Lyonell Boulton (Heriot-Watt University)

Second order Sobolev embeddings and non-linear bi-Laplacians

In this informal talk I describe recent progress on the analytic determination of the singular numbers of second order Sobolev embeddings with respect to two different Lebesgue norms, L^p and L^q for general $p, q > 1$. The argumentation relies on the study of a non-linear biharmonic eigenvalue problem on the unit segment subject to Navier boundary conditions. By establishing existence, uniqueness and symmetry of periodic solutions, I will then give an explicit description of the singular numbers in the case of homogeneous Dirichlet boundary conditions. For $q = p'$, these are given in terms of the composition of two first order embeddings and therefore they have an explicit expression in terms of known special functions. However, it is remarkable that this is not so the case when $q \neq p'$ as the eigenfunctions are related to a new family of functions which are solution of an irreducible coupled system of integral equations. The results reported are joint work with Jan Lang available at arXiv:2204.04703.