SDE Model for Powder Distribution

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Introduction

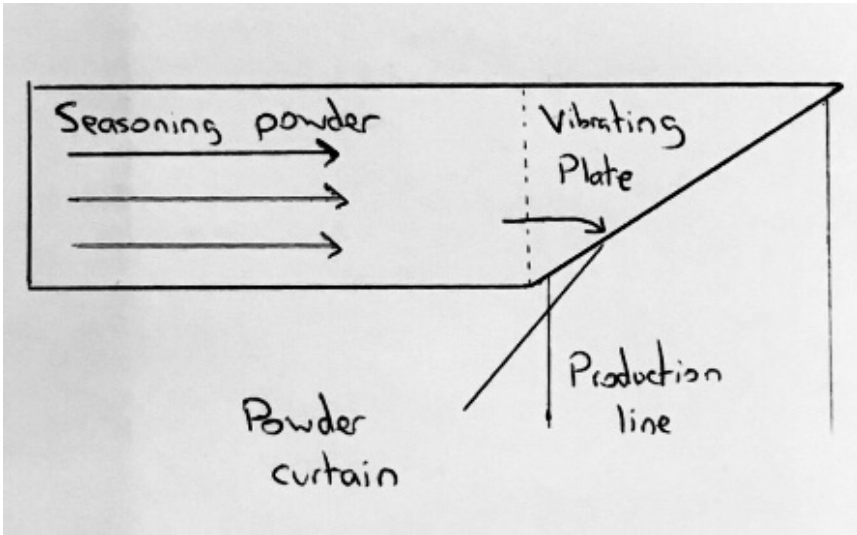
The problem

How do particles move across the shelf?

- How does the shelf vibrate?
- How do we encode/model this driving powder motion?

Potential Approaches

- Physical model?
  - Doable, but ...
  - Impractical
- Stochastic approach?
  - Noise compensating for effects we don't understand.
  - Andreas was in our group.



Encoding the Shelf - a PDE

Ideally

Model shelf as a plate.

- Solve for the vibration modes of the plate:

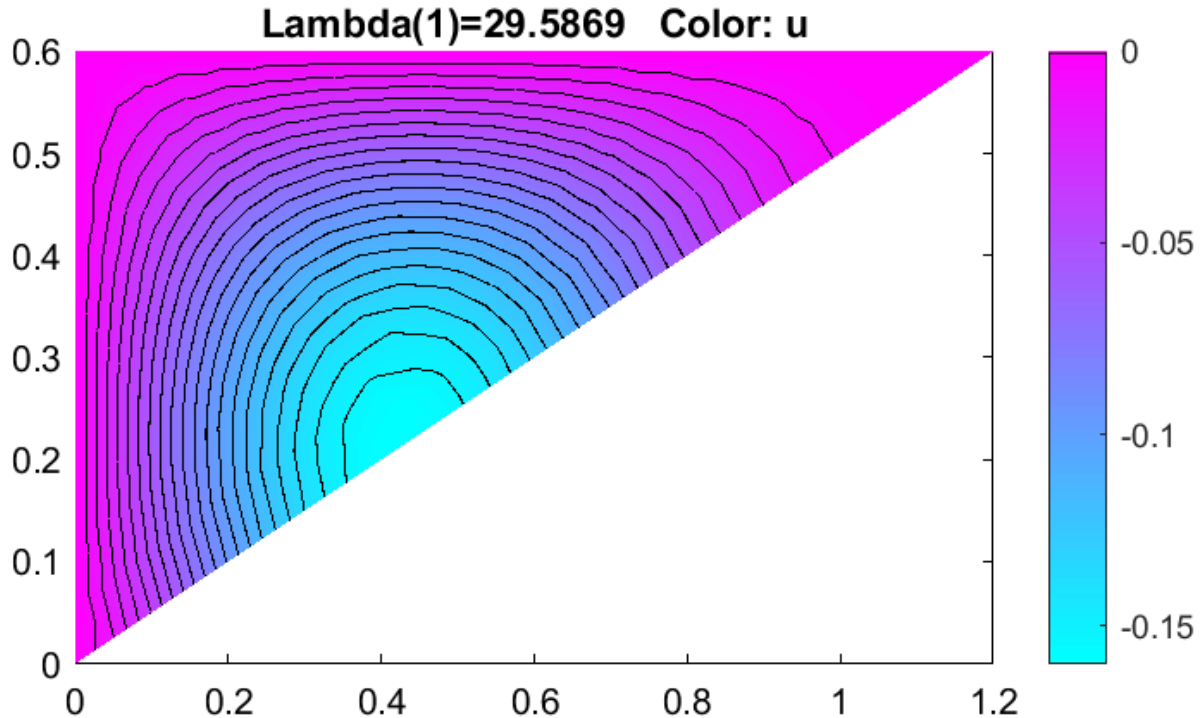
$$-\nabla^2 u - \omega^2 u = f$$

- Boundary conditions; mix of Neumann (free) and Dirichlet (fixed).

