

# Operators, Operator Families, and Asymptotics II

14–17 January 2019

DEPARTMENT OF MATHEMATICAL SCIENCES, UNIVERSITY OF BATH

## Programme of Talks

### Monday 14 January

**9:50–10:20** ARRIVAL, COLLECTION OF WELCOME PACKS, TEA/COFFEE:  
**Level 1 Atrium**

**10:20–10:25** WELCOME **Wolfson Lecture Theatre**

**10:25–11:25** IGOR VELČIĆ (UNIVERSITY OF ZAGREB, CROATIA): Operator norm long-wave asymptotics for elastic plates with rapidly oscillating periodic properties

**11:25–11:35** QUESTIONS AND CHANGEOVER

**11:35–12:35** LUIS SILVA (IIMAS-UNAM, MÉXICO): Sampling, oversampling, and subsampling in de Branges spaces arising from differential operators

**12:35–12:40** QUESTIONS

**12:40–14:00** LUNCH: **Level 1 Atrium**

**14:00–15:00** PATRICK DONDL (UNIVERSITY OF FREIBURG, GERMANY): The effect of forest dislocations on the evolution of a phase-field model for plastic slip

**15:00–15:05** QUESTIONS

**15:05 – 15:35** TEA/COFFEE: **Level 1 Atrium**

**15:35–16:35** STEPHEN SHIPMAN (LOUISIANA STATE UNIVERSITY, USA): Reducibility of the Fermi surface for periodic quantum-graph operators

**16:35–16:40** QUESTIONS



**Tuesday 15 January**

**Wolfson Lecture Theatre 4W1.7**

**9:25–10:25** MONICA MUSSO (UNIVERSITY OF BATH, UK): Gluing methods for vortex dynamics in Euler flows  
**10:25–10:30** QUESTIONS

**10:30 – 11:00** TEA/COFFEE: **Level 1 Atrium**

**11:00–12:00** SABINE BÖGLI (IMPERIAL COLLEGE LONDON, UK): Schrödinger operator with non-zero accumulation points of complex eigenvalues  
**12:00–12:10** QUESTIONS AND CHANGEOVER

**12:10–13:10** PETER HORNING (TU DRESDEN, GERMANY): Narrow ribbons in nonlinear three-dimensional elasticity  
**13:10–13:15** QUESTIONS

**13:15–14:30** LUNCH: **Level 1 Atrium**

**14:30–15:30** MIKHAIL CHERDANTSEV (CARDIFF UNIVERSITY, UK): Stochastic homogenisation of high-contrast media  
**15:30–15:35** QUESTIONS

**15:35 – 16:05** TEA/COFFEE: **Level 1 Atrium**

**16:05–17:05** MATTHIAS LANGER (UNIVERSITY OF STRATHCLYDE, UK): Spectral enclosures for boundary value problems  
**17:05–17:10** QUESTIONS



**Wednesday 16 January**

**Wolfson Lecture Theatre 4W1.7**

**9:25–10:25** NADIA ANSINI (SAPIENZA UNIVERSITY OF ROME, ITALY): Scale-bridging in complex systems: a variational approach to statics and dynamics

**10:25–10:30** QUESTIONS

**10:30 – 11:00** TEA/COFFEE: **Level 1 Atrium**

**11:00–12:00** PETR SIEGL (QUEEN'S UNIVERSITY BELFAST, UK): Non-accretive Schrödinger operators and exponential decay of their eigenfunctions

**12:00–12:10** QUESTIONS AND CHANGEOVER

**12:10–13:10** GRIGORY PANASENKO (INSTITUT CAMILLE JORDAN, SAINT ÉTIENNE/LYON, FRANCE): Reconstruction of the pressure in the method of asymptotic partial decomposition for the flows in thin domains

**13:10–13:15** QUESTIONS

**13:15–14:30** LUNCH: **Level 1 Atrium**

**14:30–15:30** NADIA SIDOROVA (UNIVERSITY COLLEGE LONDON, UK): Localisation and delocalising in the parabolic Anderson model

