SPECTRAL APPROACH TO HOMOGENIZATION OF NONSTATIONARY SCHRÖDINGER TYPE EQUATIONS

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In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a selfadjoint strongly elliptic operator A_ε, $\varepsilon > 0$, given by the differential expression $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$. Here $g(\mathbf{x})$ is a periodic bounded and positive definite matrix-valued function, and $b(\mathbf{D})$ is a first order differential operator. We study the behavior of the operator exponential e^{-itA_ε} for small ε. We prove that, as $\varepsilon \to 0$, the operator e^{-itA_ε} converges to e^{-itA^0} in the $(H^s(\mathbb{R}^d) \to L_2(\mathbb{R}^d))$-operator norm (for a suitable s). Here $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator. Sharp order error estimates are obtained. The results are applied to study the Schrödinger type equation $i\partial_t u_\varepsilon(x,t) = (A_\varepsilon u_\varepsilon)(x,t)$. Applications to the Schrödinger equation and the two-dimensional Pauli equation with singular potentials are given. The method is based on the scaling transformation, the Floquet-Bloch theory and the analytic perturbation theory.

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