Bubbles and bubbly flows

Paul Milewski

Young-Laplace equation

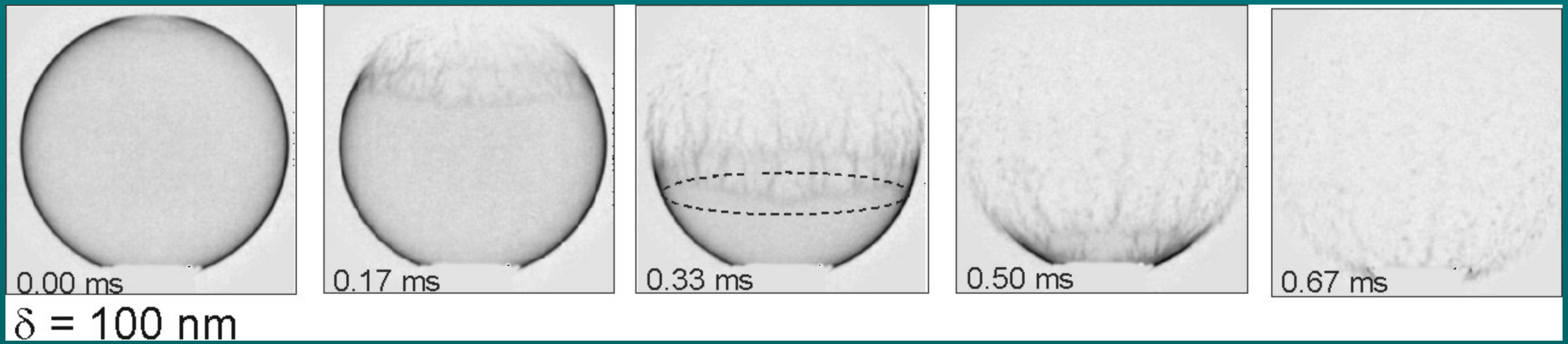
- Jump in pressure across interface. H is the mean curvature.

$$[P] = 2\gamma H; \qquad H = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\gamma = force/length$$

