Optimal force application

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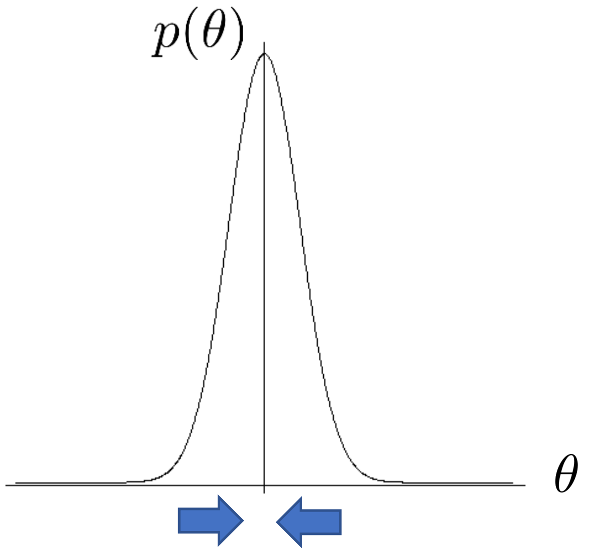
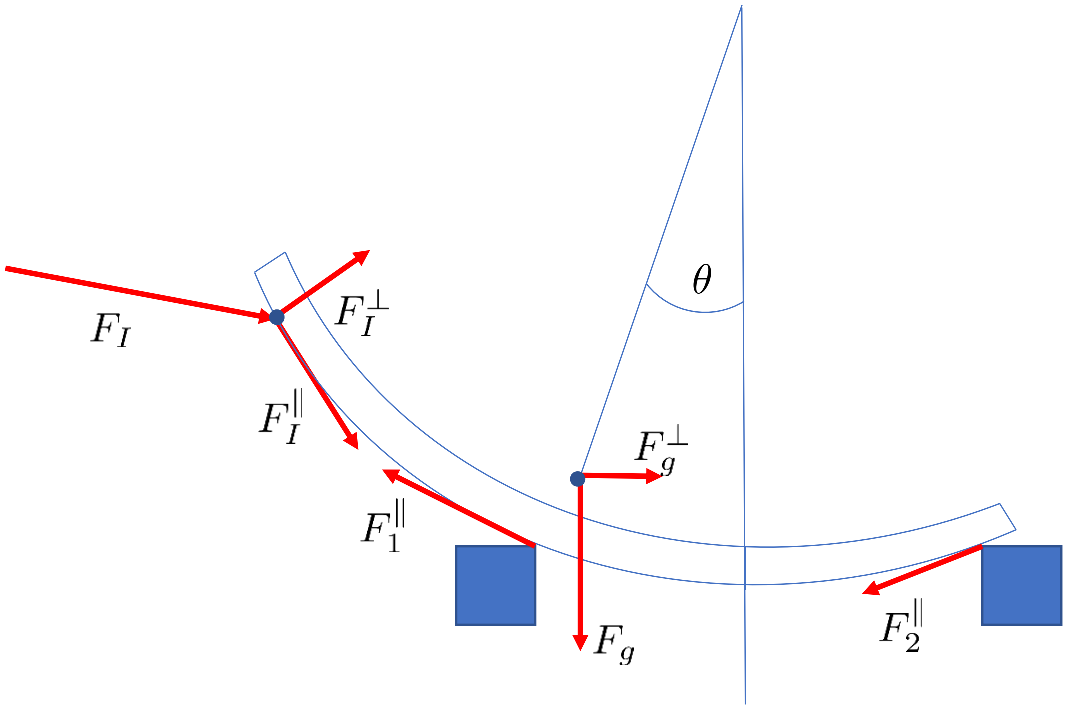
Recalling the problem



Given: $\theta(\text{crisp angle}), F_I(\text{applied force}), F_1(\text{friction}), F_2(\text{friction})$

Question: Where to apply external force optimally?

