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The application of optimal control to treatment planning in proton radiation therapy

Proton radiation therapy is a novel method of treating inoperable cancers, that uses a beam of protons to irradiate cancerous cells. Unlike traditional radiotherapy, which involves high energy photons, the rate of linear energy transfer of a proton is non-linear (forming a Bragg peak) which means that the majority of the dose is deposited in the target volume with minimal damage to the surrounding tissue [1].

In treatment planning the objective is to find the optimal beam angle, shape and energies, such that a given cost functional is minimised; this cost functional is typically given as the weighted sum of functionals which depend on the dose. Typically, each summand is the norm of the difference between the dose and a target dose, in a given region [2].

This cost functional is minimised subject to the constraint the the behaviour of the travelling protons is governed by the hyperbolic Boltzmann transport equation. This produces a triplet of equations known as the Primal, Dual and Control conditions [2]. We seek to demonstrate the stability of the solutions of a simplified form of these equations, by considering the notion of hypocoercivity, developed by Villani [3,4,5].

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