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The minimal regularity Dirichlet problem for degenerate elliptic PDEs beyond symmetric coefficients

We prove that the Dirichlet problem for degenerate elliptic equations with nonsymmetric coefficients on Lipschitz domains is solvable when the boundary data is in weighted L^p for some $p < \infty$. The result is achieved without requiring any structure on the coefficient matrix, thus allowing for nonsymmetric coefficients, in which case $p > 2$ becomes necessary. We build on the groundbreaking result obtained by Hofmann, Kenig, Mayboroda and Pipher for uniformly elliptic equations, by allowing for the bound and ellipticity on the coefficients to degenerate under the control of a Muckenhoupt weight. We also adopt an alternative strategy, which is outlined in their work, although the crucial technical estimate is not at all an obvious extension of the uniformly elliptic theory. In this approach, a Carleson measure estimate for bounded solutions is established directly. This allows us to avoid good-λ inequalities entirely, and thus apply a Dhalberg–Kenig–Stein pull-back based on an L^2-Hodge decomposition instead of an $L^{2+\epsilon}$-version. The result is then combined with an oscillation estimate for solutions, which allows us to avoid the method of ϵ-approximability, to deduce that the degenerate harmonic measure is in the A_∞-class with respect to weighted Lebesgue measure on the domain boundary. This is joint work with Steve Hofmann and Phi Le.