Inference in Bayesian Networks

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Inference

Network + Cond Prob Tbl $\rightarrow$ Marginal and Conditional Prob

Evidence, e

- Hard evidence: Observe $Y = y^*$
- Likelihood evidence: Observe $\Pr(Y = y)$

Compute $p(x|e)$

Incorporate likelihood evidence

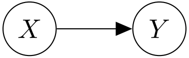
Evidence: $\Pr(Y = y)$

Add a new node with only parent the concerned variable.

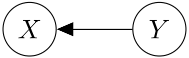
Assign probabilities $\Pr(Z = z|Y = y)$.

Arc reversal

Given: $p(x)$, $p(y|x)$

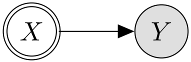


Compute: $p(y) = \sum_x p(x)p(y|x)$, $p(x|y) = p(x)p(y|x)/p(y)$



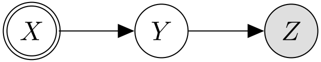
Simple Networks

Evidence: $Y = y^*$



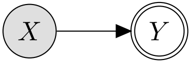
$$p(x|y^*) \propto p(x)p(y^*|x)$$
$$\pi(x|y^*) \propto \pi(x)\lambda(x|y^*)$$

Evidence: $Z = z^*$



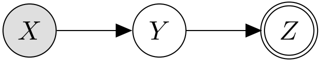
$$\pi(x|z^*) \propto \pi(x)\lambda(x|z^*)$$
$$\lambda(x|z^*) = \sum_y \lambda(y|z^*)p(y|x)$$
$$\pi(y|z^*) \propto \sum_x \pi(x|z^*)p(y|x)$$

Evidence: $X = x^\dagger$



$$\pi(y|x^\dagger) \propto \pi(x^\dagger)p(y|x^\dagger)$$

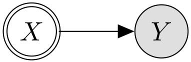
Evidence: $X = x^\dagger$



$$\pi(z|x^\dagger) \propto \sum_y \pi(y|x^\dagger)p(z|y)$$

Simple Networks

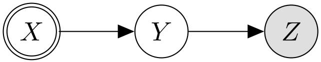
Evidence: $Y = y^*$



$$p(x|y^*) \propto p(x)p(y^*|x)$$

$$\pi(x|y^*) \propto \pi(x)\lambda(x|y^*)$$

Evidence: $Z = z^*$

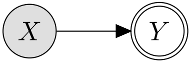


$$\pi(x|z^*) \propto \pi(x)\lambda(x|z^*)$$

$$\lambda(x|z^*) = \sum_y \lambda(y|z^*)p(y|x)$$

$$\pi(y|z^*) \propto \sum_x \pi(x|z^*)p(y|x)$$

Evidence: $X = x^\dagger$



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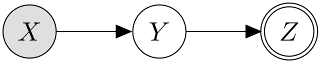
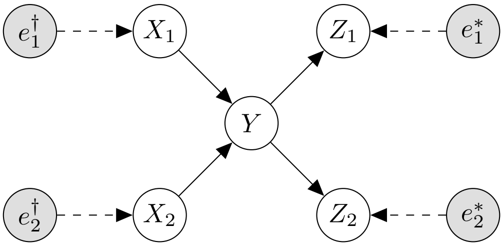


$$\pi(z|x^\dagger) \propto \sum_y \pi(y|x^\dagger)p(z|y)$$

Message passing algorithm in polytrees

Kim and Pearl (1983)



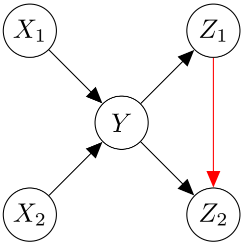
Maintain quantities $\lambda(\cdot)$ and $\pi(\cdot)$

$\lambda(y|e^*) = \sum_{z_1} p(z_1|y)\lambda(z_1|e_1^*) \sum_{z_2} p(z_2|y)\lambda(z_2|e_2^*)$

$\pi(y|e^\dagger) = \sum_{x_1,x_2} p(y|x_1,x_2)\pi(x_1|e_1^\dagger)\pi(x_2|e_2^\dagger)$

