Alexander Kiselev (St. Petersburg State University, Russia)

Norm-resolvent convergence to zero-range models with internal structure in models with strong inhomogeneities

In this talk, I will consider a prototype large-coupling transmission problem, posed on a bounded domain, containing a "low-index" inclusion located at a positive distance to the boundary.

Mathematically, this is modelled by a "weighted" Laplacian $-a_{\pm}\Delta$, where $a_+ = 1$ (the weight on the containing domain) and $a_- \equiv a$ (the weight on the inclusion) is assumed to be large, $a \gg 1$. This is supplemented by the Neumann boundary condition on the outer boundary and "natural" continuity conditions on the interface (i.e., the inclusion boundary). A formal asymptotic argument suggests that eigenvalues of this operator should converge (as $a \to \infty$) to those of the so-called "electrostatic problem" discussed in [2] and references therein (although the existing results do not quite yield spectral convergence). The operator-theoretic approach we use in [1], based on our analysis of critical-contrast periodic composites as presented in [3], allows one to improve these results in two respects: a) our estimates are of the operator-norm resolvent type, implying, in particular, the uniform convergence of the associated spectra in any compact set of $\mathbb{C}$; b) our estimates are uniform with respect to the "contrast" parameter a and are order-sharp, i.e. the rate of convergence in terms of $a \to \infty$ cannot be improved further.

Next, I will discuss the high-frequency regimes, where it proves insufficient to consider the main order term of the resolvent asymptotics only. Physically, these are related to the transitional regimes from frequencies characteristic of Rayleigh scattering (i.e., where the wavelength in the inclusion is much larger than its size) to the ones of Mie scattering (where this wavelength is comparable to the size of the inclusion). Here I will argue that an effective model akin to that of a zero-range perturbation with internal structure (introduced in 1980s by Boris Pavlov) takes place of the effective model of [1].

References

[1] K. Cherednichenko, A. Kiselev, L. Silva, 2021. Operator-norm resolvent asymptotic analysis of continuous media with low-index inclusions. *To appear in Mathematical Notes,* arXiv: 2010.13318.

[2] H. Ammari, H. Kang, K. Kim, H. Lee, 2013. Strong convergence of the solutions of the linear elasticity and uniformity of asymptotic expansions in the presence of small inclusions. *J. Differential Equations* 254: 4446–4464.

[3] K. D. Cherednichenko, Yu. Yu. Ershova, A. V. Kiselev, 2020. Effective behaviour of critical-contrast PDEs: microresonances, frequency conversion, and time dispersive properties. I. *Communications in Mathematical Physics* 375: 1833–1884.