Mathematical writing during your PhD

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Introduction

During your PhD you will reach a point where it will be necessary to write your findings up in LaTeX. If you don't know how to prepare in documents in LaTeX, then this is your first priority, there are courses given by the department, but realistically you can teach yourself within an afternoon by playing with an existing LaTeXfile.

Below you will find some tips, some things to think about as well as some standards to work your way towards during the period of your PhD supervision. Mathematical writing is not as easy as it looks. Ultimately the reason why you will write documents is to *communicate* your thoughts to others (initially your supervisor), to *archive* your thoughts for later meditation and to be *efficient* with the work that you put into typing up mathematical arguments into LaTeXthat ultimately find their way into papers or your thesis. With the advent of the electronic media and, depending on how busy your supervisor is, it can also play to your advantage to have documentation that your supervisor can refer to outside of meetings and during periods that one or both of you is away from the department.

It takes attention and thought to prepare a readable document. The first person who needs to be able to read your work is your supervisor, but you should always write your documents in a way that they can easily be re-worked, expanded, and ultimately feed into other documents that will become research papers or your PhD thesis.

When should you start documenting your calculations?

The answer to this question is: *immediately*. As soon as you feel you have a calculation that matters, a proof, a part proof, or something that you know you will want to refer to again in the future, you should commit it to LaTeX. There are different types of documents you can initiate in this respect. You may wish to save individual lengthy calculations as single documents. You may wish to collect them in a single document. You may wish to develop a running document into which you grow the cumulative progress of your thoughts as they appear chronologically. However be warned, *your supervisor does not want to see that document*, **ever**! How you *store* your thoughts in LaTeX is different to how you present them to your supervisor, as well as others.

Writing comprehensive notes vs writing a paper

A good habit to get into is to have a working document that you share regularly with your supervisor. This document is different to a research paper. Nonetheless, it should be written in a way that it has purpose, flow and clarity. It should be a document that your supervisor can navigate easily. Don't fill it with every thought you have had, make sure that you trim off the fat, keep to the point and present your material in a logical order. See also below remarks concerning notation and stylistic presentation. Casually written documents are shabby and ineffectual. Mathematics is a precision language so write precisely. Even if you have half-baked proofs, or you only have heuristic ideas that you want to communicate you can still make your document precise by emphasising clearly, for example,

- what you are *proving* or *disproving*,

- what assumptions you have made,
- what is a conjecture and what is fact
- where you have obtained facts from (i.e. cite clearly)
- where you are replicating or adapting the arguments of others

Diagrams, where possible, are highly recommended to help reinforce what you are saying. There are many free electronic packages you can download that are robust enough to produce excellent quality pictures with embedded LaTeX.

A research paper is what you write at the end once you have a package of work that tells a complete story. Your supervisor will advise you on when this moment has been attained. If you have kept a healthy working document and you keep consistent notation, you can often migrate significant amounts of LaTeX from your notes to the the paper you are writing. If you have a specific journal you want to submit to then it is worth downloading their style file, which almost all journals now offer on line.

The principles of writing a research paper are no different to the principles by which you should develop your working notes. Think about your reader! Ultimately you are writing for another mathematician who is familiar with the field, but who is reading your ideas for the first time. There is no magic formula to writing a good research paper. Some people are naturally better at it than others. Nonetheless, there are some steps you can take to make sure your paper writing is not drifting off into chaos.

Your research paper should have a very strong sense of direction, with a clear introduction indicating precisely *what it is you aim to achieve* in the publication, the *value* of doing so and where the *originality* lies in your contribution. It should be absolutely clear where the main results are situated in the paper. Moreover, you should tell your reader how you have structured the paper, so they know what to expect. Going into an endless monologue is not everyone's idea of a riveting read. A conclusion is option, some authors prefer to write a small concluding section, many don't. Most of all, you must regularly take a step back and take a holistic view of your paper. Often it is necessary to re-order the way in which you write things as you write more into the document. As irritating as this can be, it is a good way to feel your way around the document and find the natural order in which to present your story.