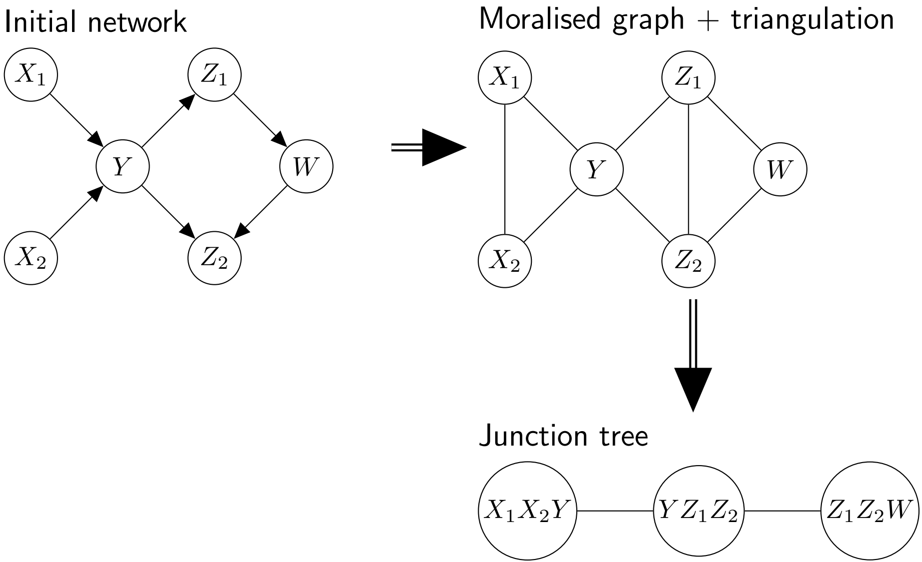
Nice tutorial: Krieg (2001)

Loops



- Iterative message passing algorithm
- Clustering
- Lauritzen & Spiegelhalter (1988): Moralised graph $\rightarrow$ junction tree.

Lauritzen & Spiegelhalter's method



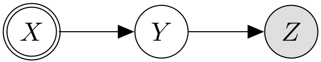
Compute joint probabilities in the nodes of the junction tree.

Direct simulation (Rejection sampling)

1. Propagate from roots to leaves. Sample from a node after all its parents are sampled.
2. Discard all samples that do not agree with the evidence.
3. Compute probabilities empirically from the remaining samples.

Importance sampling

Evidence $Z = z^*$



$$p(x|z^*) = \frac{\sum_y 1(X = x, Z = z^*) p(x)p(y|x)p(z|y)}{\sum_y 1(Z = z^*) p(x)p(y|x)p(z|y)}$$

$$= \frac{\sum_y 1(X = x, Z = z^*) \frac{p(z|y)}{q(z|y)} p(x)p(y|x)q(z|y)}{\sum_y 1(Z = z^*) \frac{p(z|y)}{q(z|y)} p(x)p(y|x)q(z|y)}$$

Likelihood weighting: Set $q(z^*|y) = 1$, and 0 o/w. No need to discard samples.

MCMC

For each node X, sample from $p(x|\text{rest}) \propto p(x|\text{pa}(x)) \times p(\text{ch}(x)|x, ...)$

- Can use Gibbs sampling if Gaussian, discrete nodes
- Metropolis-Hastings algorithm in general

Software

Package	Nodes	Method
gRain	Discrete	L & S
BUGS, JAGS, STAN	Mixed	MCMC
HUGIN	Mixed	L & S
GeNIe and SMILE	Mixed	Deterministic and Stochastic
Netica	Mixed	L & S
Bayesia	Mixed	Deterministic, Likelihood weighting

References

Kim and Pearl (1983) A computational model for causal and diagnostic reasoning in inference engines [Message passing algo in polytrees]

Krieg (2001) A tutorial on Bayesian belief networks [Online]

Lauritzen and Spiegelhalter (1988) Local Computations with Probabilities on Graphical Structures and Their Application to Expert Systems [JRSSB paper]