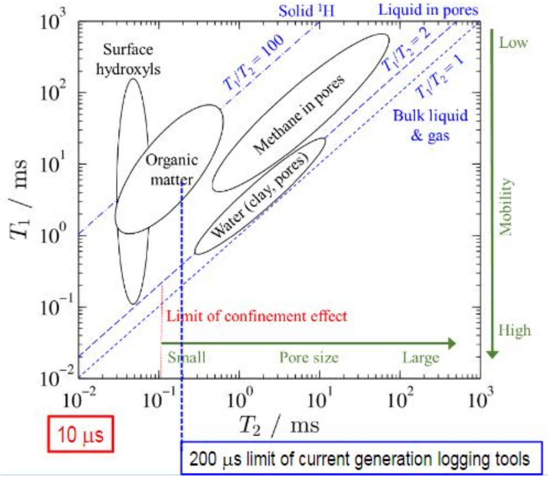
- Compare forward models for shales



Data Visualisation

- What is the most effective way to present the BayeSys output?
  - Display uncertainty
  - Meaningful at face value

Summary

- Showed we can use BayeSys infrastructure for this problem

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- Showed we can use BayeSys infrastructure for this problem
- Scope to develop and improve code
- Incorporate meaningful summary measures
- Create informative data displays

Thanks for listening

Any questions?