MA20222 **Problem Sheet 6**

Do all questions and hand in your answers to the ⋆**starred**⋆ questions as instructed by your tutor.

⋆E6.1. Show that the iteration
$$x_{n+1} = \frac{1 + 9x_n - x_n^3}{8}$$
has a fixed point in the interval $[1, 2]$, and that this fixed-point iteration is convergent.

E6.2. By verifying the conditions in the fixed-point theorem, show that the iteration
$$x_{n+1} = g(x_n), \qquad g(x) := \frac{x+1}{x+2}$$
converges to a fixed point in the interval $[0, 1]$ for any initial condition $x_0 \in [0, 1]$.

⋆E6.3. By considering a suitable equation of the form $f(x) = 0$, write down Newton's method for computing the cube root of a given number a. Use this technique to find an approximation of $25^{1/3}$ correct to four-significant figures. How many iterations do you require?

E6.4. Determine a numerical value (correct to 12 decimal places) of $1/3$ *without division* by use of Newton's method applied to $f(x) = 1/x - a$ with $a = 3$, $x_0 = 0.3$. Examine closely the notion of quadratic converge. (Before computer hardware was fully developed, this was the standard way of performing division on early computers).