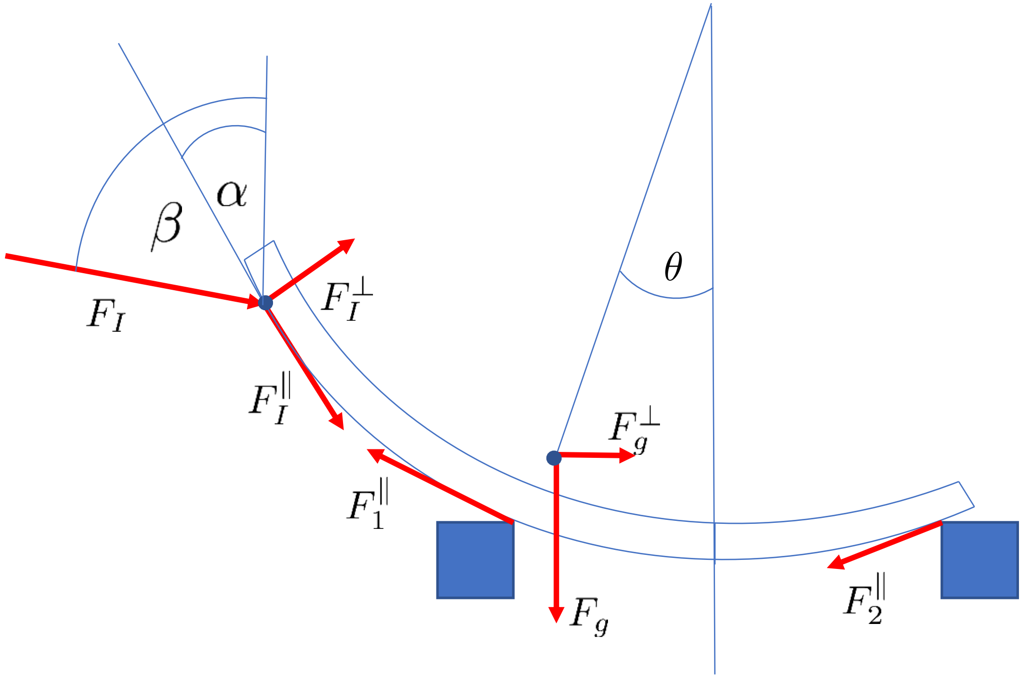
Approach: - determine the right mathematical framework
- simulate effect of applied forces
- test different scenarios



Framework for fixed θ

Mathematical framework

Given: $\theta(\text{crisp angle}), F_I(\text{applied force}), F_1(\text{friction}), F_2(\text{friction})$



Question: Where to apply external force optimally?

Approach: - determine the right mathematical framework
- simulate effect of applied forces
- test different scenarios

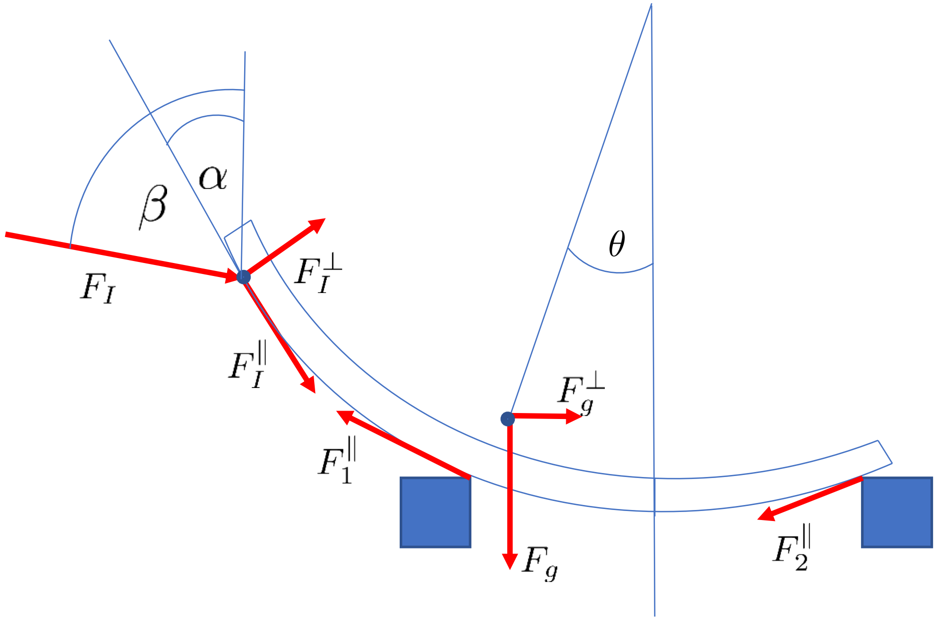
Mathematical framework: - Optimisation problem with constraints
- Find $(\alpha, \beta) \in \Omega$ such that $F_I^{\perp}(\alpha, \beta)$ and $F_I^{||}(\alpha, \beta)$ satisfy certain criterions

