Extensions of the JKR Contact Theory to Elastic Punches of Arbitrary Shapes, 2D Membranes and Thin Layers

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Contact probing is the preferable method for studying mechanical properties of very small volumes of materials. Usually contact problems are considered without taking into account adhesive forces. Indeed, adhesion between contacting bodies has usually a negligible effect on surface interactions at the macro-scale. However, nano-world is the **'*Sticky Universe*'**, and adhesion caused by van der Waals forces is a universal molecular phenomenon, hence problems of nanomechanics should normally take into account adhesive interactions. The JKR (Johnson-Kendall-Roberts) theory of adhesive contact was introduced for frictionless contact between a rigid spherical indenter and an isotropic half-space.

First the JKR theory is extended for arbitrary convex, blunt axisymmetric body, in particular to the case of the punch shape being described by monomial (power-law) punches of an arbitrary degree $d \geq 1$. The results are also extended for samples whose materials can be described by linear or linearized models having rotational isotropy of its mechanical properties like transversely isotropic or homogeneously prestressed materials. Further the frictionless JKR model is generalized to the case of no-slip boundary conditions. It is shown that regardless of the boundary conditions, the solution to the contact problems for monomial punches is reduced to the same dimensionless relations between the actual force, displacements and contact radius

Then a new theory of adhesive contact for graphene and other two-dimensional materials is presented. Analogies to both non-adhesive and adhesive contact problems for 2D stretched membranes are formulated and solved. General nonlinear relations among the actual force, displacements and contact radius between a sticky membrane and an arbitrary axisymmetric indenter are derived.

Finally contact problems for a thin compressible elastic layer attached to a rigid support are studied. Assuming the thickness of the layer is much less than characteristic dimension of the contact region and using the direct GKN (Goldenveizer-Kaplunov-Nolde) approach, it is shown that the leading order approximation to the non-adhesive contact problems is equivalent to contact problem for a Winkler-Fuss elastic foundation. The JKR approach has been generalized to the case of blunt axisymmetric indenters having arbitrary shape. The explicit expressions are derived for the values of the pull-off force and for the corresponding critical contact radius.

Professor F.M. Borodich holds several degrees in Mechanics and Applied Mathematics (MSc 1979, PhD 1984, DSc 1990) awarded by the Department of Mechanics and Mathematics at Moscow State University, and MSc degree in Civil Engineering (Moscow 1985). In 1979-1994 he worked in Moscow. In 1995-2001 he held various research and teaching positions at several universities including Cambridge, Hannover, Glasgow, and Barcelona. In 2001-2003 he was a visiting professor at Northwestern University. Since 2004, he is a Professor at Cardiff University. His research interests are in a number of areas of Engineering, Mechanics and Applied Mathematics related to modelling real-world processes. In particular, he obtained new results in static and dynamic contact problems, self-similar models and scaling in solid mechanics, nanoindentation, nanomanufacturing, nanotribology, mechanics of adhesive contact, descriptions of rough surfaces, modelling of dry friction and friction-induced oscillations, and other related topics.