Research statement

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Below I give an outline of my current research interests. This is followed by my research track record.

Research interests:

1. *Wave propagation; Spectral properties of differential operators emerging in linearised elasticity and electromagnetism.*

Here I am particularly interested in the effects of interaction of the lengthscale associated with the wave motion and the characteristic size of the heterogeneities. In the language of the operator theory, the related range of questions concerns the asymptotic behaviour of operator groups/semigroups with respect to a set of parameters (heterogeneity size, contrast between components in a composite *etc.*). This approach leads to two kinds of results:

a) Understanding "effective" characteristics of media in relation to the wave motion, *e.g.* the speed of the wave motion along a boundary ("surface wave") in a layered medium;

b) Considering special "critical" scalings between the parameters involved leads to new formulations that possess properties unattainable by homogeneous media, *e.g.* localised modes in photonic crystals).

(Collaborators: M. Cherdantsev, Y. Ershova, A. Kiselev, J. Roberts; PhD student: W. Graham)

2. *Multiscale analysis; Homogenisation of families of PDEs with non-uniformly elliptic coefficients, high-contrast media and their spectral properties.*

In the "stationary" framework (*i.e.* for equilibrium equations rather than equations of dynamics), the effective description of a composite can be viewed as a convergence proof for a family of operators in a suitable representation of the Hilbert space (linear PDEs), or for a family of integral functionals (non-quadratic, possibly non-convex) on a Banach space. The conventional notions of convergence ("G-convergence", "H-convergence", "Γ-convergence") are not sufficient (at least in the form they are known by) for control of problems involving strong scale interactions, or effects of macroscopic "resonances" at the microscale.

(Collaborators: M. Cherdantsev, P. Dondl, A. Kiselev, I. Velčić)

3. Four interconnected themes of EPSRC Fellowship "Mathematical foundations of metamaterials: homogenisation, dissipation and operator theory", which I have been awarded for July 2014 – June 2019. This is an extensive programme of research involving 2 postdoctoral researchers, 2 years each (Y. Ershova, M. Waurick):

A. *Applications of the theory of dissipative operators to estimating convergence rates in homogenisation; Norm-resolvent estimates.*

B. *Scattering theory; Effective descriptions of periodic composite media via asymptotics of their scattering matrix.*

C. *The study of systems of Maxwell equations in dissipative media and the related spatial localisation effects ("cloaking").*

D. *Time-dispersive media and their derivation by multiscale analysis.*

(Collaborators: M. Cherdantsev, A. Kiselev, S. Naboko, L. Silva, I. Velčić)

The asymptotic analysis of differential operators and boundary-value problems is a natural tool for understanding the macroscopic behaviour of inhomogeneous media. Indeed, the ratio ε between the characteristic size of the microstructure and the macroscopic length-scale, such as the typical wavelength of a propagating wave packet, is a natural parameter for asymptotic expansions of the solutions to continuum mechanics problems. This asymptotic analysis needs to account for the energy exchange across microscopic interfaces in the medium, which leads to a non-trivial dependence on the wave frequency.

Prior to the Fellowship, asymptotic methods in the analysis of composites did not fully take advantage of the existing operator-theoretic tools for the study of resolvents of boundary-value problems of mathematical physics. In particular, the idea of "reducing the problem to the boundary/interfaces" via suitable Dirichlet-to-Neumann (DN) operators, which is used for a range of inverse problems and in numerical methods, had been largely overlooked as a possible multiscale analysis technique.

Motivated by a number of previous exciting challenges in materials science and wave propagation, we set out to exploit the power of abstract techniques for the analysis of DN operators in the study of families of parameter-dependent partial differential equations (PDE). The key challenge that my group has addressed during the Fellowship was to formulate new strategies for the analysis of composites that yield ε-order-sharp effective theories. As a result, the following two major goals have been achieved:

i) Asymptotic problems of different kinds (resolvent, direct/inverse spectral, scattering) and for a variety of geometries (high-contrast composites, perforated media, thin structures), which had previously been studied within separate research streams, can now be viewed under the unifying umbrella of general operator theory. At the heart of the new technique is an asymptotic decomposition of the resolvent operator $(A_\varepsilon - z)^{-1}$ that utilises the additivity of the DN map with respect to a splitting of the composite into components. This allows us to express the asymptotic properties of A_ε, as $\varepsilon \to 0$, purely in terms of the behaviour of certain DN operators, understood as analytic functions of z.

ii) We have developed a general approach to the asymptotic analysis of composites that yields a quantitative description of macroscopic time-dispersion properties of inhomogeneous media on the basis of their microstructure. This has enabled us to pioneer an explicit link between composites and metamaterials, by interpreting the latter as time-dispersive media obtained by averaging critical-contrast composites. In this way our approach plays a fundamental role in the description of metamaterials, where properties such as negative refraction and cloaking are now simple consequences of our operator-theoretic framework.

4. *Variational methods in homogenisation; Γ-convergence of functionals describing composite media in nonlinear elasticity.*

I have developed a technique of "two-scale Γ-convergence", which is convenient for understanding the effective behaviour of composites whose constituents are described by non-convex energy densities that allow a high degree of contrast between deformation gradients at different material points and/or in different directions. I also work on the analysis of the behaviour of heterogeneous plates in the "bending" regime for the scaling between elastic energy and plate thickness. I have shown that when these are small in comparison with the period of the composite, an additional constraint emerges in the effective description, which corresponds to the intuitive notion that the plate mid-surface is an isometric immersion at the miscroscale.