Constraint #1

With

$$\|F_1^{\|} + F_2^{\|}\| < \|F_I^{\|}\|$$

we obtain rotation



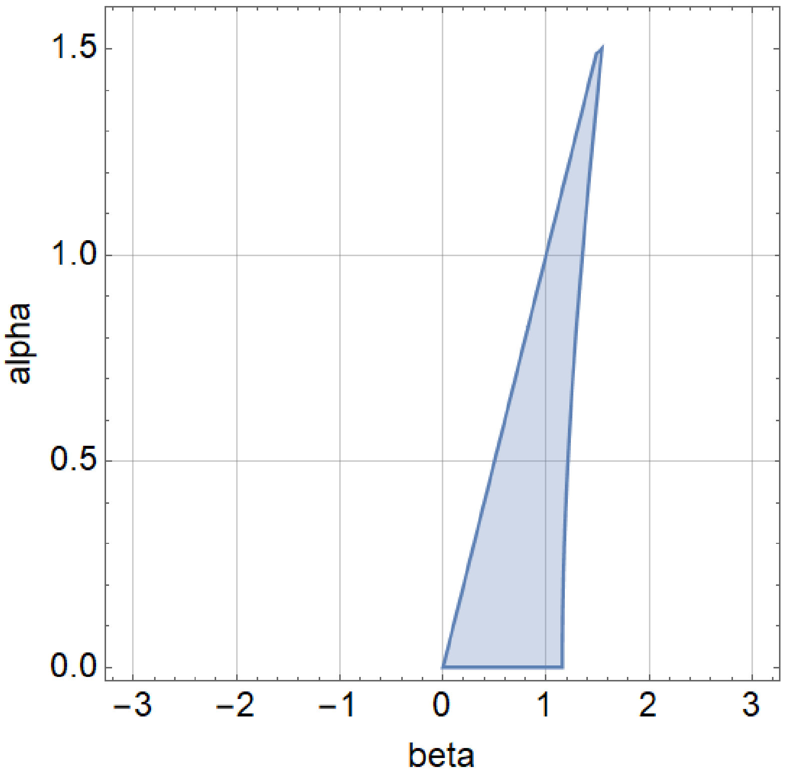
This translates into

$$\frac{\|F_1^{\|}+F_2^{\|}\|}{\|F_I\|} < \frac{\cos(\beta)}{\cos(\alpha)}$$

with

$$\alpha < \beta$$

for outer forces



Problem with residual force

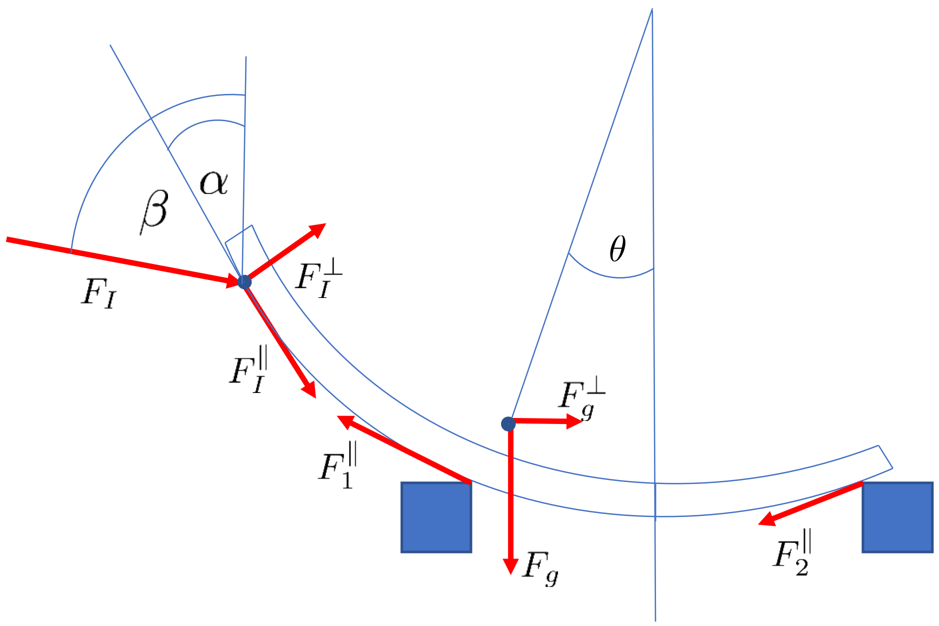
Blue: $\|F_I^\perp\|$ small