- Dimensionless numbers

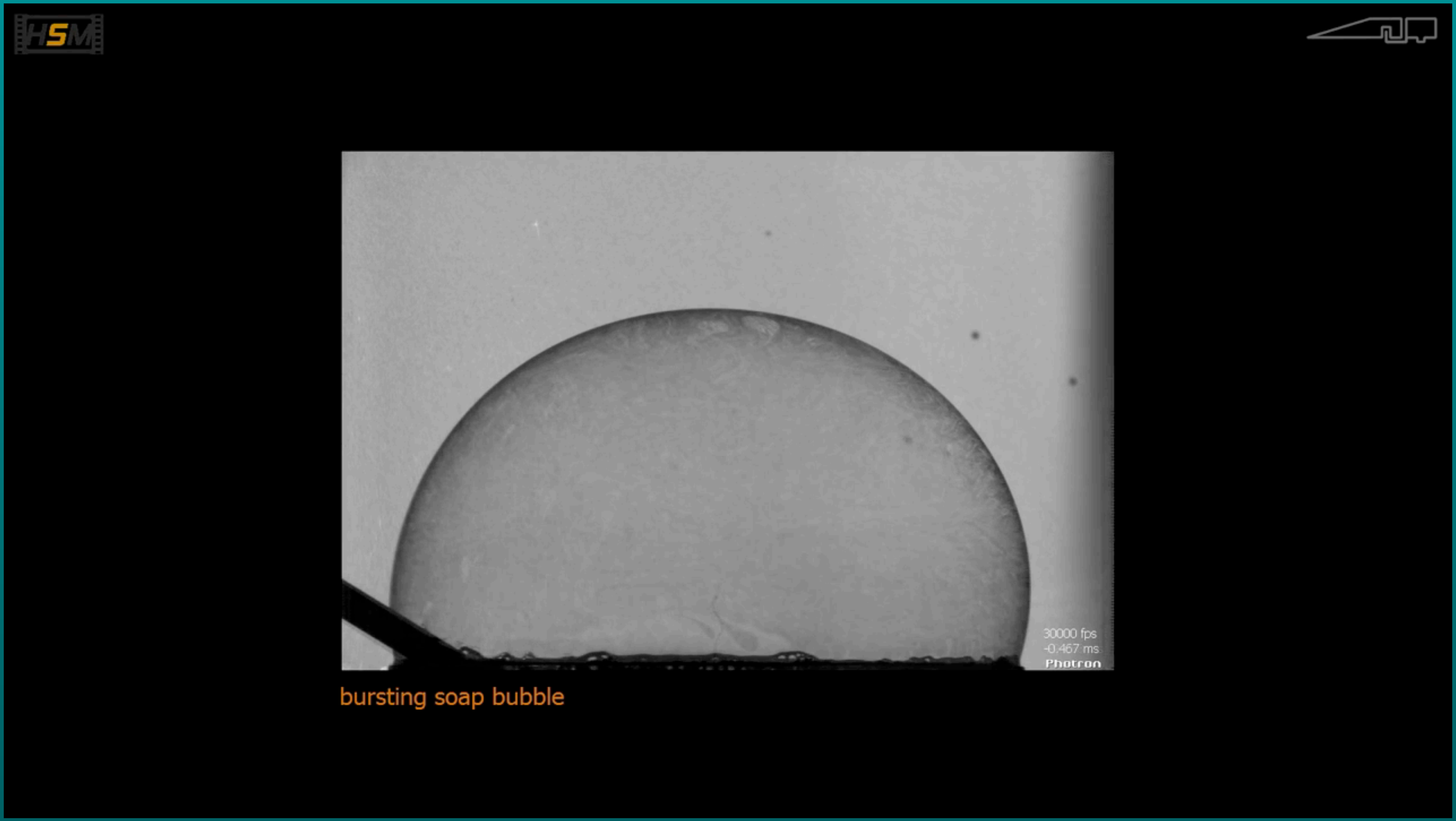
$$Bo = \frac{\Delta\rho g R^2}{\gamma}, \qquad Ca = \frac{\mu U}{\gamma}, \qquad We = \frac{\rho U^2 R}{\gamma}$$



Bursting Bubble

Energy per unit volume

$$\rho U^2 \sim \gamma / D, \qquad U \sim (\gamma / \rho D)^{1/2}$$



Single bubbles surrounded by fluids

- The Rayleigh - Plesset equation describes the evolving radius of a spherical bubble surrounded by a viscous incompressible fluid.

