"Packet of Neutrons" $N(\underline{r}, \underline{\Omega}, t)\, dV\, d\Omega$

shift time by Δt

prob collision $= \sigma(\underline{r})\, v\, \Delta t$ (Defⁿ)

[$\sigma(\underline{r})$ = prob of neutron collision per unit distance]

Number remaining $= N(\underline{r}, \underline{\Omega}, t)\,(1 - \sigma(\underline{r})\, v\, \Delta t)$ (A)

[$\frac{1}{\sigma}$ = mean free path $\to \infty$ when $\sigma \to 0$]

Number entering packet from collisions

$$\int_{S^2} \underbrace{\sigma(\underline{r})\, f(\underline{r}, \underline{\Omega}' \to \underline{\Omega})}_{\text{prob of transfer from } \underline{\Omega}' \to \underline{\Omega}}\, v\, N(\underline{r}, \underline{\Omega}', t)\, d\Omega'\, \Delta t \qquad \text{(B)}$$

So $N(\underline{r} + \underline{\Omega}\, v\, \Delta t, \underline{\Omega}, t + \Delta t) = \text{(A)} + \text{(B)} + \underbrace{Q(\underline{r}, \underline{\Omega})\, \Delta t}_{\text{Source}}$

$$= N(\underline{r}, \underline{\Omega}, t)\,(1 - \sigma(\underline{r})\, v\, \Delta t)$$

$$+ \int_{S^2} \sigma(\underline{r})\, f(\underline{r}, \underline{\Omega}' \to \underline{\Omega})\, v\, N(\underline{r}, \underline{\Omega}', t)\, d\Omega'\, \Delta t$$

$$+ Q(\underline{r}, \underline{\Omega})\, \Delta t$$

prob of collision for neutron of angle $\underline{\Omega}'$ to produce one of angle $\underline{\Omega}$

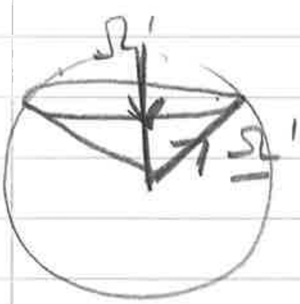
Rearrange, $\Delta t \to 0$

$$\left(\underbrace{\frac{\partial N}{\partial t} + v\underline{\Omega}.\nabla N}_{\text{"Total derivative"}} + \sigma v N \right)(\underline{r}, \underline{\Omega})$$

$$= \int_{S^2} \underbrace{\sigma(\underline{r}) f(\underline{r}, \underline{\Omega}' \to \underline{\Omega})}_{\text{sum of scattering + fission}} v N(\underline{r}, \underline{\Omega}', t)\, d\Omega'$$

$$+ Q(\underline{r}, \underline{\Omega})$$

Scattering



$$\frac{1}{4\pi}\sigma_s(\underline{r}, \underline{\Omega}'.\underline{\Omega})$$