You should be cut-throat-efficient about writing mathematical arguments that are only absolutely necessary to achieve the results you are presenting. At the same time, cut-throat-efficiency should not come at the expense of clarity. How much you put into explaining your arguments is an art form. You don't need to assume your reader is an idiot, but you also should not assume that they are as immersed in the problem as you have been for the last months or years. Sometimes it is the simplest of arguments that cause enormous bottlenecks to many readers if they are not exposed clearly enough. In the worst case, you can create the false impression that your argument is simply wrong which will result in a loss of trust from your reader. Don't waffle, don't overpopulate your document with side discussion. But more importantly, **don't brain dump**!

Brain dumping

Brain dumping is a phenomenon that many mathematics masters and PhD students often engage in without realising it. The journey to proving a result is a long one and has almost surely meant passing through many dead ends from which you have learned a great deal. Although, after such a treacherous mathematical journey, you will feel you have earned the right to tell the reader about every step of your adventure that led to the conclusions you are presenting, you actually haven't. That is brain dumping!

The reader (especially your supervisor) is not interested in the gory details of your journey, they are interested in understanding things as quickly and as clearly as possible. Accordingly, the reader is not necessarily interested in experiencing the mathematical discovery of things in the same sequential order that you did. You are obliged to present the quickest, cleanest and clearest route to proof once you know how. You can explain verbally to your supervisor about how you fought your way from A to B during your supervision meeting if you think there is some value in it.

It is also not uncommon that, after slogging your way through a horrendously difficult proof, you find a much smarter and cleaner way of achieving the same or stronger result. Indeed, your supervisor, on reading your first proof, may retort with a much quicker method. When that happens you have no choice but to formally adopt it in your working document or paper. Don't allow your ego to be bruised. Be proud that you have successfully engaged in quality scientific dialogue around your ideas to the extent that you have inspired a response from another mathematician which has moved things forward. The original proof should not be forgotten, however. It may later provide inspiration for other things and should be kept either in your working document as an 'alternative proof', or as a separate document.

Notation

Notation is the biggest challenge of all when writing new mathematics. Often your work will be a continuation or application of theories that are already in the literature. In that case, it is dangerous to completely reinvent new notation. If it is generally acknowledged to work well, it is better to keep close to the notation that is pre-existing in the literature.

That said, the very fact that writing up research implies that you are presenting an original contribution and so new notation will be inevitable. Chose your notation wisely. Don't overburden or overcomplicate notation trying to say too much about what it represents. For example, often it is not necessary to indicate dependency of certain quantities if that dependency is irrelevant to the general use of the notation.

One of the worse crimes you can commit when mathematical writing is to be *sloppy* with your notation. What does sloppy notation mean? Examples include:

- Not defining things and expecting the reader to guess what they mean (this is a tell-tale sign of brain-dumping!).
- Defining important things implicitly rather than explicitly or hidden in text so it is difficult to find.
- Having more than one notation for the same thing (another tell-tale sign of brain dumping).
- Writing dependencies of one variable on another in different ways e.g. $\varphi(t)$ vs. φ_t. Sometimes it is necessary to interchange between two different ways of writing something, but in that case, you should warn the reader you are going to do it.
- Not taking pride and care in how you write things, e.g.

  $$\mathbb{E}(\prod_i f[X_i])$$

  is disgusting and lazy but

  $$\mathbb{E}\left[\prod_{i=1}^{\infty} f(X_i)\right].$$

  shows more care and consideration for the reader's perspective and how their eyes will move over the symbols, simultaneously examining and deciphering what they mean.
- In contrast to the previous bullet point, sometimes it is necessary to keep notation to a bare minimum because of the size of a computation. However, you should then warn the reader that you are doing it deliberately and make absolutely clear that your minimalistic notation cannot be understood in an ambiguous way.
- Being inconsistent with how you use standard notation, e.g. using dx, $\mathrm{d}x$, $\lambda_1(dx)$ all for Lebesgue measure at different places in your document, or writing $F(dx)$ as well as $dF(x)$ in the same document.

Most importantly, *mathematics should look beautiful.* Consistency in how you use your notation, its *shape* and *spacing* is as much about the inherent mathematical beauty in what you are presenting as the meaning of the symbols. If your mathematical writing looks ugly and uncomfortable, then this is what your reader will feel about what you are saying. Although mathematics is supposed to be purely logical reasoning and not something cosmetic, don't forget it is read by humans who are complex and emotional beings (yes, even mathematicians!).