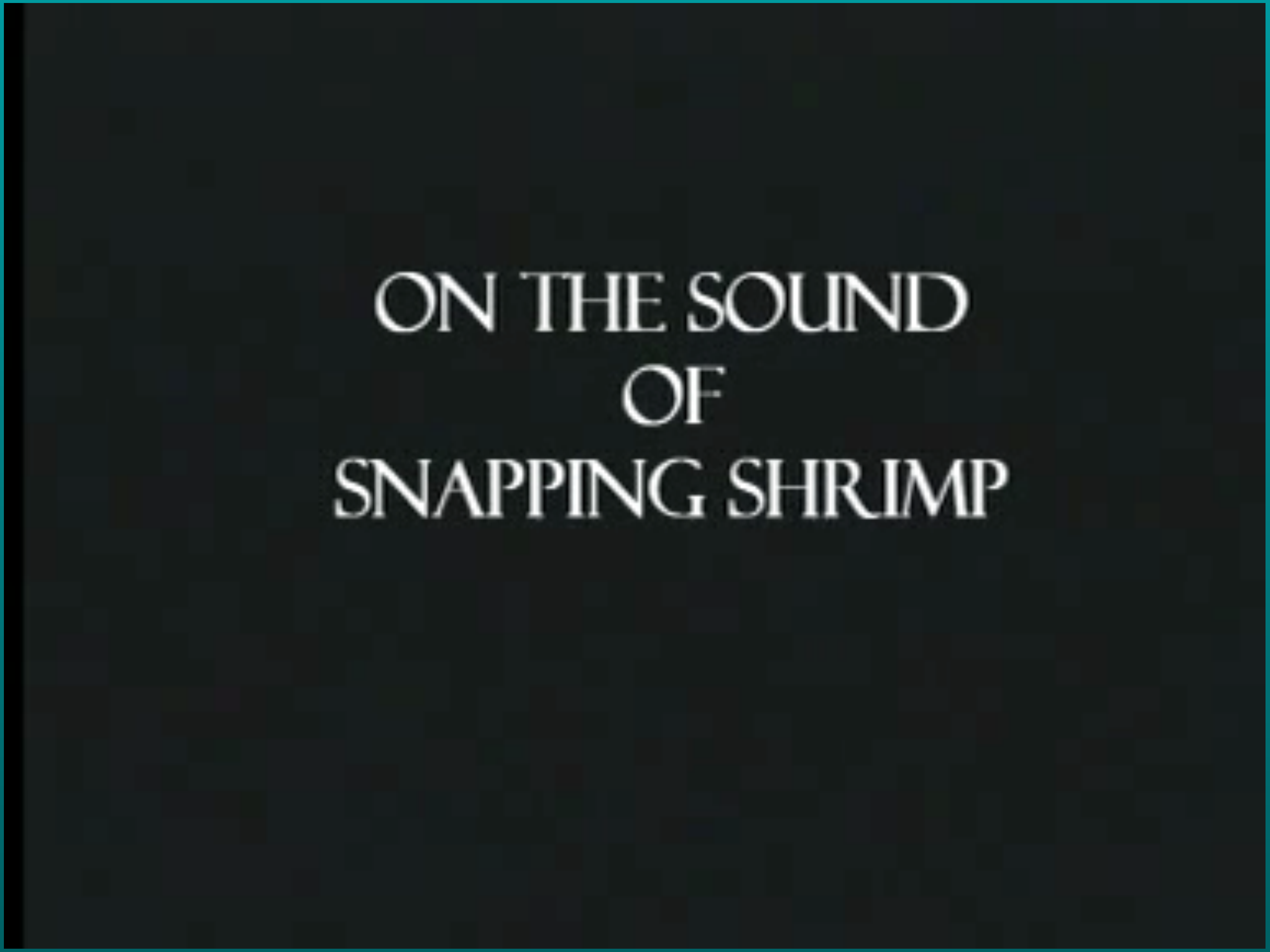
$$\frac{1}{\rho_L}(P_b(t) - P_\infty(t)) = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + 4\nu_L \frac{1}{R}\frac{dR}{dt} + \frac{2\gamma}{\rho_L R}$$

- From it one can derive the natural frequency of oscillation of a bubble. (Thermodynamic part is called Minnaert frequency.)

$$\omega = \frac{1}{R_0 \rho_L^{1/2}}\left(3kP_0 - \frac{2\gamma}{R_0}\right)^{1/2}, \qquad P/\rho^k = const.$$

- A more complicated differential equation describes the bubble in a compressible fluid - needed for consistent studies of the emission of sound by bubbles [Keller & Miksis JASA 1980]

Collapsing bubbles and the pistol shrimp



Sonoluminescence!