fission

depends on energy

$$\frac{\chi}{4\pi} \nu(\underline{r})\, \sigma_f(\underline{r})$$

number of neutrons produced

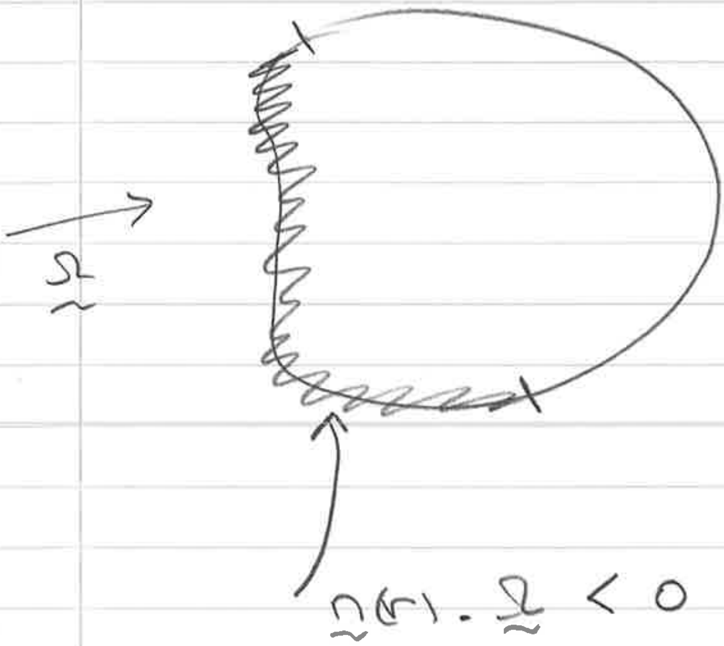
$$\cancel{\sigma(\underline{r}) = \frac{1}{4\pi}\int \sigma_s(\underline{r}, \underline{\Omega}'.\underline{\Omega})\, d\Omega' + \chi \nu(\underline{r})\sigma_f(\underline{r}) + \sigma_c(\underline{r})}$$

~~Exercise: indept of $\underline{\Omega}$~~

Consider ${}^{e}T\Psi = g$ (Fixed $\underset{\sim}{\Omega}$) (A)

First order hyperbolic PDE

Constant "Velocity" $\underset{\sim}{\Omega}$



1 $\Psi = 0$ when $\underset{\sim}{n}(r) \cdot \underset{\sim}{\Omega} < 0$ Zero incoming flux (B)

"Vacuum BC's"

["Reflective bc" — Scheben (1.7)]

Problem (A), (B) "well posed" .... to come.

(BB4

1D Model Problem $\quad D = (0,1)$

isotropic

$$\underset{\sim}{\Omega} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix} \qquad \mu = \cos\theta$$

suppress x, y

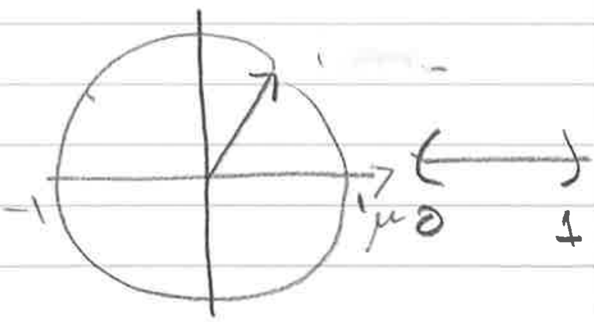
$$\mu \frac{d\psi}{dz} + \sigma\psi = \frac{\sigma_s}{2}\int_{-1}^{1} \psi(z,\mu)\, d\mu$$

$$+ \frac{\nu\sigma_f}{2}\int_{-1}^{1} \psi(z,\mu)\, d\mu$$

$$+ Q$$

$\psi(0,\mu) = 0 \qquad \mu > 0$

$\psi(1,\mu) = 0 \qquad \mu < 0$



$$\left[\sigma,\ \sigma_s,\ \sigma_f = \text{functions of } z\right]$$

BB5

Proof of (4)

$$\varphi(\underline{r}) = \frac{1}{4\pi}\int_0^{d(\underline{r},\underline{\Omega})}\int_{S^2} \exp(-\sigma s)\, g(\underline{r}-s\underline{\Omega})\, d\underline{\Omega}\, ds$$

$$= \int_0^{d(\underline{r},\underline{\Omega})}\int_{S^2} \frac{\exp(-\sigma\|\underline{r}-\underline{r}'\|_2)}{4\pi\|\underline{r}-\underline{r}'\|_2^2}\, g(\underline{r}')\, d\underline{r}'$$

$\underset{\sim}{r}' = \underset{\sim}{r} - s\underline{\Omega}$, $s = \|\underline{r}-\underline{r}'\|$

$dr' = s^2\, d\underline{\Omega}$