What we actually did

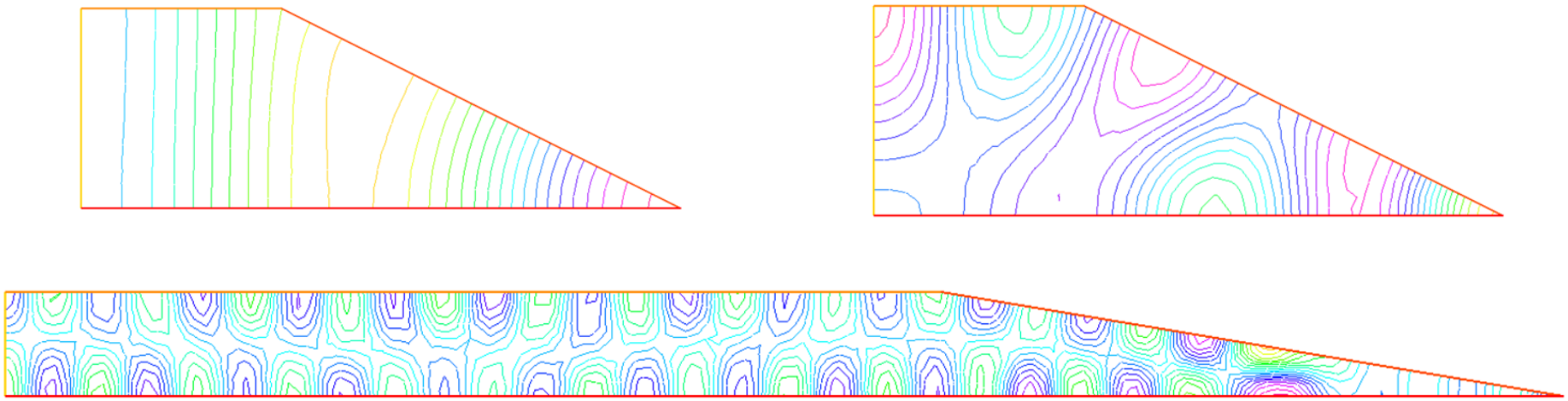
- Solve $-\nabla^2 u = \omega^2 u$ instead.
- Get exact modes; $u(x, y) = \sin\left(\frac{n\pi x}{a}\right) \sin(m\pi y)$.

Shelf Vibrations



Shelf Vibrations

Modes of vibration for the plate, solved using Finite Elements.



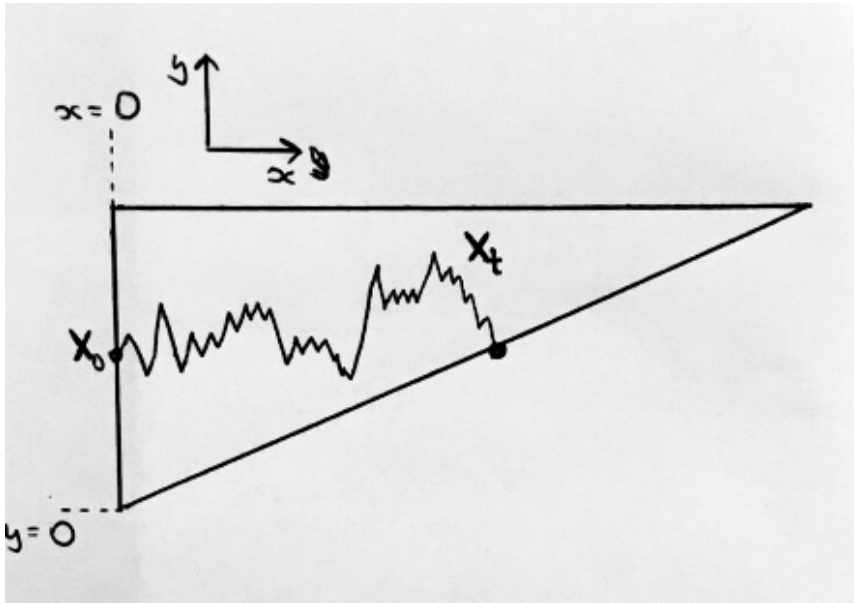
Numerical solutions could be fed into the SDE model, if desired.