**15:30–15:35** QUESTIONS

**15:35 – 16:05** TEA/COFFEE: **Level 1 Atrium**

**16:05–17:05** GIOVANNI S. ALBERTI (UNIVERSITY OF GENOA, ITALY): Essential spectrum for Maxwell's equations

**17:05–17:10** QUESTIONS



Thursday 17 January

Wolfson Lecture Theatre 4W1.7

**9:25–10:25** DIRK HUNDERTMARK (KARLSRUHE UNIVERSITY OF TECHNOLOGY, GERMANY): Bounds on quantum systems with Coulomb repulsion at the edge of their existence

**10:25–10:30** QUESTIONS

**10:30 – 11:00** TEA/COFFEE: **Level 1 Atrium**

**11:00–12:00** RAFAEL DEL RÍO (IIMAS-UNAM, MÉXICO): Inverse spectral problems for mass-spring systems

**12:00–12:05** QUESTIONS

**12:05–13:00** LUNCH: **Level 1 Atrium**

END OF MEETING



## Abstracts of Talks

GIOVANNI S. ALBERTI (UNIVERSITY OF GENOA, ITALY): Essential spectrum for Maxwell's equations

This is joint work with M. Brown, M. Marletta and I. Wood. We study the essential spectrum of operator pencils associated with anisotropic Maxwell equations

$$\begin{cases} \operatorname{curl} H = -i(\omega\varepsilon + i\sigma)E & \text{in } \Omega, \\ \operatorname{curl} E = i\omega\mu H & \text{in } \Omega, \\ E \times \nu = 0 & \text{on } \partial\Omega, \end{cases}$$

with electric permittivity  $\varepsilon$ , magnetic permeability  $\mu$  and conductivity  $\sigma$ , in a finitely connected unbounded domain  $\Omega$ . The main result is that the essential spectrum of the Maxwell pencil is the union of two sets: namely, the spectrum of the pencil  $\operatorname{div}((\omega\varepsilon + i\sigma)\nabla \cdot)$ , and the essential spectrum of the Maxwell pencil with constant coefficients. We expect the analysis to be of more general interest and to open avenues to investigation of other questions concerning Maxwell's and related systems.

NADIA ANSINI (SAPIENZA UNIVERSITY OF ROME, ITALY): Scale-bridging in complex systems: a variational approach to statics and dynamics

In this talk I give an overview of my research in Multiscale Analysis of complex systems in Materials Science (perforated domain, thin films, phase transitions) and Variational Evolution Problems. Variational techniques and global minimisation have been proven to be very successful in many applications in materials science. The notion of  $\Gamma$ -convergence has been introduced to study the asymptotic behaviour of (global) minimizers of energy functionals in the limit when the parameters (related to the multiscale nature of the problem) get small. Even if  $\Gamma$ -convergence may fail in giving the correct description of the effect of local minimizers, variational techniques can be still applied to follow the pattern of the local minimizers of energy functionals. In this seminar I will present also some recent results on microstructure evolution.

SABINE BÖGLI (IMPERIAL COLLEGE LONDON, UK): Schrödinger operator with non-zero accumulation points of complex eigenvalues

In the 1960s Pavlov studied Schrödinger operators on the half-line with potentials that decay at infinity, subject to Robin boundary conditions at the endpoint. Using inverse spectral theory, he proved the existence of a real potential and a non-selfadjoint boundary condition so that the Schrödinger operator has infinitely many non-real eigenvalues that accumulate at an arbitrary prescribed point of the essential spectrum (the positive half-line). Since then, it has been an open question whether these results can be modified so that the non-selfadjointness is not coming from the boundary conditions but from a non-real potential. In this talk we consider Schrödinger operators on the whole Euclidean space (of arbitrary dimension) or on the half-space, subject to real Robin boundary conditions. I will present the construction of a non-real potential that decays at infinity so that the corresponding Schrödinger operator has infinitely many non-real eigenvalues accumulating at every point of the essential spectrum.

MIKHAIL CHERDANTSEV (CARDIFF UNIVERSITY, UK): Stochastic homogenisation of high-contrast media

Using a suitable stochastic version of the compactness argument of [V. V. Zhikov, 2000. On an extension of the method of two-scale convergence and its applications. *Sb. Math.*, **191**(7–8), 973–1014], we develop a probabilistic framework for the analysis of heterogeneous media with high contrast. We show that an appropriately defined multiscale limit of the field in the original medium satisfies a system of equations corresponding to the coupled “macroscopic” and “microscopic” components of the field, giving rise to an analogue of the “Zhikov function”, which represents the effective dispersion of the medium. We demonstrate that, under some lenient conditions within the new framework, the spectra of the original problems converge to the spectrum of their homogenisation limit.

