Bubble Formation in Crisps

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The problem

- Crisps are formed from a mixture of water and potato flakes.
- The ingredients are mixed, flattened and then fried.
- In the course of the heating process bubbles are formed.
- Ideally roughly equal sized bubbles are located uniformly in the chip, however this is not always the case.

Figure: "Good" and "bad" chips

- We would like to understand how the mixing and heating processes affect the bubble structure.
- This week: explored possibilities of modelling mixing and bubble expansion.

Methods

Mixing

- Importance: If the water-potato mixture is not well-mixed, we expect to have areas with big bubbles and other areas with no bubbles at all after frying.
- Idea: Consider a map and apply it repeatedly, so that it mimics the effect of a real mixing process.
- Examine the properties of this dynamical system.

Bubble expansion

- Understand the evolution of a single bubble
- Possible model for two bubbles taking interaction into account
- In both cases we aim to determine the radial expansion of the bubble(s) in the dough, using a system of ODEs, and to understand the randomness coming from manufacture.

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Model of mixing

- Baker's map, a measure-preserving transformation of the unit square.
- Apply the map repeatedly for the points of a lattice, and observe the long time behaviour of this process.
- The long time limit gives an initial condition for the heating process: neighbouring water cells are grouped and represent bubbles whose initial radius is proportional to the number of cells in the group.
- Simple model, but 2D can be realistic in chips production.

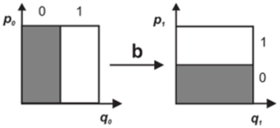

FIG. 1: The baker map propagates a pair of vertical rectangles onto a pair of horizontal rectangles. Inside each separate region the evolution is linear.

Figure: "Squash, cut, stack."

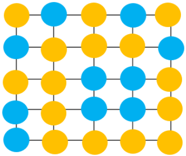

Figure: Potato and water particles on the lattice

Model of mixing

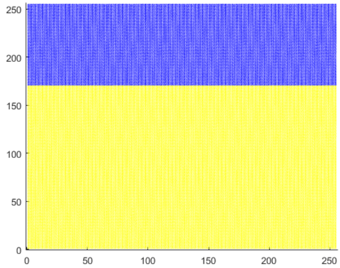

Figure: Separated groups initially

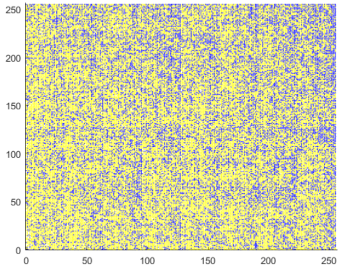

Figure: After 10 iterations of the Baker's map

- What does it mean to be well-mixed?
- First idea: proportion of water in any sample = proportion of water in the whole mixture

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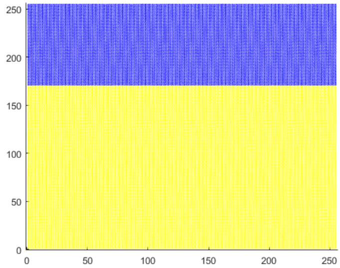

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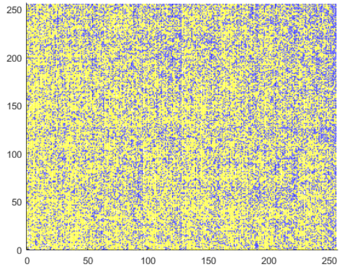

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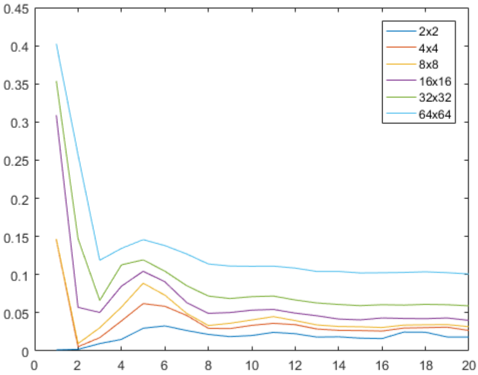

Figure: Mean absolute deviation from 1/3 when looking at the ratio of water particles within squares of different size.

Model of bubble expansion

- Before heating: water pockets are surrounded by a potato mix (dough).
- During heating: a random proportion of water pockets form bubbles, whose radii evolve over time.
- Initial radii are set based on the end state of the Baker's map.
- Use the Rayleigh–Plesset equation: an ODE which governs the dynamics of a spherical bubble in an infinite body of incompressible fluid.
- Assumptions:
  - The dough is a viscous, incompressible fluid, and therefore has constant density.
  - Potato is infinitely large compared to the size of a bubble.