The SDE

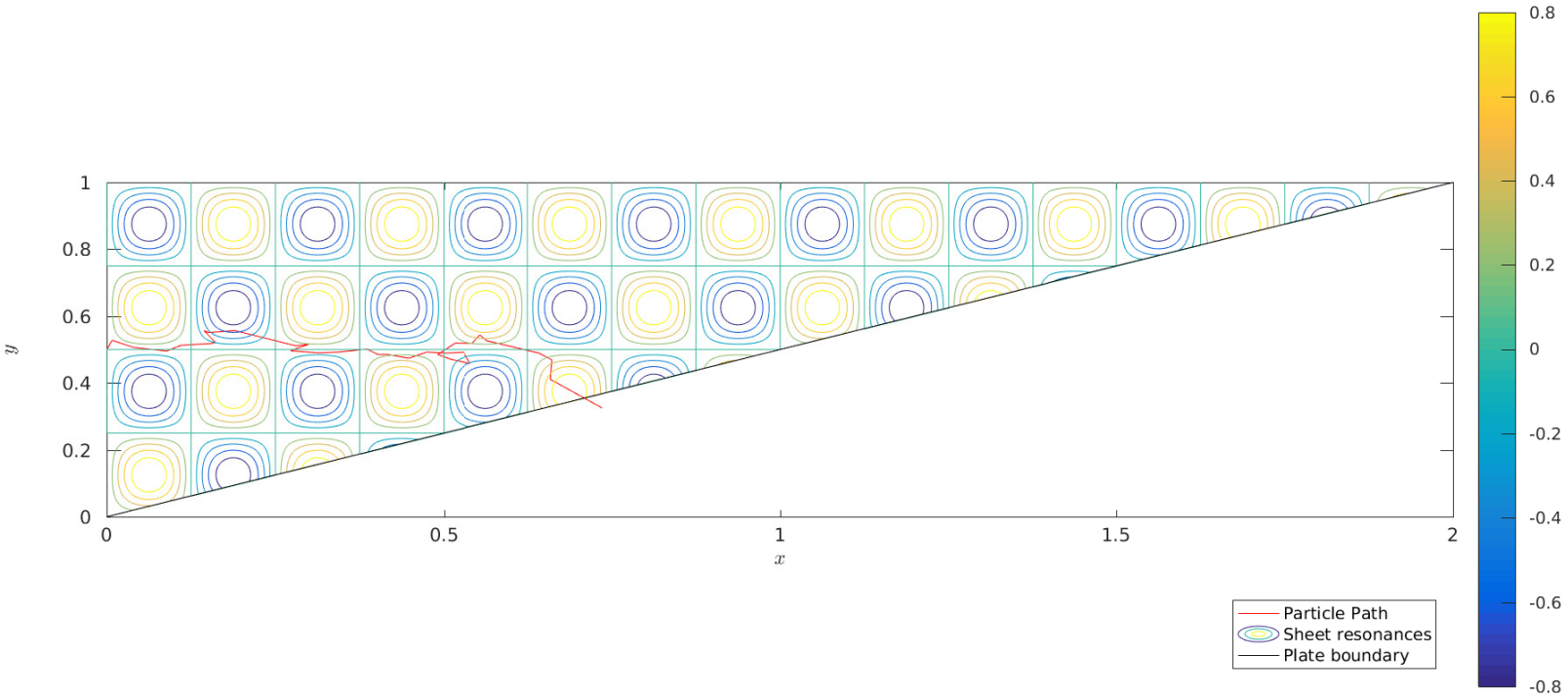
The Variables

$$\mathrm{d}\boldsymbol{X}_t = \mu\left(\boldsymbol{X}_t\right)\mathrm{d}t + \sigma\left(\boldsymbol{X}_t\right)\mathrm{d}\boldsymbol{W}_t$$

- $\boldsymbol{X}_t$: (x, y) position
- μ : drift, scales with distance from $x = 0$
- σ : noise, varies with sheet vibration profile u