PATRICK DONDL (UNIVERSITY OF FREIBURG, GERMANY): The effect of forest dislocations on the evolution of a phase-field Model for plastic slip

We consider a phase field model for dislocations introduced by Koslowski, Cuitino, and Ortiz in 2002. The model describes a single slip plane and consists of a Peierls potential penalizing non-integer slip and a long range interaction modeling elasticity. Forest dislocations are introduced as a restriction to the allowable phase field functions: they have to vanish at the union of a number of small disks in the plane. Garroni and Müller proved large scale limits of these models in terms of Gamma-convergence, obtaining a line-tension energy for the dislocations and a bulk term penalizing slip. This bulk term is a capacity stemming from the forest dislocations. In the present work, we show that the contribution of the forest dislocations to the viscous gradient flow evolution is small. In particular it is much slower than the timescale for other effects like elastic attraction/repulsion of dislocations, which, by recent results due to del Mar Gonzales and Monneau/Patrizi and Valdinoci is already slower than the time scale from line tension energy. Overall, this leads to an effective behavior like a gradient flow in a wiggly potential. On the other hand, when adding a driving force in the direction of increasing slip, one needs to spend the energy to overcome the obstacles. The forest dislocations thus act like a dissipation for increasing the amount of slip, but their effect on the propagation is absent when the amount of slip is decreasing.

DAVIT HARUTYUNYAN (UNIVERSITY OF CALIFORNIA, SANTA BARBARA, USA): On the asymptotics of the smallest eigenvalue of Lamé and related operators in thin domain

We study the asymptotes of the smallest eigenvalue of the Lamé operator in linear elasticity in thin domains in terms of the domain thickness. This problem is equivalent to finding optimal constants in Korn's inequalities. To that end, we introduce a new auxiliary problem and study it. The related operator for the new problem is an interpolation between the Lamé operator and the one arising from Korn's second inequality. I will present new classical result in this direction, and discuss the nonlinear problem. This is recent progress in the direction of determining the geometric rigidity of thin domains.

PETER HORNING (TU DRESDEN, GERMANY): Narrow ribbons in nonlinear three-dimensional elasticity

In the 1930s Sadowsky showed the existence of a developable Moebius strip. He proposed that the configuration assumed by such a strip can be computed by minimizing the bending energy. He further argued that the bending energy density is proportional to the square of the mean curvature of the surface, i.e., its Willmore energy density.

Wunderlich later formally justified the energy functional proposed by Sadowsky. His analysis showed that the Sadowsky functional can be recovered under an assumption of non vanishing curvature of the centerline of the strip.

First, we provide a rigorous derivation of the limit energy of such an inextensible, isotropic, elastic strip as the strip width goes to zero. Our analysis makes no a priori assumptions on the curvature of the centerline. The functional obtained in this way agrees with the classical Sadowsky functional, but only when the curvature of the centerline of the strip is large enough.

Then we present a derivation of this modified Sadowsky functional starting from three dimensional nonlinear elasticity. We discuss some of the difficulties which arise due to the absence of a hard isometry constraint in the original three dimensional model.

This is joint work with L. Freddi, M.G. Mora and R. Paroni.

DIRK HUNDERTMARK (KARLSRUHE UNIVERSITY OF TECHNOLOGY, GERMANY): Bounds on quantum systems with Coulomb repulsion at the edge of their existence

One of the best methods on getting upper bounds on the asymptotic exponential decay of wave functions for quantum systems is the Agmon method. This method works well for bound states with energies strictly below the infimum of the essential spectrum, and it is well-known not to work when one has an eigenvalue at the bottom of the essential spectrum. In a sense, one needs a "safety distance" of the eigenvalue and the bottom of the essential spectrum, in order that Agmon method can work. Otherwise, without this safety distance, one faces the problem of having to divide by zero when applying Agmon's method. So this case no explicit bounds on the decay rate of the eigenfunction are known, in general.

