(Vis 1

Boltzmann's Transport Equation

[Bell & Glasstone §1]

$\underset{\sim}{r} \in D \subset \mathbb{R}^3$, $\quad \underset{\sim}{\Omega} \in S^2$ = unit sphere

$t \geq 0 \qquad$ (energy $E = \frac{1}{2} m v^2$)

v = speed

Angular flux

$$\Psi(\underline{r}, \underline{\Omega}, t) = v \underbrace{N(\underline{r}, \underline{\Omega}, t)}$$

neutrons/unit volume at $\underset{\sim}{r}$ travelling in direction $\underset{\sim}{\Omega}$ at time t

(E, v are really variables – Thesis p10)

$$\frac{1}{v}\frac{\partial \Psi}{\partial t} + \underset{\sim}{\Omega} . \nabla \Psi + \sigma \Psi$$

Transport

$$= \frac{1}{4\pi} \int_{S^2} \sigma_s (\underset{\sim}{r}, \underset{\sim}{\Omega} \cdot \underset{\sim}{\Omega}') \Psi(\underline{r}, \underline{\Omega}') \, d\Omega'$$

Scattering

$$+ \frac{\chi}{4\pi} \nu(\underline{r}) \sigma_f(\underline{r}) \int_{S_2} \Psi(\underline{r}, \underline{\Omega}') \, d\Omega'$$

Fission

$$+ Q(\underline{r}, \underline{\Omega})$$

Source

"Collisions" ⇒ "Scattering", "Fission", "Capture"

"Isotropic" $\sigma_s(\underline{r}, \underline{\Omega}, \underline{\Omega}') = \sigma_s(\underline{r})$

"Homogeneous" $\sigma, \sigma_s, \sigma_f$ Constants.

"Steady State"

$$\underset{\sim}{\Omega} \cdot \nabla\Psi + \sigma\Psi = \frac{1}{4\pi}\int_{S^2} \sigma_s(\underline{r}, \underline{\Omega}' \cdot \underline{\Omega})\Psi(r, \Omega')d\Omega'$$

$$+ \frac{\chi}{4\pi}\nu(\underline{r})\sigma_f(\underline{r})\int_{S^2}\Psi(\underset{\sim}{r}, \underset{\sim}{\Omega}')d\Omega'$$

$$+ Q(r, \underline{\Omega}) \qquad (1)$$

$$\boxed{T\Psi = S\Psi + F\Psi + Q}$$

"Source Problem"

Boundary Condition?

W.r.t. $\underset{\sim}{r}$ only!

Integro-differential equation

"Criticality problem"

$$(T - S)\psi = \lambda F \psi \qquad (1)$$

+ vacuum boundary condition

Theorem (Schaben Cor 2.29)

problem (1) has smallest positive real eigenvalue λ_1 with non-negative eigenfunction. [Krein-Rutman Theorem]

Problem

Design reactor so that $\lambda_1 \simeq 1$

More relevant

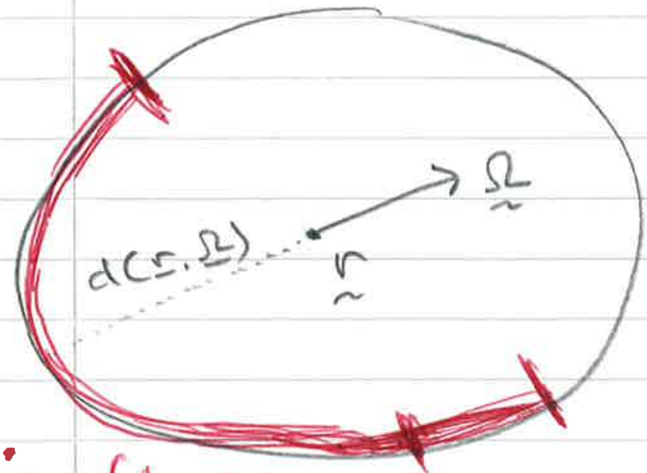
Effect of perturbations in material properties on stability of a reactor?

(Vis 4)

Exact Solution of $\mathcal{T}\Psi = g(\underline{r})$ (3)

Scheben Lemma 2.1

Convex domain



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Characteristics !

$$(3) \equiv -\frac{d}{ds}\left[\Psi(\underline{r}-s\underline{\Omega},\underline{\Omega})\exp(-\sigma s)\right]$$

$$= g(\underline{r}-s\underline{\Omega})\exp(-\sigma s) \qquad \text{(A)}$$

$$\Psi(\underline{r},\underline{\Omega}) = \int_0^{d(\underline{r},\underline{\Omega})} \exp(-\sigma s)\, g(\underline{r}-s\underline{\Omega})\, ds$$

Solves (A) : Calculation

+ UNIQUENESS

[Integrate (A) from $s=0$ to $s=d(\underline{r},\underline{\Omega})$]

Exercise : Works for $\sigma = \sigma(\underline{r})$?

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Consequence [Scheben, Lemma 2.3]

$$\phi(\underline{r}) := \frac{1}{4\pi} \int_{S^2} \Psi(\underline{r}, \underline{\Omega})\, d\Omega$$

$$= \frac{1}{4\pi} \int_{D} \frac{\exp\left(-\sigma \|\underline{r} - \underline{r}'\|_2\right)}{\|\underline{r} - \underline{r}'\|_2^2}\, g(\underline{r}')\, d\underline{r}' \qquad (4)$$

$g \longmapsto \phi$ is represented by a symmetric positive, positive definite weakly singular integral operator!

Exercise what is corresponding result when $\sigma = \sigma(\underline{r})$?