Boundary Perturbations of planar waveguides: oscillating boundary, windows, non-periodic perforations

G. Cardone

Dep. of Engineering, University of Sannio, Benevento, Italy

We consider an elliptic operator in a planar infinite strip perturbed in different way:

- by substituting one side of the boundary by a fast oscillating curve.

  We assume that both the period and the amplitude of the oscillations are small and impose the Dirichlet condition on the upper boundary and Dirichlet, Neumann or Robin boundary condition on the oscillating boundary. In all cases we describe the homogenized operator, establish the uniform resolvent convergence of the perturbed resolvent to the homogenized one, and prove the estimates for the rate of convergence. These results are obtained as the order of the amplitude of the oscillations is less, equal or greater than that of the period. It is shown that under the homogenization the type of the boundary condition can change.

- by infinite numbers of "windows":

  We impose the Dirichlet condition on the upper boundary and frequent alternation boundary condition on the lower boundary. The alternation is introduced by the periodic partition of the boundary into small segments on which Dirichlet and Neumann (the "windows") conditions are imposed in turns. We study the cases when the homogenization gives the Dirichlet or Neumann condition instead of the alternating ones, and we establish the uniform resolvent convergence and the estimates for the rate of convergence; moreover, we study the spectrum of the perturbed operators.

- by a perforation by small holes along a curve.

  We impose mixed classical boundary conditions (Dirichlet, Neumann and Robin) on the holes. Assuming that the perforation is non-periodic and satisfies rather weak assumptions, we describe all possible homogenized problems. Our main result is the uniform resolvent convergence of the perturbed operator to a homogenized one in various operator norms and the estimates for the rate of convergence. On the basis of the norm resolvent convergence, we prove the convergence of the spectrum.

These results are part of joint works with D. Borisov, R. Bunoiu, T. Durante, L. Faella and C. Perugia.