There are quantum systems with a long range Coulomb repulsion, which have eigenvalues at the edge of the essential spectrum. For example, Helium with strong enough Coulomb repulsion is such an example, as shown in an old work of Hofmann-Ostenhof and Simon. Recently, Frank, Lieb, and Seiringer gave an alternative proof of existence of a bound state at the edge of the essential spectrum in such cases. In these cases is expected, that the eigenfunction decays sub-exponentially, more precisely, like  $\exp(-C\sqrt{|x|})$ . In a joint work with Michal Jex and Markus Lange we give a simple proof of this conjecture.

MATTHIAS LANGER (UNIVERSITY OF STRATHCLYDE, UK): Spectral enclosures for boundary-value problems

In this talk I will consider spectral enclosures for certain boundary value problems. In particular, I will consider elliptic problems with non-local and non-self-adjoint boundary conditions and Schrödinger operators with potentials that are supported on hypersurfaces. In the latter example I shall also focus on the case when the coupling constant converges to zero.

MONICA MUSSO (UNIVERSITY OF BATH, UK): Gluing methods for vortex dynamics in Euler flows

A classical problem for the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. is that of finding regular solutions with highly concentrated vorticities around  $N$  moving *vortices*. The formal dynamic law for such objects was first derived in the 19th century by Kirkhoff and Routh. We devise a gluing approach for the construction of smooth  $N$ -vortex solutions. We capture in high precision the core of each vortex as a scaled finite mass solution of Liouville's equation plus small, more regular terms.

This work is in collaboration with J. Dávila, M. del Pino, J. Wei.

GRIGORY PANASENKO (INSTITUT CAMILLE JORDAN, UNIVERSITY OF LYON, FRANCE): Reconstruction of the pressure in the method asymptotic partial decomposition for the flows in thin domains

The method of asymptotic partial decomposition of thin domains was introduced in [1]. For the Stokes or Navier-Stokes equations in thin tube structures it is presented in [2], [3], [4]. This method uses the variational formulation “without pressure” and provides a special restriction of this formulation on the Sobolev subspace of vector-valued functions having the Poiseuille-like shape within the thin tubes at some small distance from the bifurcations. However for its numerical implementation a weak formulation “with pressure” is needed. The main result of the talk is related to the reconstruction of the pressure in the frame of this method. This reconstruction algorithm uses some PDEs on the graphs described in [5]. The result is obtained in collaboration with C. Bertoglio and C. Conca.

[1] Panasenko G., 1998. Method of asymptotic partial decomposition of domain, *Mathematical Models and Methods in Applied Sciences*, 8(1), 139–156.

[2] Panasenko G. P., 1998. Partial asymptotic decomposition of domain: Navier-Stokes equation in a tube structure, *C. R. Acad. Sci. Paris*, 326, Série IIB, 893–898.

[3] Panasenko G., 2005. *Multi-Scale Modelling for Structures and Composites*, Springer, Dordrecht.

[4] Panasenko G., Pileckas K., 2015. Asymptotic analysis of the non-steady Navier-Stokes equations in a tube structure. I. The case without boundary layer-in-time. *Nonlinear Analysis, Series A, Theory, Methods and Applications*, 122, 125–168.

[5] Panasenko G., Pileckas K., 2014. Flows in a tube structure: equation on the graph, *Journal of Mathematical Physics*, 55, 081505.

RAFAEL DEL RÍO (IIMAS-UNAM, MÉXICO): Inverse spectral problems for mass-spring systems

This talk is about a Borg type inverse spectral problem for vibrating linear systems of point masses connected by springs. From the natural frequencies of vibration of the original system and a perturbation of it, we show how the masses and elastic coefficients of the springs can be reconstructed. To accomplish this, rank three perturbations of Jacobi matrices are considered and their associated Green's functions explicitly described in terms of spectral data. We give necessary and sufficient conditions for two given sets of points to be eigenvalues (natural frequencies) of the original and modified system respectively. This is joint work with Luis Silva and Mikhail Kudryavtsev.