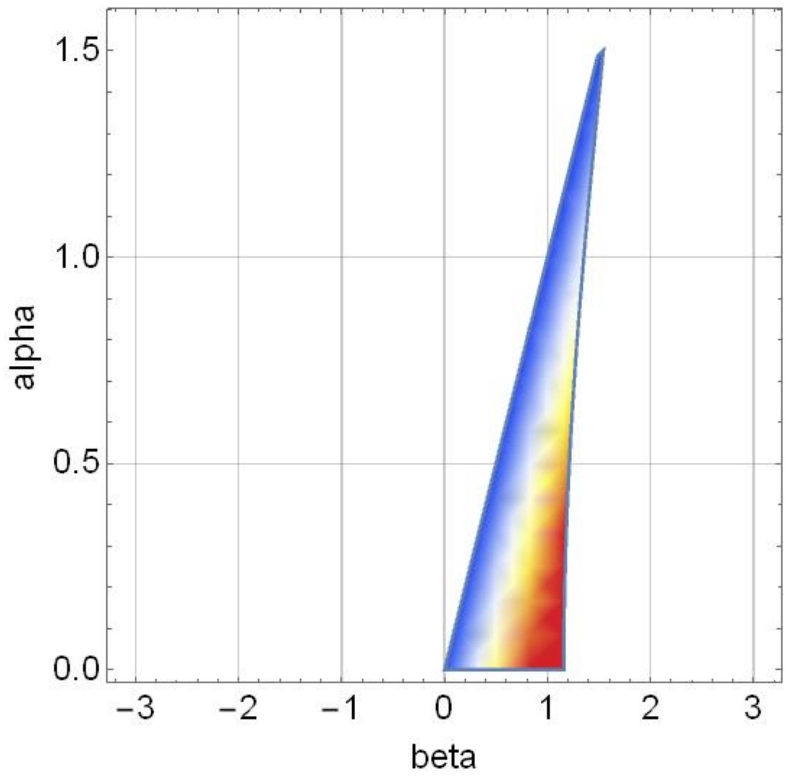
Red: $\|F_I^\perp\|$ large

Normal force

$$F_I^\perp$$

can cause cracking or lifting of the crisp



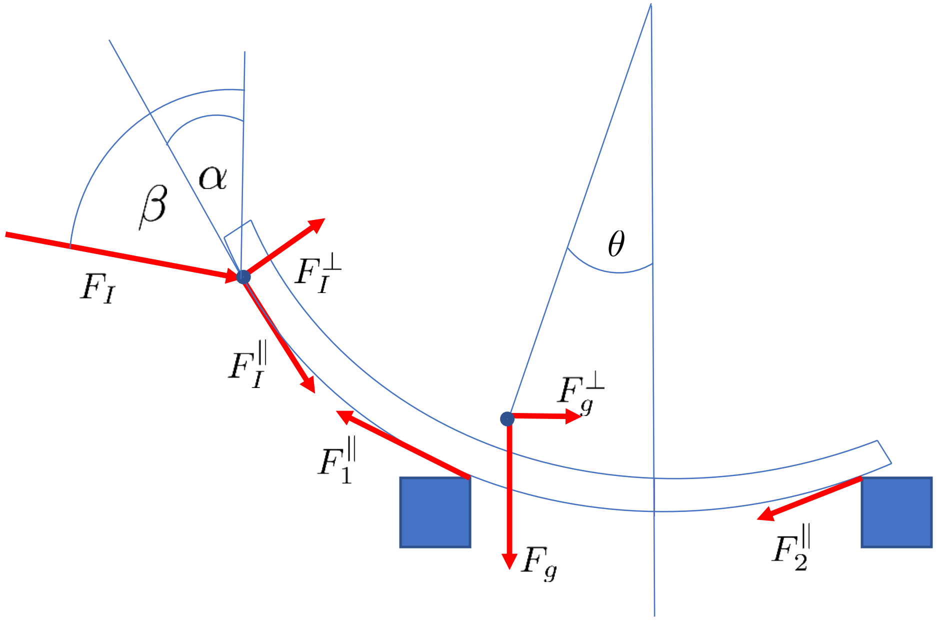


Constraint #2

Enforcing

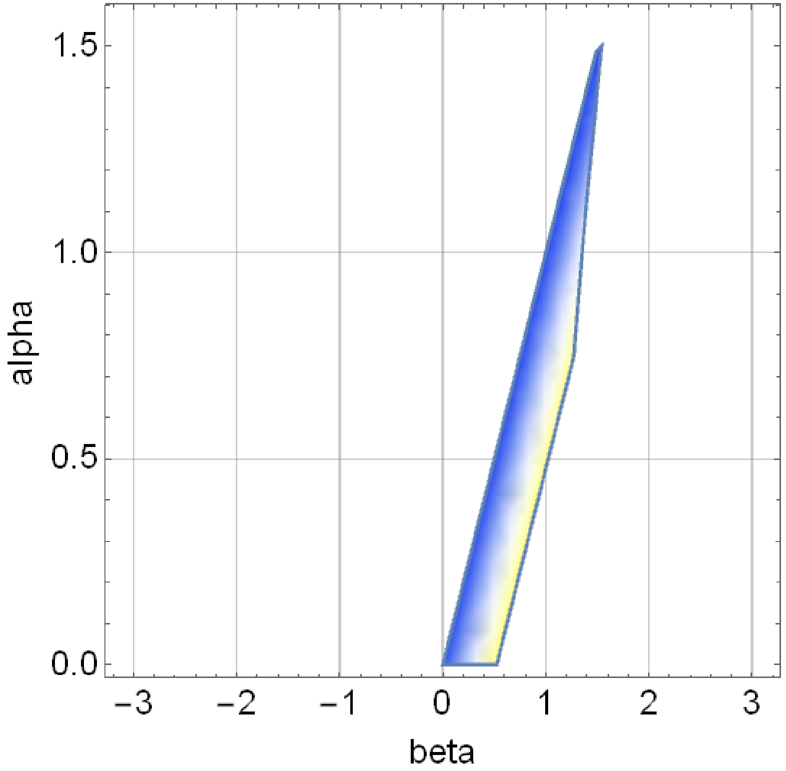
$$\|F_I^{\perp}\| \leq F_{max}$$

prevents cracking or lifting of the crisp



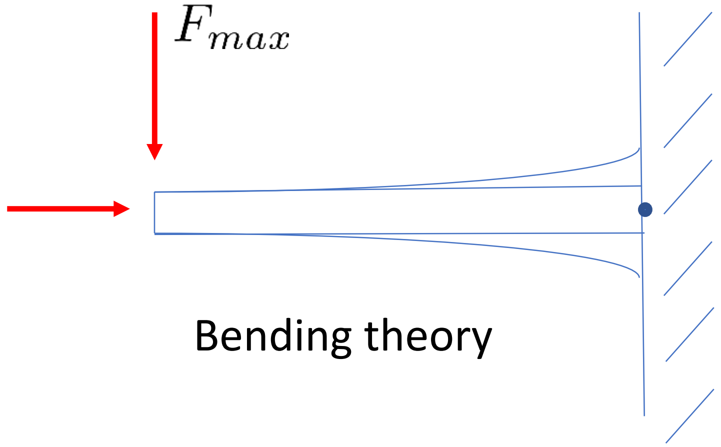
This translates into

