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Invariant subspaces of elliptic systems

Consider an elliptic self-adjoint pseudodifferential operator A acting on m-columns of half-densities on a closed manifold M, whose principal symbol is assumed to have simple eigenvalues. We show existence and uniqueness of m orthonormal pseudodifferential projections commuting with the operator A and provide an algorithm for the computation of their full symbols, as well as explicit closed formulae for their subprincipal symbols. Pseudodifferential projections yield a decomposition of $L^2(M)$ into invariant subspaces under the action of A modulo $C^\infty(M)$. Furthermore, they allow us to decompose A into m distinct sign definite pseudodifferential operators.

We use our pseudodifferential projections to show that the spectrum of A decomposes, up to an error with superpolynomial decay, into m distinct series, each associated with one of the eigenvalues of the principal symbol of A. These spectral results are then applied to the study of propagation of singularities in hyperbolic systems.

Time permitting, I will explain how similar results cannot be obtained by (microlocal) diagonalisation of our system.

The talk is partly based on joint works with G. Rozenblum, N. Saveliev and D. Vassiliev.