NADIA SIDOROVA (UNIVERSITY COLLEGE LONDON, UK): Localisation and delocalising in the parabolic Anderson model

The parabolic Anderson problem is the Cauchy problem for the heat equation on the integer lattice with random potential. It describes the mean-field behaviour of a continuous-time branching random walk. It is well-known that, unlike the standard heat equation, the solution of the parabolic Anderson model exhibits strong localisation. In particular, for a wide class of iid potentials it is localised at just one point. However, in a partially symmetric parabolic Anderson model, the one-point localisation breaks down for heavy-tailed potentials and remains unchanged for light-tailed potentials, exhibiting a range of phase transitions.

LUIS SILVA (IIMAS-UNAM, MÉXICO): Sampling, oversampling, and subsampling in de Branges spaces arising from differential operators

An important mathematical technique in signal analysis and image processing is sampling theory. Kramer-type formulae for sampling functions in de Branges spaces (dB spaces) are obtained by combining properties of canonical selfadjoint extensions of the multiplication operator and the reproducing kernel. By recurring to the property that dB subspaces of dB spaces are totally ordered by inclusion, we extend the concepts of oversampling and subsampling to a wide class of dB spaces related to Schrödinger operators. Moreover, we obtain a fine tuning of sampling kernels on the basis of recent results for dB spaces arising from differential operators

PETR SIEGL (QUEEN'S UNIVERSITY BELFAST): Non-accretive Schrödinger operators and exponential decay of their eigenfunctions

We consider non-selfadjoint electromagnetic Schrödinger operators on arbitrary open sets with complex scalar potentials whose real part is not necessarily bounded from below. Under a suitable sufficient condition on the potentials, we introduce a Dirichlet realization as a closed densely defined operator with non-empty resolvent set and show that the eigenfunctions corresponding to discrete eigenvalues satisfy an Agmon-type exponential decay. Remarks on the spectral convergence of domain truncations and completeness of eigenfunctions will be given.

The talk is based on:

- [1] D. Krejčířík, N. Raymond, J. Royer and P. Siegl: *Non-accretive Schrödinger operators and exponential decay of their eigenfunctions*, Israel Journal of Mathematics, 221, (2017) 779-802
- [2] S. Bögli, P. Siegl, and C. Tretter: *Approximations of spectra of Schrödinger operators with complex potentials on  $\mathbb{R}^d$* , Communications in Partial Differential Equations 42, (2017), 1001-1041

STEPHEN SHIPMAN (LOUISIANA STATE UNIVERSITY, USA): Reducibility of the Fermi surface for periodic quantum-graph operators

The Fermi, or Floquet, surface for a periodic Schrödinger operator on a metric graph (a quantum graph) at a given energy level is an algebraic variety that describes all complex wave vectors admissible by the periodic operator at that energy. Its reducibility is intimately related to the construction of embedded eigenvalues supported by local defects. The rarity of reducibility is reflected in the fact that a generic polynomial in several variables cannot be factored. The "easy" mechanism for reducibility is symmetry. However, reducibility ensues in much more general and interesting situations. This work constructs different classes of non-symmetric periodic quantum graphs whose Floquet surface is reducible for all energies. One class of graphs is obtained by coupling two identical copies of a periodic quantum graph by edges to form a bilayer graph. Reducibility ensues when the coupling edges have potentials belonging to the same asymmetry class in the sense that their "spectral A-functions" are identical. Another class consists of bilayer graphs in which the single layer is bipartite, including graphene. This applies to both AA-stacked and AB-stacked graphene.

IGOR VELČIĆ (UNIVERSITY OF ZAGREB, CROATIA): Operator norm long-wave asymptotics for elastic plates with rapidly oscillating periodic properties

In this talk we analyse a system of partial differential equations describing the behaviour of an elastic plate with periodic moduli in the two planar directions. We assume that the displacement gradients of the points of the plate are small enough for the equations of linearised elasticity to be a suitable approximation of the material response. Following the application of an appropriate version of the Floquet transform, we analyse the operator-norm resolvent behaviour of the operators in each fibre of the resulting direct integral, as the period and the plate thickness go to zero simultaneously. The convergence estimates we obtain are uniform with respect to both the Floquet parameter and the plate thickness, which yields order-sharp resolvent estimates for the convergence of the original plate problems as the plate thickness goes to zero. This is a joint work with Kirill Cherednichenko (University of Bath).

