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A singularly perturbed transmission problem for the Helmholtz equation

Let Ω^i, Ω^o be bounded open connected subsets of $\mathbb{R}^n$ that contain the origin, and let $\Omega(\epsilon) \equiv \Omega^o \setminus \epsilon\overline{\Omega^i}$ for small $\epsilon > 0$. We consider a linear transmission problem for the Helmholtz equation in the pair of domains $\epsilon\Omega^i$ and $\Omega(\epsilon)$ with Neumann boundary conditions on $\partial\Omega^o$. Under appropriate conditions on the wave numbers in $\epsilon\Omega^i$ and $\Omega(\epsilon)$ and on the parameters involved in the transmission conditions on $\epsilon\partial\Omega^i$, the transmission problem has a unique solution $(u^i(\epsilon,\cdot), u^o(\epsilon,\cdot))$ for small values of $\epsilon > 0$. Here $u^i(\epsilon,\cdot)$ and $u^o(\epsilon,\cdot)$ solve the Helmholtz equation in $\epsilon\Omega^i$ and $\Omega(\epsilon)$, respectively. We prove that if $x \in \Omega^o \setminus \{0\}$, then $u^o(\epsilon, x)$ can be expanded into a convergent power expansion of ϵ, $\kappa_n \epsilon \log \epsilon$, $\delta_{2,n} \log^{-1} \epsilon$ for ϵ small enough. Here $\kappa_n = 1$ if n is even and $\kappa_n = 0$ if n is odd and $\delta_{2,2} \equiv 1$ and $\delta_{2,n} \equiv 0$ if $n \geq 3$.

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