$$\sin(\beta - \alpha) \leq \frac{F_{max}}{\|F_I\|}$$



Further considerations

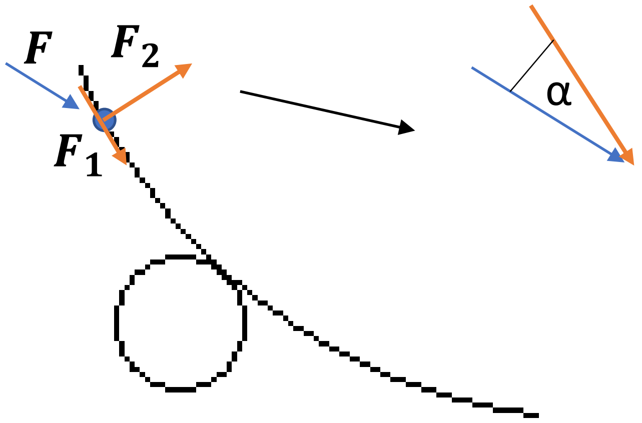
- Generalisation to wider range of $(\alpha, \beta) \in \Omega$
- Determine the exact friction terms $F_1^{||}$ and $F_2^{||}$
- Include the microstructural properties of the crisps to determine F_{max}



- Extension into three dimensions
- Include distribution of initially misaligned θ