Entry #: V 75

Shocking Bubbles

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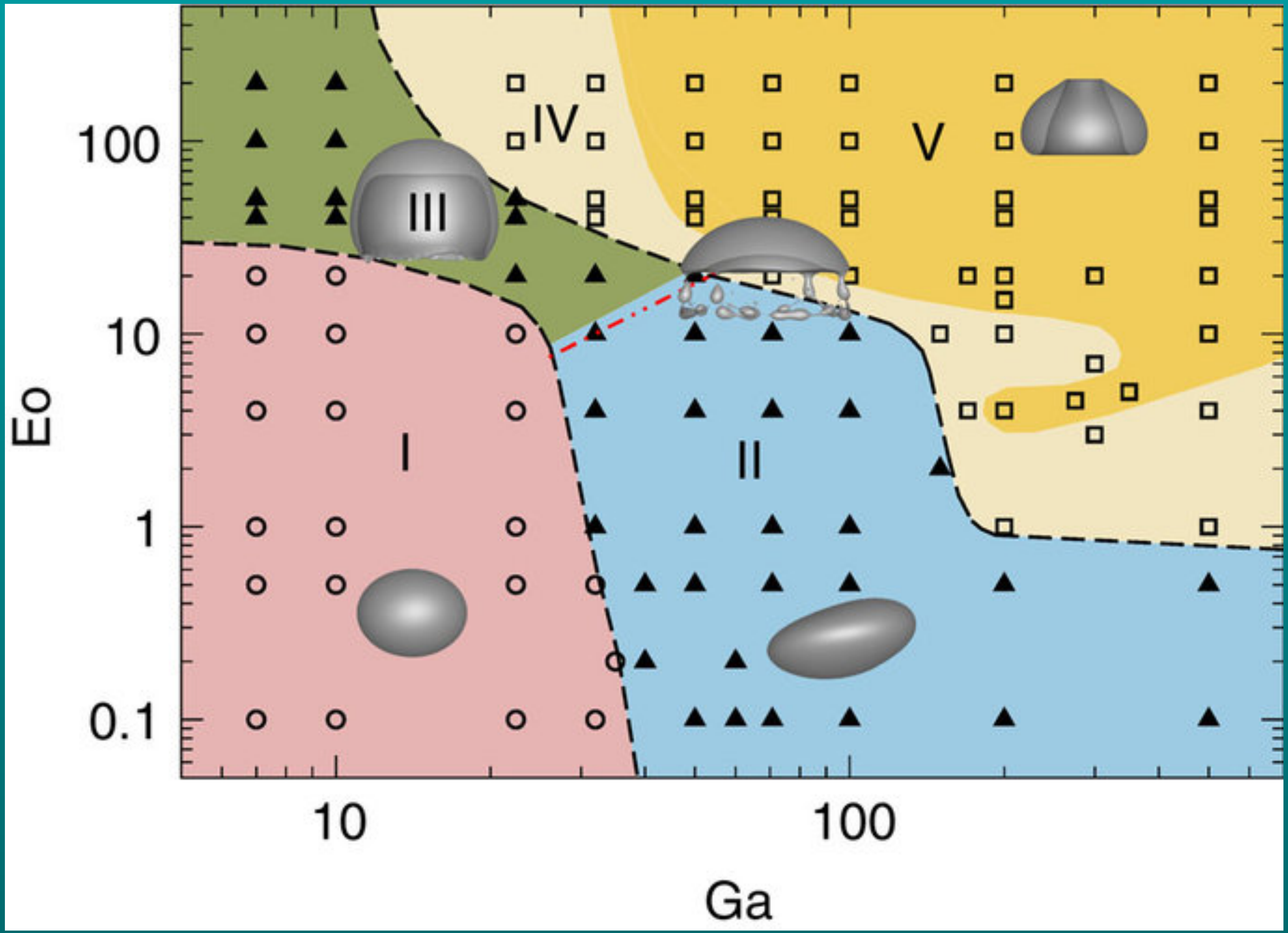
Rising bubbles - the champagne problem

- Even a seemingly simple problem can require some careful modelling.
- For low Re (<0.1) the drag on a bubble is 2/3 that of on a solid sphere. Unfortunately the rise has Re = O(1-20) - Guinness to Moet & Chandon.

$$F = \frac{1}{2}\rho U^2 A_C C_D, \qquad C_D = \frac{16}{Re}, \qquad \text{Stokes' Law}$$