Mathematical grammar vs English grammer

Many young mathematicians have had limited exposure to literary writing because their education has largely been invested in learning mathematics and developing mathematical prowess. This is not the place to deal with English grammar, but assuming you have a reasonable grasp of that, there are some important points about how you interlace mathematics into English grammar. Here are some, but not all, of them.

- Never, ever, **ever**, start a sentence with a mathematical symbol. This is stylistically abhorrant.
- To ensure you are adhering to correct English grammar, you should read out loud in your head sentences interlaced with mathematics. In this way, you can be sure that you are treating certain notation as nouns, and certain operations as verbs. You can also be sure that your sentence structure adheres to the norms of English grammar, e.g. if you are using subclauses, '*If, ..., then ...*' constructions etc. For example the sentence

  If we have that

  $$P[f](x) := \int_0^1 f(x,y)\mathrm{d}y = 3, \qquad x \in \mathbb{R},$$

  then f is degenerate.

  will be read out loud in your head as: *If we have that the operator P acting on f, defined to be the integral in y, from zero to one, of f with respect to Lebesgue measure is equal to three, for each real x, then f is degenerate.* Accordingly you can be satisfied that it reads well.
- Every sentence in English needs a verb in each clause. In light of the previous bullet point, one should be careful to ensure that the verb is present in English and not represented as a mathematical operation. In the previous bullet point example sentence, some authors may have written:

  If

  $$P[f] := \int_0^1 f(x)\mathrm{d}x = 3,$$

  then f is degenerate.

  As such := (defined to be) is being treated as a verb. However, this is stylistically bad as the verb is not visible in English.
- Don't double define. E.g. *Suppose we define $f(x) := \sin(x)$*. Similarly, don't say in English what you are already saying symbolically. The only exception to that is when a verb is needed in the sentence which is in principle already symbolically displayed. For example, write

  Suppose we multiply $f \times g$ and its value is equal to h, then.....

  or

  Suppose we have $f \times g = h$, then .....

  but don't write

  Suppose $f \times g = h$, then .....

  as the verb in the main clause is apparently missing.
- When you define functions, be clear about their domain and their range, by working this into the English surrounding their definition. Similarly when equalities are true for a range of values, be sure to say so in a grammatically correct way.

Completeness of arguments

Don't fall into the trap of writing to the point that you think you are convinced things are true. Don't be lazy, finish the argument to the bitter end. A proof is a proof and each step must be correctly justified. But also see earlier remarks. If you are unsure how to finish an argument or have run out of time, then say so in your document. Indicating in a tidy and coherent way what more there is to do and why your proof is lacking completion.

Citations and repeating others' arguments

There are endless documents on how to correctly cite and create bibliographies. I am not intending to dwell on the technicalities of that here. Moreover, I am not going to stress one style over another in this respect. However, it is worth saying something about the culture of citing others.

Science has begun to move at an alarmingly fast pace. Contributions come from all over the globe and are increasingly accessible via the internet. There is no longer an excuse for missing out a reference, and other mathematicians become irritated when they are not deservingly cited.

Sometimes there are too many papers to cite when discussion a particular method or technique or history to a problem. In that case one should judiciously choose who to cite. However, in other situations when the preceding history of your mathematical contribution has a handful of clearly identified contributors, make sure you cite them in a way that is commensurate with their contributions.

Sometimes, for the sake of clarity, there is value in repeating (parts of) arguments that have already appeared in other papers. Similarly, there are occasions when making minor changes to an existing line of reasoning or methodology can form part of your proof. In that case, you must make reference to where such arguments have appeared before, or give credit to the authors who first introduced the genre of reasoning.

Don't be lazy when you are citing other author's work. Set up your bibliography properly from the very start and use the \cite{ } command accordingly. Don't say "by John's strong convergence result..." (unless this is how it is formally known, in which case it should be capitalised). If you get into the habit of citing correctly as you type up your notes, then recycling text into a research paper or the final version your thesis will be much easier.

Do as I say and not as I do

Finally, your supervisor may not observe all of the above advice. This does not mean you have the right to copy their sloppyness. Your supervisor may also speak with a Mancunian accent and swear a lot, but this does not mean you need to speak with a Mancunian accent and swear a lot too.