Solving Inverse Imaging Problems using Generative Machine Learning Methods

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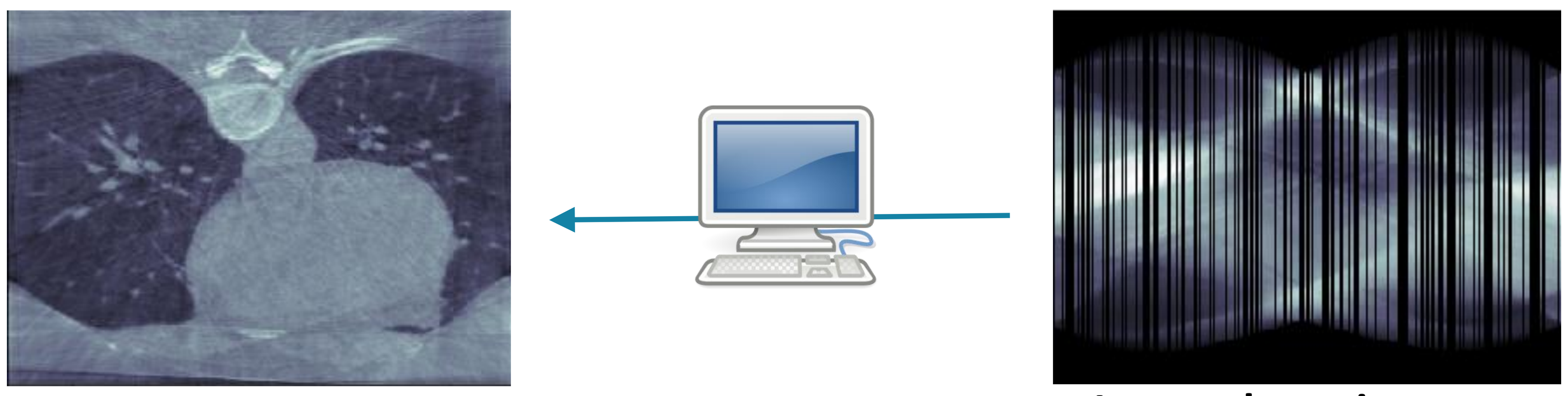
Motivation: Inverse Problems

An inverse imaging problem is the task of reconstructing an unknown image from observed, potentially noisy, measurements. For example in CT scan of the heart and lungs:

Heart and lungs **Sinogram**

In CT imaging we want to limit the dose of radiation given to a patient, but the lower the dose the fewer measurements and less data to work with. This may result in a poor quality reconstruction.



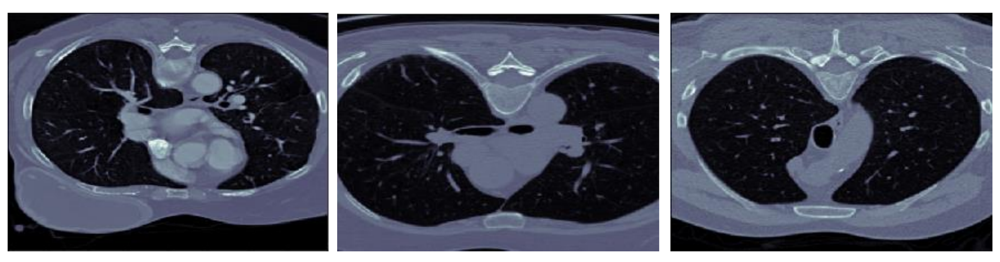
Reconstruction **Lower dose sinogram**

Inverse problems are everywhere in modern science and engineering, including medical imaging, signal processing and geophysics. Our results have wide applications.

