

BSc and MSci EXAMINATIONS (MATHEMATICS)

May-June 2013

This paper is also taken for the relevant examination for the Associateship.

M4P54/M5P54

Differential Topology (Mastery question)

Date: 7th May 2013

Time: 10am – 1pm

Credit will be given for all questions attempted but extra credit will be given for complete or nearly complete answers.

Calculators may not be used.

- M. (i) Let Y be a topological space and $\varphi : S^{n-1} \rightarrow Y$ a continuous function. Give the definition of the topological space $Y \cup_{\varphi} B^n$ formed by attaching an n -cell to Y by φ , and of a finite cell complex of dimension n . Explain briefly how to compute the singular homology groups $H_k(X)$ of a finite cell complex through cellular homology.
- (ii) Let $Z \subset \mathbb{R}^8$ be the union of $\{(x_1, \dots, x_8) : \sum_{i=1}^4 x_i^2 = \sum_{i=5}^8 x_i^2 = 1\}$ (homeomorphic to $S^3 \times S^3$) and a 4-ball $\{(x_1, x_2, x_3, x_4, 1, 0, 0, 0) : \sum_{i=1}^4 x_i^2 \leq 1\}$. By identifying a cell complex homeomorphic to Z , or otherwise, compute the singular homology $H_k(Z)$ for all k , showing that $b_0(Z) = b_3(Z) = b_6(Z) = 1$ while the other Betti numbers vanish. (*Hint: Consider a map $f : B^3 \times B^3 \rightarrow S^3 \times S^3$ of the form $(x, y) \mapsto (g(x), g(y))$, where $g : B^3 \rightarrow S^3$ maps the boundary to $(1, 0, 0, 0) \in S^3$ and is a homeomorphism on the interior.*)
- (iii) Is Z homotopy equivalent to any closed manifold?