- Bubble volume growth is due to change in hydrostatic pressure (easy to model), dynamic pressure (harder to model unless Re small) and mass growth rate in the droplet.
- The liquid is usually supersaturated with the gas and therefore, at a gas-liquid interface, one has a flux of mass. To obtain it one calculates an advection-diffusion process in the bulk for the concentration of dissolved gas with boundary condition at the interface. [For a Physics approach see Zhang & Xu 2008]

Drag and rising bubble - mass conserved



Larger bubbles (Bond)

Viscosity increases (relative to gravity)

Tripathi, M. K. *et al.* Dynamics of an initially spherical bubble rising in quiescent liquid. *Nat. Commun.* 6:6268 doi: 10.1038/ncomms7268 (2015).

Other problems - Taylor bubbles.

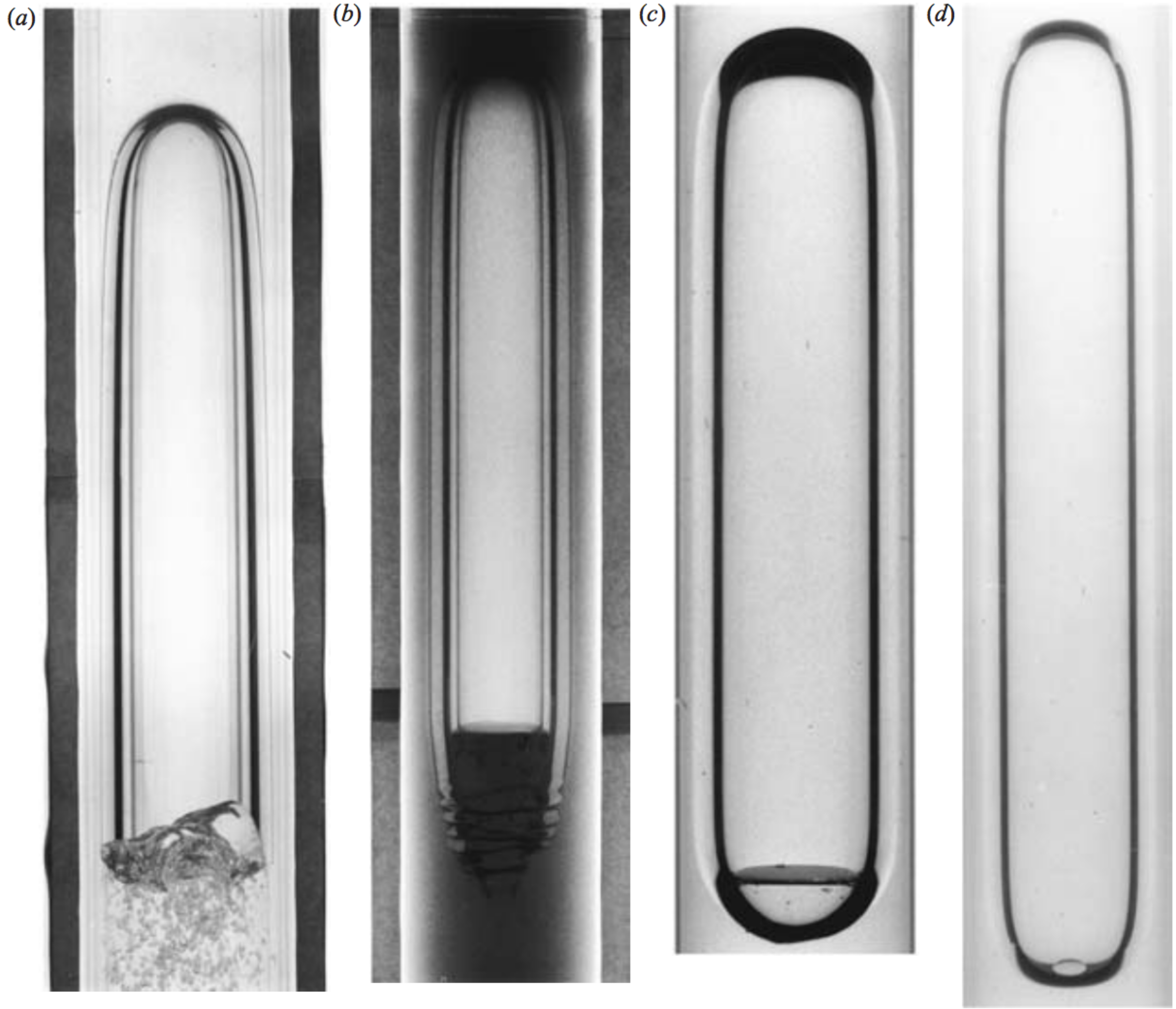


FIGURE 5. Photographs of Taylor bubbles rising through 76.2 mm inside-diameter pipe filled with different viscosity liquids: (*a*) water; (*b*) Purolub 150 oil (480 mPa s); (*c*) silicone oil (1300 mPa s); (*d*) silicone oil (3900 mPa s).

Taylor Bubbles, Viana et al JFM 2003

Bubbly flows - two phase models

- The modelling here is as a gas phase and a liquid phase. There is no transfer between the two (ie no dissolved gas) and a low concentration of bubbles is assumed (ie no bubble-bubble interactions). From Biesheuvel & van Wijngaarden 1984:

