Exclusion process modelling for the mixing problem

Kit, Tim, Marcus, Gianluca, Andrea, John, Matthias

ITT5

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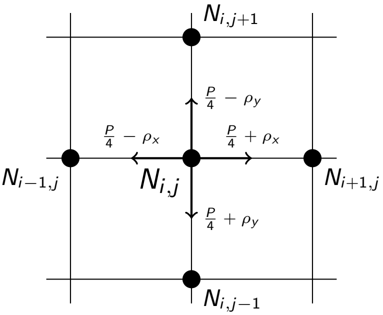
Recalling the problem

- "Seed Shaking":
  - Looking to optimise an existing process for tumbling-based seed coating.
  - Further desire to better understand granular flow and granular mixing.
- Our approach:
  - ASEP models can directly give numeric results.
  - Further "weakly asymmetric" limits can give a continuous PDE model for the mean flow.
  - The PDE gives a direct description of the macroscopic forces (hopefully some understanding) and faster numerical simulation.
  - Individual-based simulation can verify the PDE model.

Defining a microscopic interaction

- The Markov chain $(N_n)_n$ moves as an exclusion process (right). The flexible biases can model gravity or mixing forces.
- Collision between a particle with relative coating C_p and another particle with C_o induces transfer of coating $rC_p(1 - C_o)$.
- To create a nontrivial limit we rescale the system with weak asymmetry $\rho/P \propto \Delta \propto \sqrt{\tau} \to 0$.



Lattice size Δ

Time scale τ

N maps into $\{0, 1\}$

C into $[0, 1]$

The Macroscopic Description

Define

$$\frac{P\Delta^2}{4} \to D \ , \ 2\Delta\rho \to \alpha$$

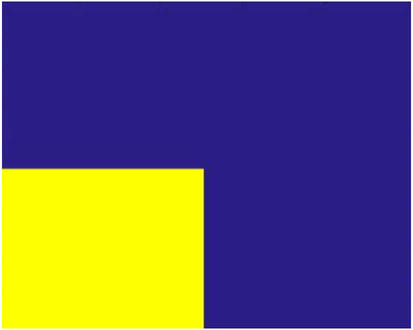
then in the limit we have coupled PDEs

$$N_t = \alpha \cdot \nabla N + D\nabla^2 N \tag{1}$$

$$\begin{aligned} C_t &= 2rDC(N_y C_y + N_x C_x) \\ &+ C(1 - r + rC)(\alpha \cdot \nabla N + D\nabla^2 N) \\ &+ (1 - (1 - r)N)(\alpha \cdot \nabla C + D\nabla^2 C) \end{aligned} \tag{2}$$

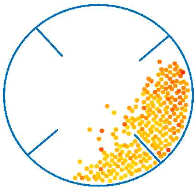
The same equation also describes flow with a time dependent forcing direction $\alpha(t)$

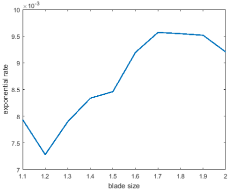
PDE solution

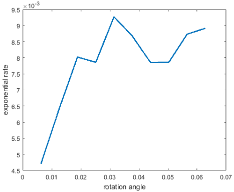


Agreement between finite-difference PDE solution and individual-based simulations (video).

Simulation for specific answers







- If we are committed to a specific context it is very possible to answer questions on this simulation by Monte Carlo.
- Can identify fairly confidently a plateau in the mixing improvements from increasing blade size and rotation speed.

Looking forward

- Short term: solving the coating PDE.
- Extend to three spatial dimensions.
- Lattice models to show segregation?
- Could extend beyond reflecting boundary.
- Coating exchange mechanics could be better informed by experiment.