Generative Machine Learning Methods

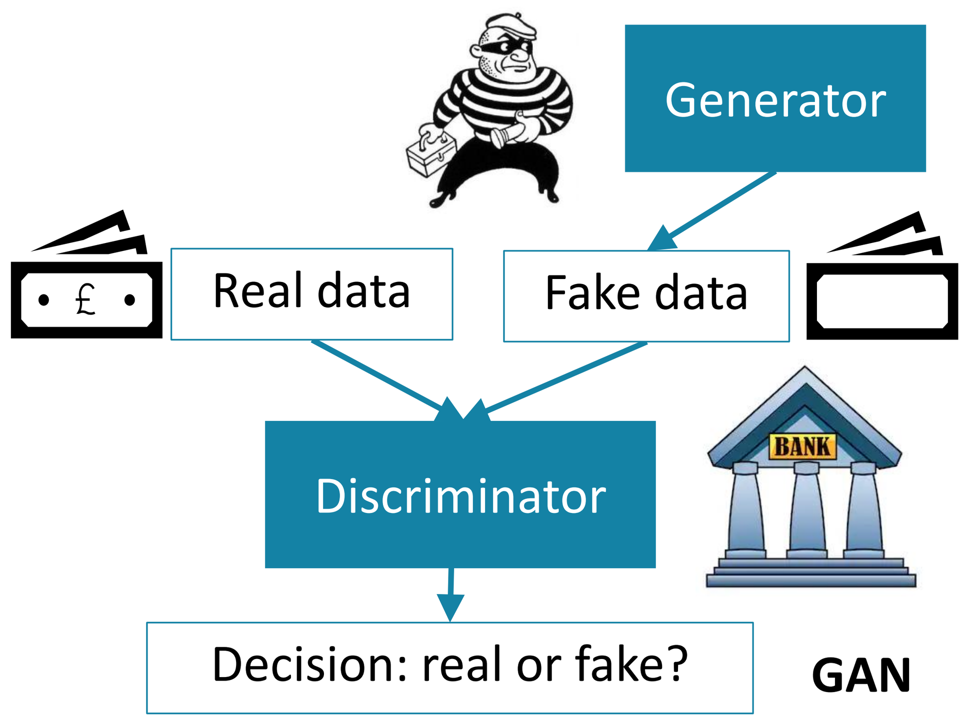
Common reconstruction techniques add extra information by assuming some properties of the images, e.g. that they are mostly smooth with some edges.

What if instead, extra information could be learnt from other, similar images?



Examples of similar CT images

- If there was a way to look at every feasible image, then the reconstruction would be the image that bests fits the data.
- A generative model can generate images similar to examples it has seen before. They produce a range of feasible images.
- A Generative Adversarial Network (GAN) [Goodfellow, 2014] consists of a generator that produces fake images and a discriminator sorts real from fake data. They compete until the fake images are indistinguishable from the real images. The generator can then be used on its own to generate images.

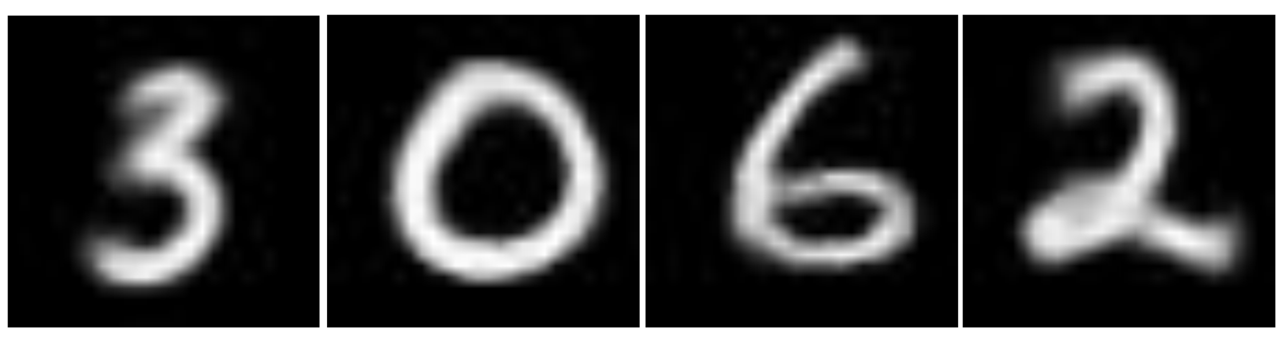


GAN

Reconstruction of Handwritten Digits

A generative model can be trained to produce hand written digits.

