Marta Lewicka (University of Pittsburgh)

Quantitative immersability of Riemann metrics and the infinite hierarchy of prestrained shell models

We present results that relate the following two contexts:

(i) Given a Riemann metric G on a thin plate, we study the question of what is its closest isometric immersion, with respect to the distance measured by energies E^h which are modifications of the classical nonlinear three-dimensional elasticity.

(ii) We perform the full scaling analysis of E^h, in the context of dimension reduction as the plate's thickness h goes to 0, and derive the Gamma-limits of $h^{-2n}E^h$ for all n. We show the energy quantization, in the sense that the even powers $2n$ of h are the only possible ones (all of them are also attained).

For each n, we identify conditions for the validity of the corresponding scaling, in terms of the vanishing of Riemann curvatures of G up to appropriate orders, and in terms of the matched isometry expansions. Problems that we discuss arise from the description of elastic materials displaying heterogeneous incompatibilities of strains that may be associated with growth, swelling, shrinkage, plasticity, etc. Our results display the interaction of calculus of variations, geometry and mechanics of materials in the prediction of patterns and shape formation.