When does the norm of a Fourier multiplier dominate its L^{∞} norm?

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It is well known that the L^{∞} norm of a Fourier multiplier on $L^{p}(\mathbb{R}^{n})$, $1 \leq p \leq \infty$ is less than or equal to its norm. The standard proof of this fact extends with almost no change to weighted L^{p} spaces provided the weight w is such that w(x) = w(-x) for all $x \in \mathbb{R}^{n}$. It is natural to ask whether the norm of a Fourier multiplier on a weighted L^{p} space still dominates its L^{∞} norm if the weight does not satisfy the above condition. If w satisfies the Muckenhoupt A_{p} condition, then the L^{∞} norm of a Fourier multiplier on $L^{p}(\mathbb{R}, w)$, 1 is less than or equal to its norm times a

multiplier on $L^p(\mathbb{R}, w)$, 1 is less than or equal to its norm times aconstant that depends only on <math>p and w. This result first appeared in 1998 in a paper by E. Berkson and T.A. Gillespie where it was attributed to J. Bourgain. It was extended to more general function spaces over \mathbb{R} by A. Karlovich (2015). We prove that the above estimate holds with the constant equal to 1 for function spaces over \mathbb{R}^n under considerably weaker restrictions. We also show that our result is in a sense optimal and that there exist weighted L^p spaces with many unbounded Fourier multipliers.

The talk is based on a joint work with Alexei Karlovich (Lisbon).