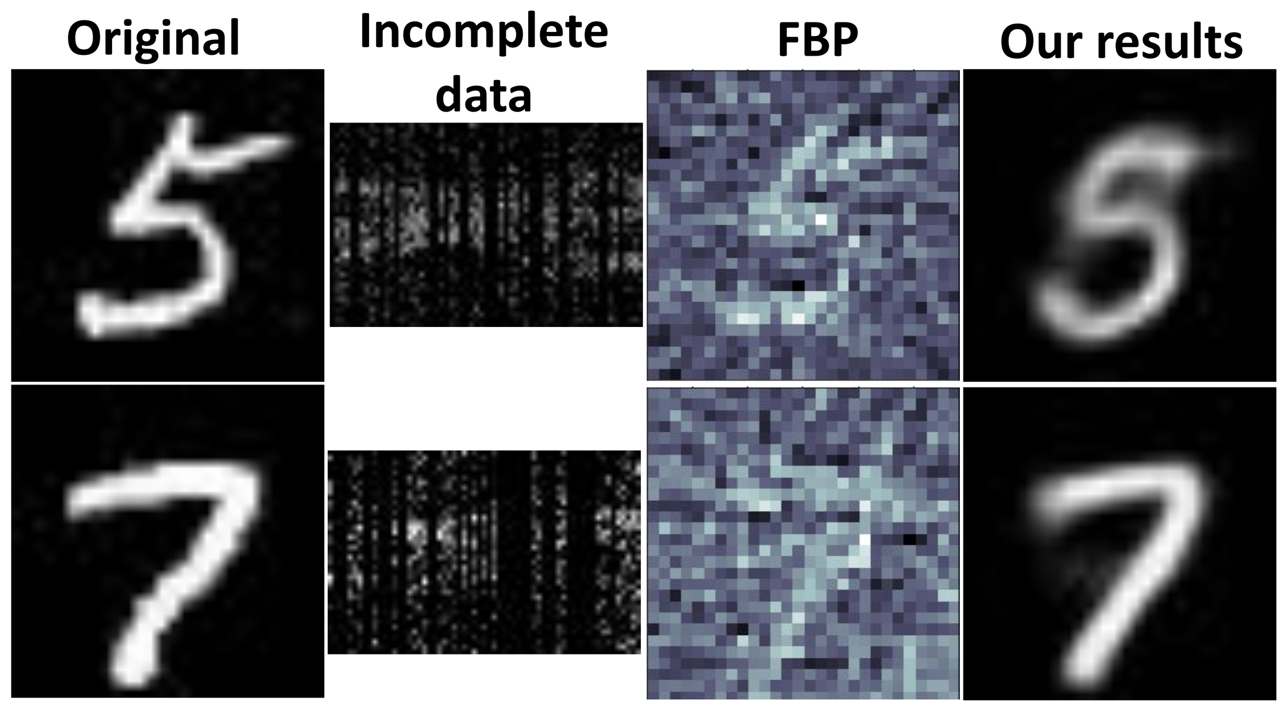
Examples of generated digits



Images can be reconstructed by searching through the images produced by the generative model to find the one that best fits the data.

It works with noisy and incomplete data even when the standard reconstruction method, filtered back projection (FBP), does not.

Original **Incomplete data** **FBP** **Our results**



Mathematics

Inverse Problem Background:

- Let $A: X \to Y$ be a process that takes an image $x \in X$, to data $y \in Y$.
- The inverse problem takes data, y, corrupted by unknown noise, ϵ, and calculates an image, x, such that:

$$y = A(x) + \epsilon.$$

- This is usually done by solving:

$$\min_x \|A(x) - y\|^2$$

Using a generative machine learning method:

- A generative model can learn a distribution of feasible images.
- A generator, $G: Z \to X$, takes points in a known lower dimensional latent space, Z, and outputs feasible images.
- It is possible to search over the latent space to solve the inverse problem:

$$\boxed{z^* = \arg\min_{z \in Z} \|A(G(z)) - y\|.}$$

- The reconstructed image is found by calculating

$$x^* = G(z^*).$$

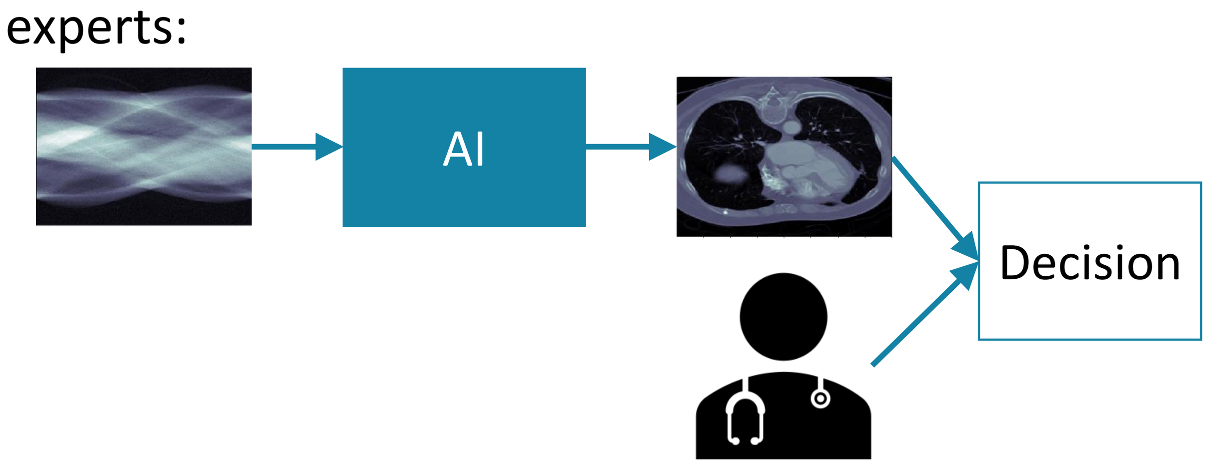
Conclusions and Future Work

We have demonstrated how using generative models in image reconstruction can provide better results.

Incorporating generative networks allows us to combine artificial intelligence and expert knowledge when making decisions.

- Black box artificial intelligence makes decisions without explanations:

- Generative methods can assist rather than replace experts:



Future work will focus on:

- Mathematical results on the feasibility of finding the best image produced by the generator to match the data.
- Mathematical approaches to checking and ensuring that the original image can be produced by the generator.

References

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