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Model of bubble expansion - Single bubble

Simple model ignoring interaction between bubbles:

- Solve the Rayleigh–Plesset equation for $R(t)$.
- Apply the solution to all initial bubbles coming from the mixing.
- Stop the growth when bubbles would collide.
- The process stops when all bubbles stopped growing.
- For the simulation bubbles were uniformly distributed in a unit square (in the future we want to initialise with the Baker's map).

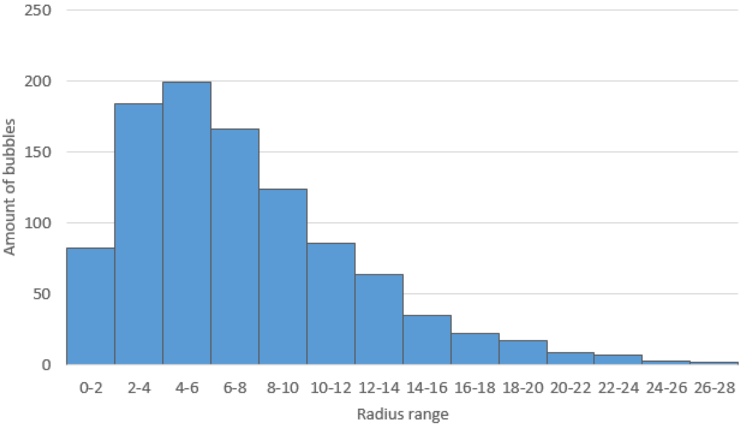
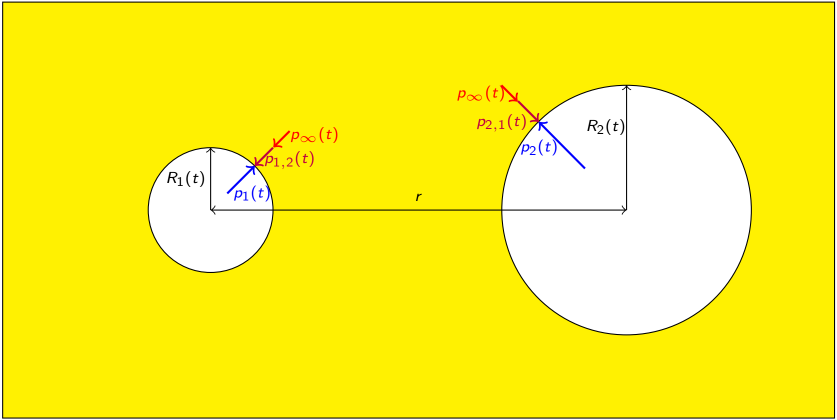


Figure: Distribution of bubble sizes

- Rayleigh–Plesset ODE system including interaction:
- The pressure on the surface of a bubble increases as the size of the other bubble increases.



The approach is to solve the coupled modified Rayleigh-Plesset equations for $(R_1(t), R_2(t))$ given that all other constants and functions are known:

$$R_1 \frac{d^2 R_1}{dt^2} + \frac{3}{2}\left(\frac{dR_1}{dt}\right)^2 + \frac{4\nu_L}{R_1}\frac{dR_1}{dt} + \frac{2\gamma}{\rho_L R_1} + \frac{p_\infty - p_{1,2} - p_1}{\rho_L} = 0$$

$$R_2 \frac{d^2 R_2}{dt^2} + \frac{3}{2}\left(\frac{dR_2}{dt}\right)^2 + \frac{4\nu_L}{R_2}\frac{dR_2}{dt} + \frac{2\gamma}{\rho_L R_2} + \frac{p_\infty - p_{2,1} - p_2}{\rho_L} = 0$$

where $p_{1,2} = \frac{p_2 R_2^2}{(r-R_1-R_2)^2}$ and $p_{2,1} = \frac{p_1 R_1^2}{(r-R_1-R_2)^2}$.

- $R_1(t)$ - Radius of bubble 1 at time t
- $R_2(t)$ - Radius of bubble 2 at time t
- ρ_L - Density of the surrounding liquid
- ν_L - Kinematic viscosity of the surrounding liquid
- p_∞ - Constant external pressure infinitely far from each bubble
- $p_1(t)$ - Pressure inside bubble 1 at time t
- $p_2(t)$ - Pressure inside bubble 2 at time t

The additional terms $p_{1,2}$ and $p_{2,1}$ are approximations of the additional contribution to the external pressure p_∞ on bubbles 1 and 2 respectively, due to the expansion of the other bubble.

Future plan

- Final goal: Comprehensive picture of how mixing and heating affects bubble structure.
- Include the stochasticity of mixing and (later) heating.
- Mixing model: Randomise Baker's map by choosing the cutting point uniformly on $(0, 1)$, study the long time behaviour and possibilities of measuring how well-mixed a state is.
- Bubble expansion model:
  - Link the initial condition to the mixing model
  - Fit a model including all bubbles by considering the growth of interacting pairs.
  - Include the interaction of three bubbles.
  - Possible other approach: probabilistic lattice model