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AN INTERLACING CRITERION
FOR MATRIX-VALUED MEROMORPHIC HERGLOTZ FUNCTIONS

This talk is about meromorphic Herglotz functions, i.e., functions that are meromorphic on all of \mathbb{C} , real-valued on \mathbb{R} and whose values in the upper half-plane have non-negative imaginary part. They have the characteristic property that their zeroes and poles are all real, simple, and interlacing: between any two zeroes there is a pole and between any two poles there is a zero. Conversely, any meromorphic function that is real-valued on \mathbb{R} and has real, simple, and interlacing zeroes and poles is a meromorphic Herglotz function if it does not grow too fast.

Now consider an $(n \times n)$ -matrix-valued meromorphic Herglotz function, with the goal of establishing a similar interlacing criterion. One needs to find the right interpretation of “zeroes and poles”, and give a suitable generalisation of the term “interlacing”. I will show that such notions exist and that a matrix-valued real meromorphic function with real entries on \mathbb{R} is Herglotz if and only if it satisfies an interlacing condition and does not grow too fast.