The essential numerical range for unbounded linear operators

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We study the approximation of a linear operator T by compressing it to an n-dimensional subspace of the domain of T (called Galerkin/finite section/projection method). It is well known that, in the limit $n \to \infty$, the eigenvalues of the $n \times n$ matrix T_n may accumulate at a point that does not belong to the spectrum of T. The occurrence of such a spurious eigenvalue is commonly known as *spectral pollution*. In this talk we present methods to identify regions in the complex plane that enclose the set of spectral pollution (as tightly as possible). A useful tool is the notion of *essential numerical range* $W_e(T)$ which was introduced in the late 1960s for *bounded* T. We discuss the generalization of this notion to *unbounded* operators.