(2.4) $$\frac{\partial}{\partial t}\rho_g \alpha + \frac{\partial}{\partial x}(\rho_g \alpha U_g) = 0, \qquad (8.1)$$

(2.5) $$\frac{\partial}{\partial t}(1-\alpha) + \frac{\partial}{\partial x} U_\ell (1-\alpha) = 0, \qquad (8.2)$$

(6.1) $$\rho_\ell (1-\alpha)\left\{\frac{\partial U_\ell}{\partial t} + U_\ell \frac{\partial U_\ell}{\partial x}\right\} = -\frac{\partial}{\partial x}\langle p\rangle - \frac{\partial}{\partial x}\alpha\rho_\ell\left[\left(\frac{dR}{dt}\right)^2 + \tfrac{1}{2}(U_g - U_0)^2\right] - \rho_\ell(1-\alpha)g, \qquad (8.3)$$

(7.4) $$\frac{1}{2}\frac{\partial}{\partial t}\rho_\ell \tau (U_g - U_0) = \rho_\ell \tau \frac{\partial U_0}{\partial t} - 12\pi R\mu(U_g - U_0) + \rho_\ell \tau g, \qquad (8.4)$$

(5.4) $$(1-\alpha)U_\ell + \alpha U_g = U_0, \qquad (8.5)$$

(7.1) $$\rho_g R^3 = \text{constant}, \quad \frac{p_g}{\rho_g} = \text{constant}, \qquad (8.6)$$

(4.6) $$\langle p\rangle = (1-\alpha)\langle p\rangle_\ell + \alpha\langle p\rangle_g - \frac{2\gamma}{R}\alpha, \qquad (8.7)$$

(4.7) $$\langle p\rangle_g = p_g = \langle p\rangle + \frac{2\gamma}{R} + \rho_\ell\left[\frac{3}{2}\left(\frac{dR}{dt}\right)^2 + R\frac{d^2R}{dt^2} - \tfrac{1}{4}(U_g - U_0)^2\right]. \qquad (8.8)$$

- Re=∞
- 1-D
- Incomp. liquid
- gravity
- Stokes' small bubble drag
- α=volume fraction
- isothermal
- τ=volume of bubble

