Control cost for heat-like equations in the (de-)homogenization limit

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Abstract:
The controlled heat equation with bounded potential V in $L^2(\mathbb{R}^d \times [0,T])$ is

$$\partial_t w - (\Delta + V)w = \mathbf{1}_S f, \quad w(0) \in L^2(\mathbb{R}^d).$$

Here $S \subset \mathbb{R}^d$ is called the control set, and $f \in L^2(S \times [0,T])$ is the control function. If $S \subset \mathbb{R}^d$ is in some sense "thick", then the above system is null-controllable for all times $T > 0$ which means that for every initial state $w(0) \in L^2(\mathbb{R}^d)$, there exists a control function f driving the system to zero at time T. We derive sharp upper bounds on the norm of the required control input f (the control cost) in terms of the model parameters T, V, and in terms of parameters describing the geometry of $S \subset \mathbb{R}^d$.

Our proofs rely on two ingredients: (1) An abstract estimate on the control cost in abstract control problems. (2) A scale-free quantitative unique continuation principle, recently proved together with Nakic, Tautenhahn, and Veselić. Combining these two ingredients allows us to study the behaviour of the control cost in certain limits. More precisely, we study its asymptotic behaviour when the large scale density of the control set $S \subset \mathbb{R}^d$ remains constant while local fluctuations in the density become either small (homogenization) or large (de-homogenization).

Similar results hold (with obvious restrictions) for analogous systems on bounded domains.

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