Initial Results



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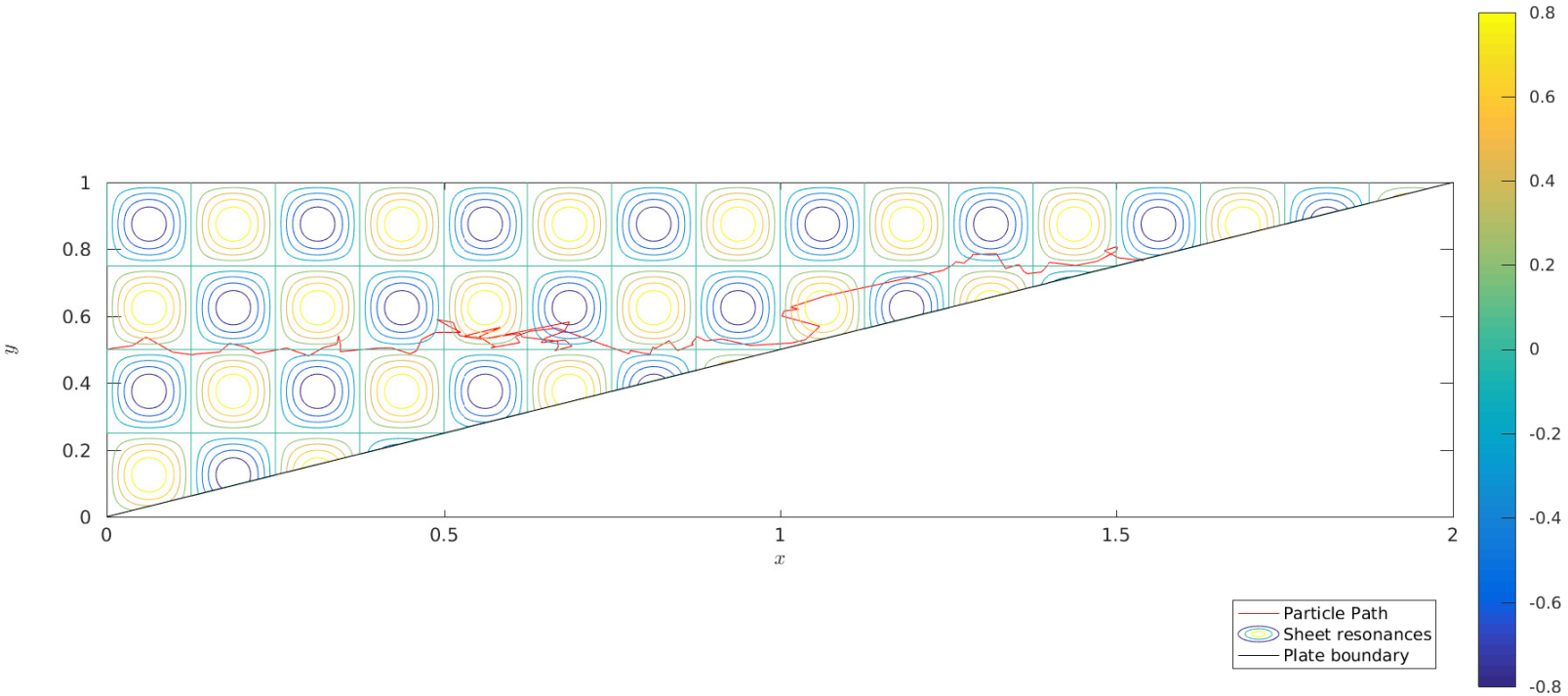