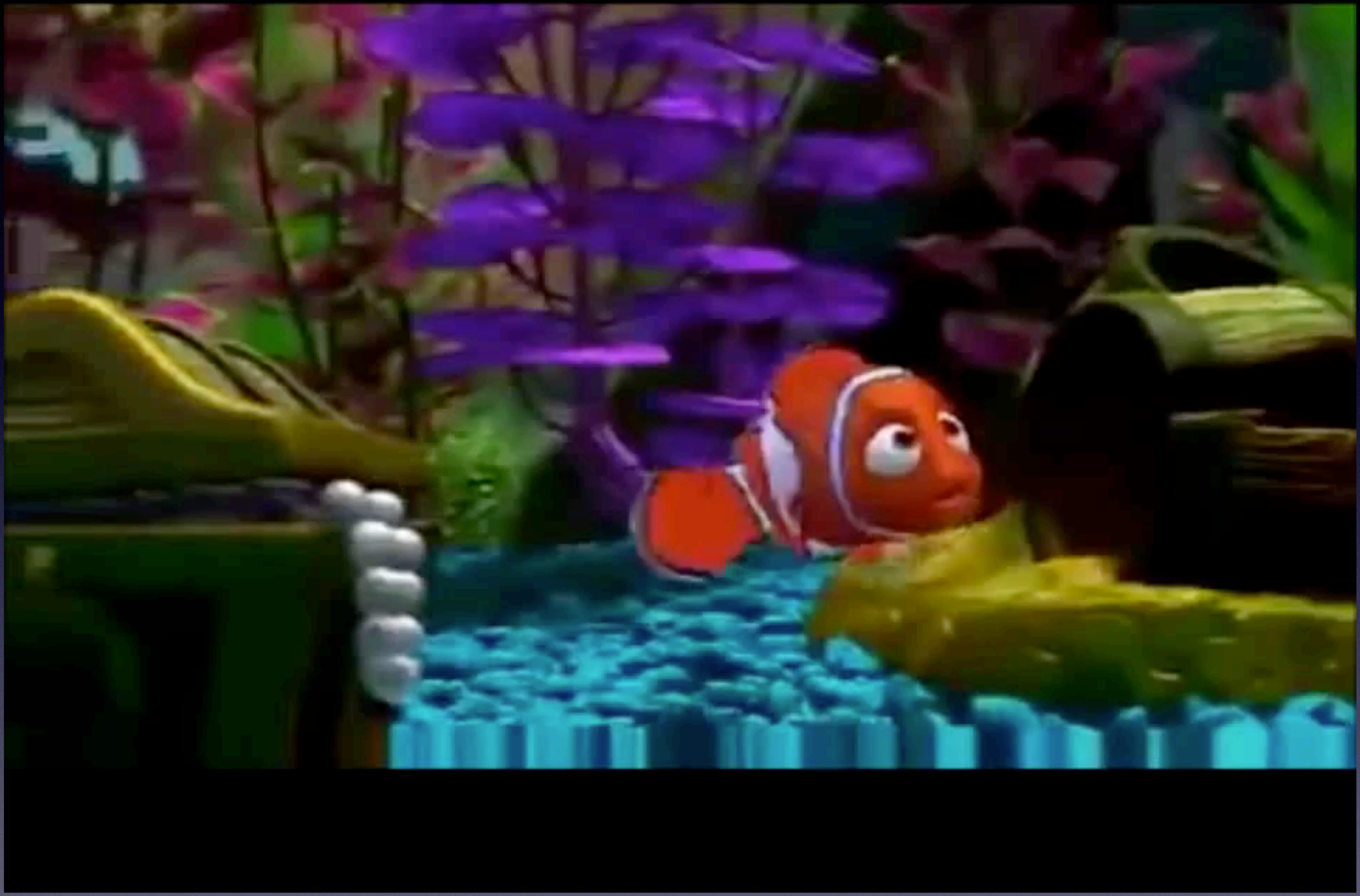
Properties

- Taking P to be the same in gas and liquid phase (the nondispersive limit) , we end up with a system of 4 hyperbolic PDEs with characteristic speeds:

$$0, \quad U_g, \quad \frac{1}{2}(U_g + U_l) + \alpha(U_g - U_l) \pm c_0$$

$$c_0^2 = \frac{p}{\rho_l \alpha(1 - 3\alpha)}$$

- Bubble dynamics, “concentration wave”, 2 sound waves.



Thank you!