SCHRÖDINGER OPERATORS WITH δ'-LIKE POTENTIALS

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We address the problem on the right definition of the one-dimensional Schrödinger operator with potential δ', where δ is the Dirac delta-function. While Schrödinger operators with δ'-*interactions* have become a standard solvable model in quantum mechanics and have been used in both physical and mathematical literature since the 1980-ies to describe e.g. various types of particle interactions, it is not clear how a δ'-potential should be defined.

For an arbitrary real-valued function V from the Faddeev-Marchenko class $L^1(\mathbb{R};(1+x)dx)$ and $\varepsilon > 0$, we study the operator family

$$S_\varepsilon := -\frac{d^2}{dx^2} + \frac{1}{\varepsilon^2} V\Big(\frac{x}{\varepsilon}\Big)$$

of Schrödinger operators as $\varepsilon \to 0$. If the potential V satisfies the conditions

$$\int_{\mathbb{R}} V(\xi)\, d\xi = 0, \qquad \int_{\mathbb{R}} \xi V(\xi)\, d\xi = -1, \tag{1}$$

then the functions $\varepsilon^{-2} V(x/\varepsilon)$ converge to δ' in the sense of distributions, so the limit S_0 of S_ε as $\varepsilon \to 0$ if exists might be taken as a mathematically motivated realization of the Schrödinger operator with *potential* δ'.

In the talk we will justify existence of the limit of S_ε as $\varepsilon \to 0$, identify the limiting operator S_0 and discuss whether this limit is the operator we are looking for. We shall also explain how the stationary scattering theory for S_ε gives physical justification of our conclusions.

The talk is based on a joint project with Yu. Golovaty (Lviv, Ukraine).