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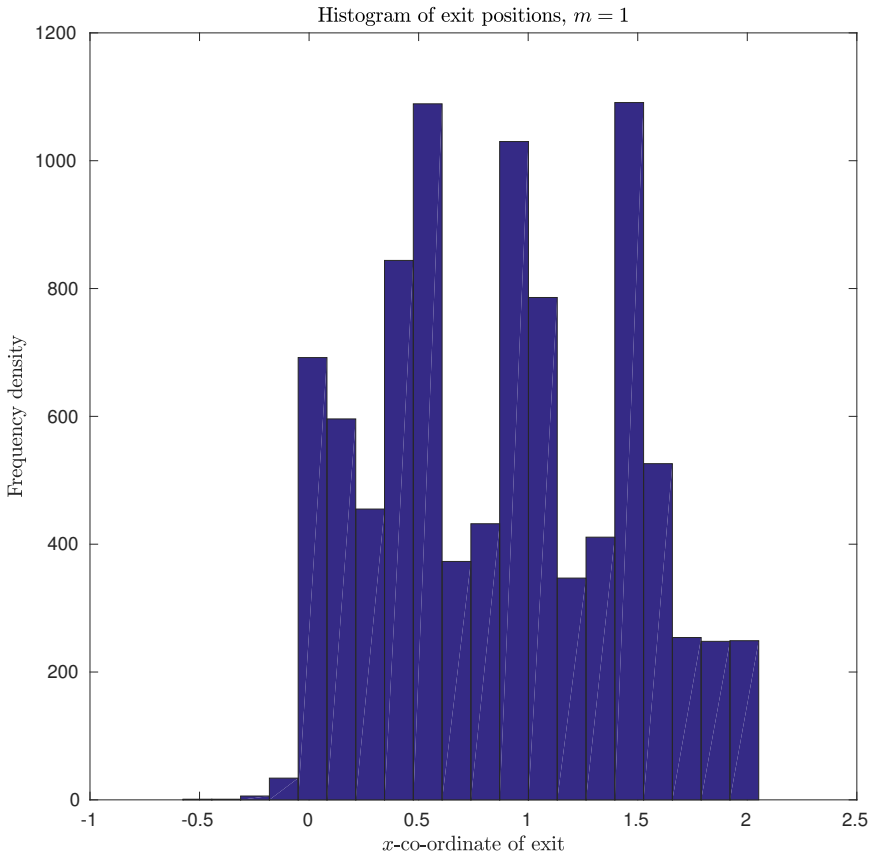

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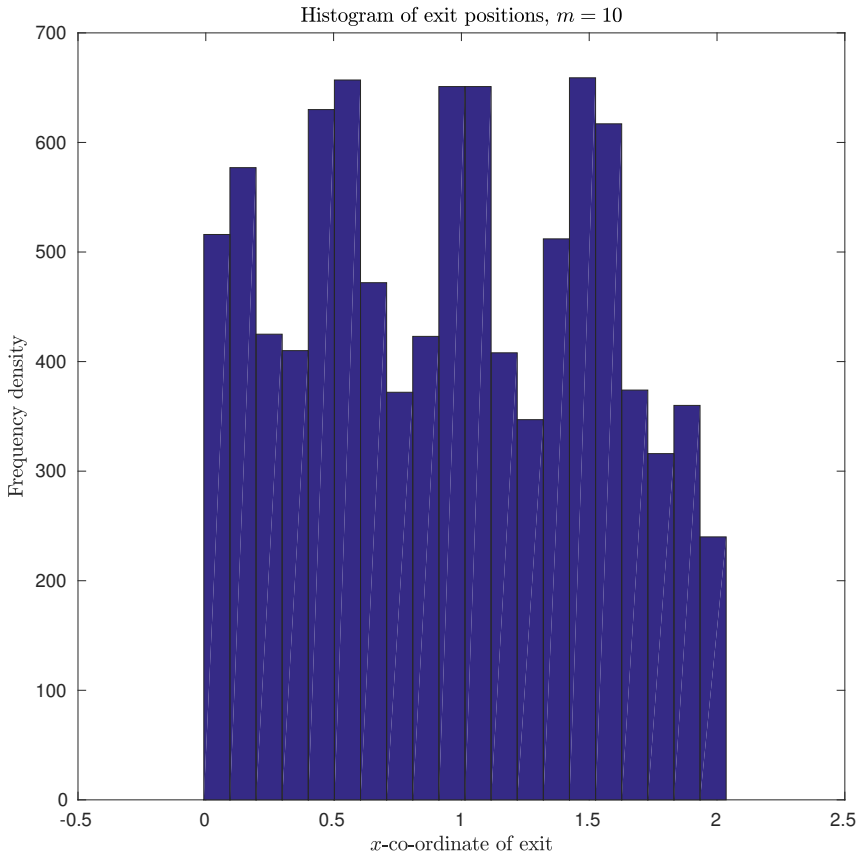

Future work

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Problems

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- Is the stochastic noise term sufficient to describe the effects of vibration?
- What is the correct underlying PDE model for shelf vibration?

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- Determine how shelf vibration affects particle velocity; is it deterministic, stochastic, or both?
- Use the Fokker-Planck equation to calculate distribution of particles on the shelf.

The End

Thanks for listening!