(Collaborators: M. Cherdantsev, S. Neukamm)

5. *Metamaterials: negative refraction, transformation media, thin structures.*

This is a variety of approaches aimed at obtaining new differential equations that describe continuous media with constitutive parameters taking negative values (usually in a certain frequency interval). It has been a subject of intensive analytical effort recently to show that, say, periodic elastic composites with voids whose volume fraction is close to one, exhibit on the one hand some Cosserat-type properties, and on the other hand, have spectra with infinitely many band gaps opening at high frequencies. Another example involves a combination of a special parameter-dependent change of variables and a homogenisation procedure in order to produce a chiral effective medium.

(Collaborators: Y. Ershova, A. Kiselev; PhD student: S. D'Onofrio)

6. *Liquid crystals: asymptotic analysis of stable singularities in liquid crystals and homogenisation.*

This research direction involves combining advanced techniques in homogenisation of multiscale media, analysis of stable singularities in liquid crystals and existence theorems in calculus of variations. We analyse the overall, averaged, properties of two types of multi-scale media, namely periodic composites and thin structures, in order to derive a new class of energy functionals in the theory of liquid crystals. This class includes some of the functionals used in the existing approaches but also contains new expressions for the energy density, which have not been previously studied by mathematical methods. We expect the related general description for the energy to be useful in understanding how the inclusion of nanoparticles affects the performance of liquid crystal devices. We also investigate the existence of minimisers for the derived class of functionals, study their mathematical properties and propose new applications based on these properties.

(Collaborators: A. Majumdar, J. Sivaloganathan; PhD student: A. Pim)

Research track record:

My research is in the general area of asymptotic analysis of solutions to differential equations arising in the study of continuous media (in particular, electromagnetism and elasticity). The notion of an asymptotic expansion, when embedded into the current knowledge in neighbouring disciplines (operator theory, calculus of variations) is a source of new analytical techniques I develop, which provide valuable insight into frontline problems of physics and engineering.

As an early-career researcher, I published a series of works on the derivation of effective material properties of multi-scale media, both in the linear and nonlinear context, and of the effect of heterogeneities and boundaries on wave propagation. The main highlights of that period (during which I worked at Oxford, Cambridge and Cardiff) are summarised as follows:

- The development of a new method, two-scale Γ-convergence, for the analysis of a wide class of nonlinear multiscale problems, including those with degeneracies.
- A series of new results for the existence of guided, surface, and interface waves in linearised elasticity, and the analysis of their properties.
- The development of a rigorous framework for phenomenological theories of size effects in the elasticity (linearised and nonlinear) and plasticity of continuous media with microstructure.

Building on the above work and expanding my expertise led to a better understanding of the mathematical structures involved in averaging and has generated several research hypotheses for the multiscale analysis of composite media. This has enabled me to push the boundaries of this area and lead the use of operator-theoretic techniques stemming from the generalised perspective on Green's identity for boundary-value problems for differential equations. Particular achievements since 2012:

- New asymptotic approach to periodic problems with degeneracies; proof of sharp operator-norm resolvent estimates; new rationale for homogenisation via "operator asymptotics".
- Bending of periodic nonlinearly elastic plates: settling the case where plates are described by isometric surfaces on the microscale; derivation of a new set of nonlinear constraints.
- Establishing a link between abstract results of operator theory (Krein formula, functional model for non-selfadjoint operators) and the asymptotic analysis of differential operators in materials science.
- Construction of the functional model for extensions of symmetric operators and solution of the inverse scattering problems for quantum graphs.

- Stochastic formulation of the homogenisation problem for degenerate PDEs and new results for the overall behaviour and spectral properties of high-contrast composites.
- Establishing a link between time/frequency dispersion in the equations of continuum mechanics and averaging (coarse-graining) in heterogeneous media.

My research group currently consists of three PhD students (S. D'Onofrio, W. Graham, A. Pim), and a UUKi Rutherford Fellow (R. Castaneira). All my former postdocs (M. Cherdantsev, Y. Ershova, K. Kuliev, M. Waurick) now have permanent university positions, and another (S. Cooper) holds the Willmore Fellowship at Durham University. One of my former students, J. Evans, successfully obtained his PhD in 2015 and another, M. Lewis, has submitted his PhD thesis for examination.

I closely work with several internationally recognised leaders in applied analysis and spectral theory: University of Freiburg (P. Dondl), University of Zagreb (I. Velčić), NAM-IIMAS, Mexico (L. Silva), St. Petersburg (S. Naboko).

I have established and run the weekly research seminar "Asymptotics, Operators and Functionals", which in 3 years has hosted around 60 speakers, including colleagues from Bath (across Mathematical Sciences, Physics and Engineering), as well as about 30 visitors from UK and abroad. The format of the seminar, where the talks are aimed at going into some detail on a research topic, has had a stimulating effect on my research group, the department, and the research field as a whole: a number of speakers have reported following up on the seminar discussions in their subsequent research.

In addition, I organised the international workshop on the project-related subjects "Operators, Operator families and Asymptotics" (`http://people.bath.ac.uk/kc525/OOFA/OOFA.html`) at the University of Bath in May 2016. The workshop featured 16 prominent speakers from UK, Europe, US and Mexico, nominated by a programme committee of world-leading experts and about 30 non-speaking participants. I aim at developing this into a triennial workshop series, and the second instalment is taking place in January 2019.

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