Mikhail Cherdantsev (Cardiff University)

Superexponential decay of defect eigenfunctions in one-dimensional high-contrast media

Following a number of recent studies of resolvent and spectral convergence of non-uniformly elliptic families of differential operators describing the behaviour of periodic composite media with high contrast, we study the corresponding one-dimensional version that includes a "defect": an inclusion of fixed size with a given set of material parameters. It is known that in the purely periodic case without the defect the spectrum and its limit, as the period ε goes to zero, has a band-gap structure. We consider a sequence of eigenvalues λ_ε that are induced by the defect and converge to a point λ_0 located in a gap of the limit spectrum for the periodic case. We show that the corresponding eigenfunctions are "extremely" localised to the defect, in the sense that the localisation exponent behaves as $\exp(-\nu/\varepsilon)$, $\nu > 0$, which has not been observed in the existing literature. As a consequence, we argue that l_0 is an eigenvalue of a certain limit operator defined on the defect only. In the two- and three-dimensional configurations whose one-dimensional cross-sections are described by the setting considered, this implies the existence of propagating waves that are localised to the defect. We also show that the unperturbed operators are norm-resolvent close to a degenerate operator on the real axis, which is described explicitly. This is joint work with Kirill Cherednichenko and Shane Cooper.