3D - 2D: Potato slices area compactness

Alex Cox, Benjamin Dadoun, Yvonne Krumbeck, Babasola Oluwatosin, Jason Wood

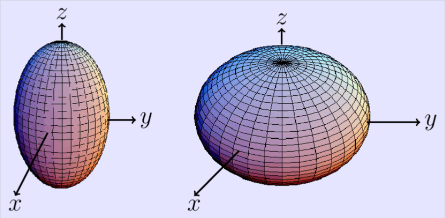
SAMBa ITT 10

June 14, 2019

Problem Statement

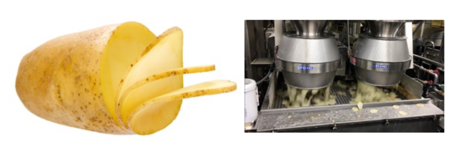
Potato chips are similar in shape but are not identical. When they are sliced they fall onto a flat conveyer. The question then arises as to what it the best way to pack them onto the conveyer so that there is minimal spacing between them.

Geometry of potatoes



Goal

Prevent slices from overlapping



Goal

Prevent slices from overlapping

Minimise spacing

Steps

Ways of tackling the arising problem

- Sample shapes and place these shapes at random centres within a plane
- Estimating overlaps
- From these simulations get "efficiency distribution" and "overlap distribution"

Sample shapes from the distribution

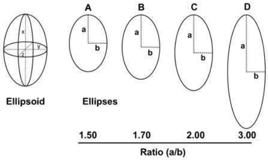


Figure : Representation of the shapes

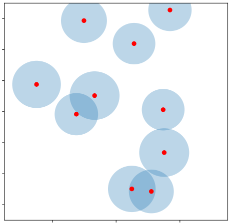


Figure : Illustration of possible placement of the slices

Estimating number of Overlaps

Here, we are interested in estimating the number of $0, 1, 1^+$ intersection. We commence by illustrating how to estimate the expected number of circles with no intersect and the problem will be broken down into sub-problems.

Estimating number of Overlaps

Here, we are interested in estimating the number of $0, 1, 1^+$ intersection. We commence by illustrating how to estimate the expected number of circles with no intersect and the problem will be broken down into sub-problems.

Sub-problem One

Suppose we have two circles C, C' in the square, we determine whether they intersect

Estimating number of Overlaps

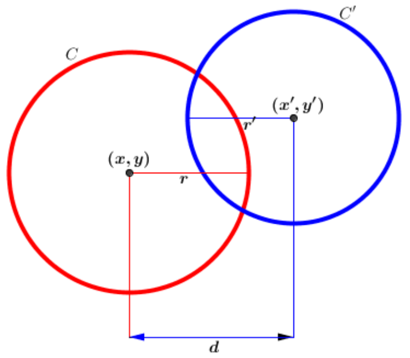
Here, we are interested in estimating the number of $0, 1, 1^+$ intersection. We commence by illustrating how to estimate the expected number of circles with no intersect and the problem will be broken down into sub-problems.

Sub-problem One

Suppose we have two circles C, C' in the square, we determine whether they intersect

Let C be centered at the point (x, y) and have radius r, and C' be centered at (x', y') with radius r'. The two circles overlap iif the distance between the two centers is at most $r + r'$, i.e., if

$$(x - x')^2 + (y - y')^2 \leq (r + r')^2$$



Sub-problem two

Suppose the first circle is given by C. Compute the probability $p(C)$ that the second circle will not intersect C.

Sub-problem two

Suppose the first circle is given by C. Compute the probability $p(C)$ that the second circle will not intersect C.

We perform different trials, where in each trial, we randomly drop the second circle C' and test whether C, C' intersect. Count the number of trials where they do not intersect and divide by the number of trials.

Sub-problem three

If we have N circles and suppose the first circle is C. Compute the probability that none of the remaining $N-1$ circles will intersect C.

Sub-problem three

If we have N circles and suppose the first circle is C. Compute the probability that none of the remaining $N-1$ circles will intersect C.

We note that each circles are randomly dropped and therefore, by independence,

$$p^0(C) = \underbrace{p(C) \times \cdots \times p(C)}_{(N-1) times} = p(C)^{N-1}$$

Sub-problem four

Compute the probability that if you drop N circles randomly, none of the last $N - 1$ circles has any intersection with the first circle.

Sub-problem four

Compute the probability that if you drop N circles randomly, none of the last $N-1$ circles has any intersection with the first circle.

If we let C be a random variable and the probability none of the last $N-1$ circles has any intersection with the first circle is denoted by q_1. We have $q_1 = E[p^0(C)]$, where the expectation is taken over C. simulating with different trials, where in each trial you randomly choose a circle C, then you compute $p^0(C)$ (using solution 3). Average the results over all of the trials. This gives the estimate of the probability q_1 [1].

Sub-problem five

Compute the probability q_i that, if you drop N circles randomly, the i^{th} circle has no intersection with any of the other $N-1$ circles.

Sub-problem five

Compute the probability q_i that, if you drop N circles randomly, the i^{th} circle has no intersection with any of the other $N - 1$ circles.

By symmetry, $q_i = q_1$.

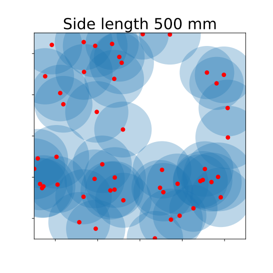
Back to the main problem

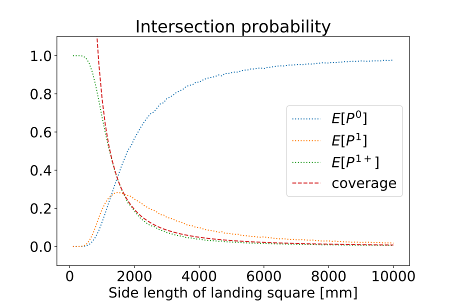
The original problem. Compute the expected number of circles that have no intersection with any other circle, if you drop N circles randomly.

By linearity of expectation, this is

$$q_1 + q_2 + \cdots + q_N = N.q_1$$

Overlap plot





Future Directions

Questions to consider

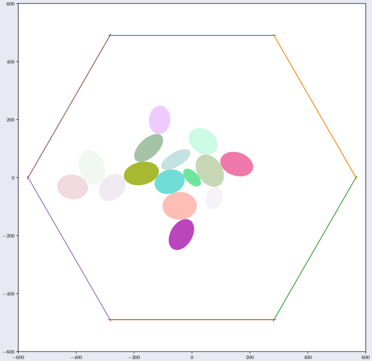


Figure : Placing an ellipse in the center then bringing an ellipse in from a random point on the edge

References

D.W- Probability of circles intersecting. “https://stats.stackexchange.com/questions/17954/probability-of-circles-intersecting”. Accessed on: 2019-06-13

The End