Framework for normally distributed θ

Applying an external force to a crisp



An external force applied to a crisp

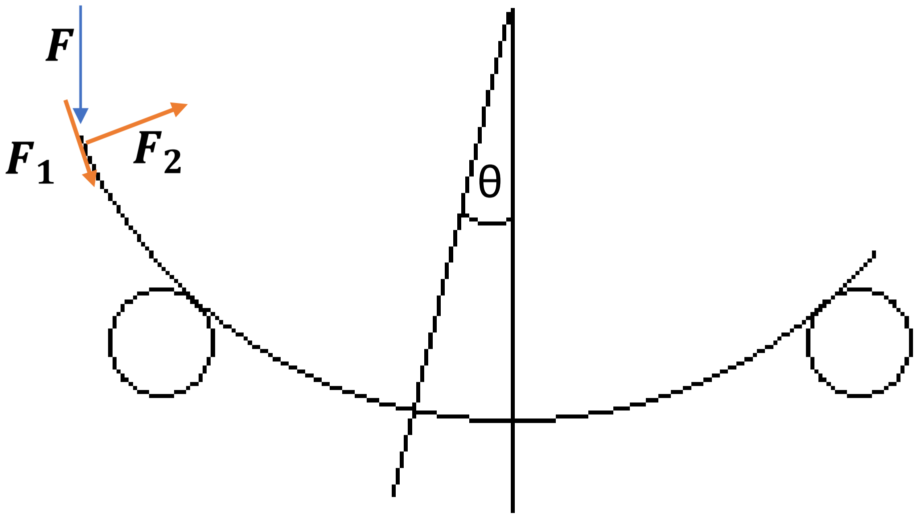
Problems with additional non-necessary force on the crisp:

- Can cause the crisp to break
- Can lift the crisp off of the support

$$F_1 = F\cos\alpha$$
$$F_2 = F\sin\alpha$$
$$F_2 = F_1 \frac{\sin\alpha}{\cos\alpha}$$

$\Rightarrow$ We need to minimise $\frac{\sin\alpha}{\cos\alpha}$

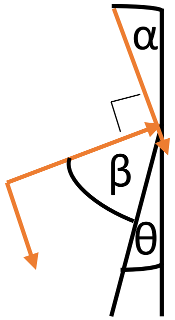
Model with only vertical applied force



Vertical force applied on a crisp with angular displacement

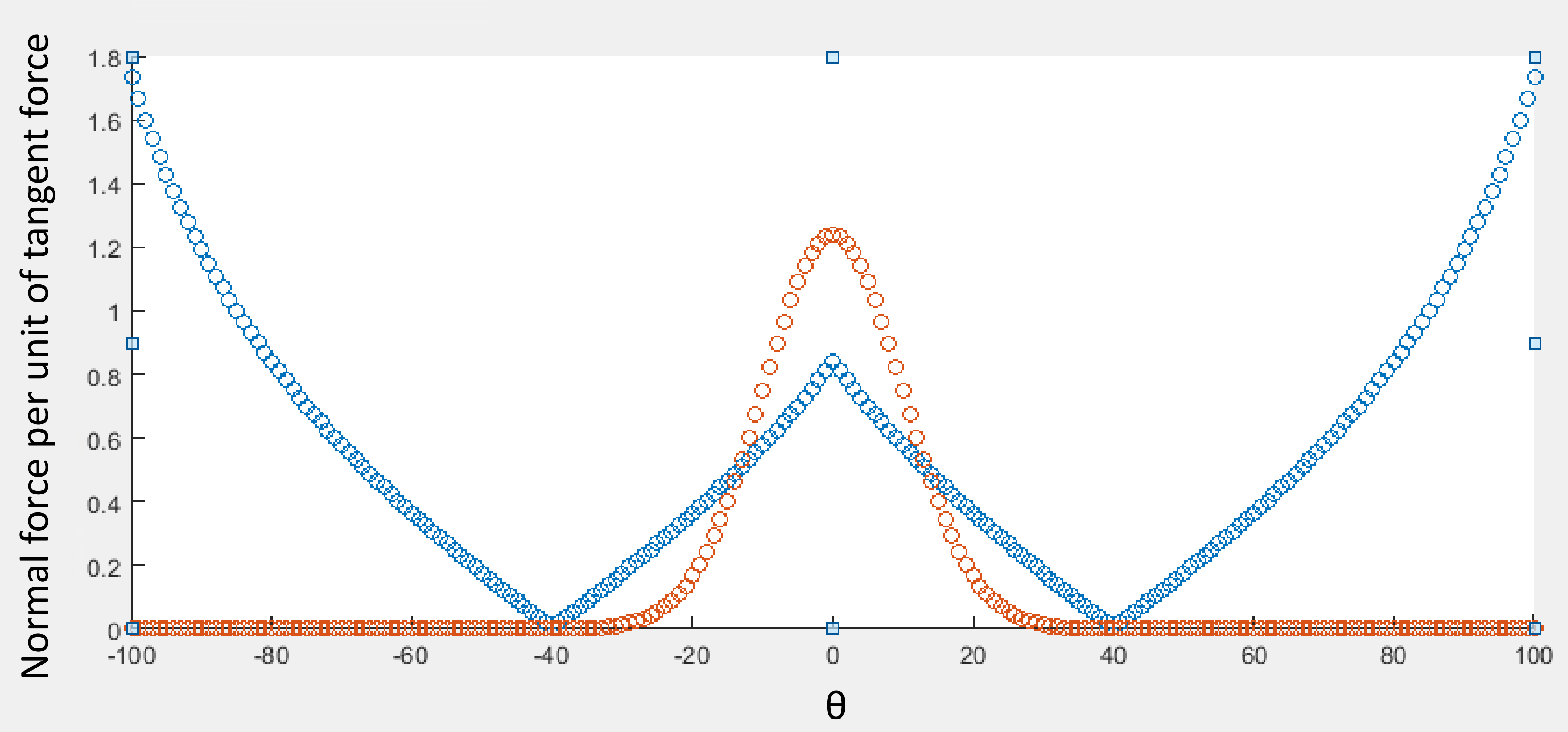
Assumptions:

- External force is always vertical pushing the crisp down
- External force is at the highest point of the crisp



We see that α and θ are related through $\alpha = |90 - \beta - \theta|$

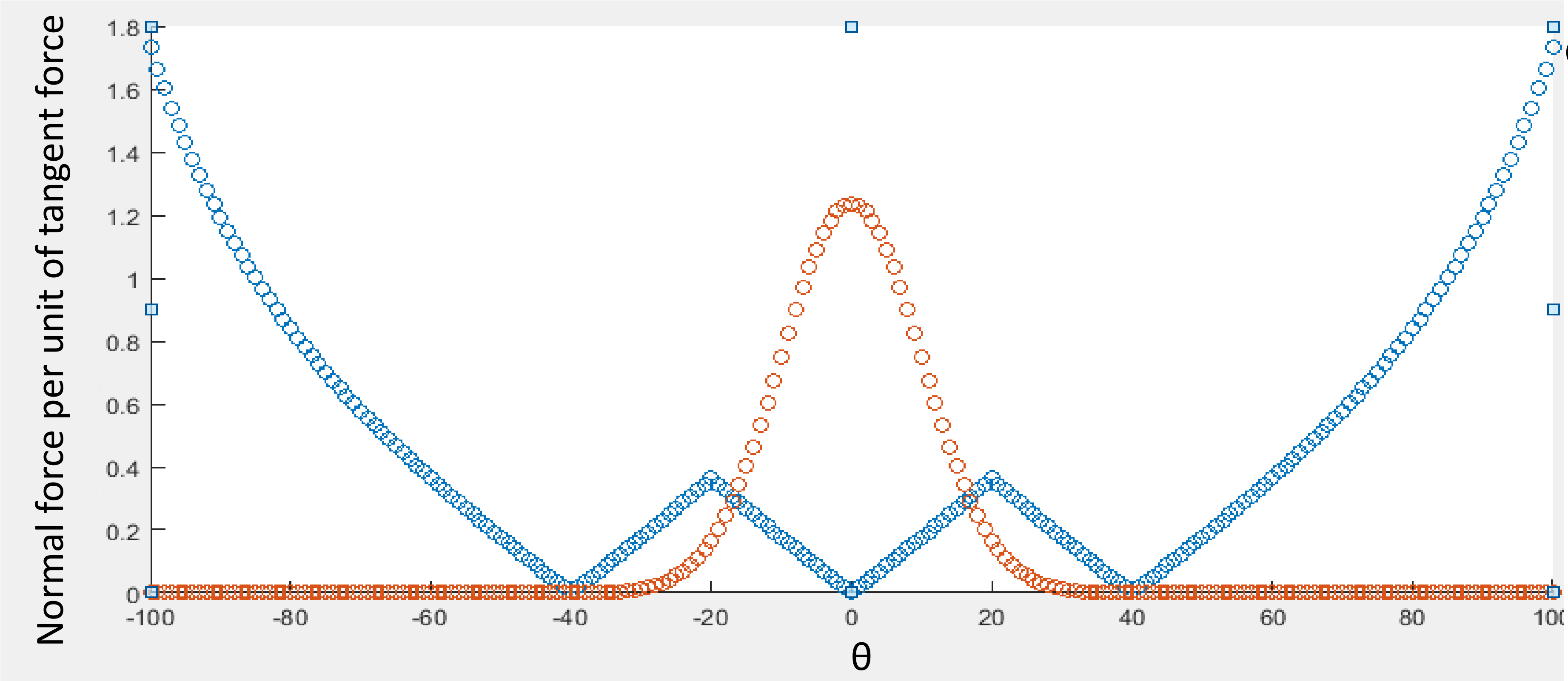
Non-necessary force applied per unit of tangent force against crisp angular displacement



Crisp arc length used - 100˚

Expected normal force per unit of tangent force = 0.6341

Non-necessary force applied per unit of tangent force against crisp angular displacement



Crisp arc length used - 100˚

Expected normal force per unit of tangent force = 0.1356

Questions to consider

- How much force is needed to break a crisp?
- How do we optimising both reducing the angular displacement and crisp loss due to non-necessary forces?
- How effective is using a slightly alternate shape?

Thank you for your attention!