A STUDY OF THE MENAI SUSPENSION BRIDGE

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Abstract: This paper describes relevant information pertaining to the Menai Suspension bridge in several aspects. It will critically analyse the aesthetics, strength, serviceability and durability of the bridge. Furthermore, the paper describes how the loadings, such as dead, superimposed dead, live, temperature and wind loadings affect the structure of the bridge. There will also be consideration of the construction process of the bridge and how that may be improved.

Keywords: cable, aesthetic, deck, wind, load

1 General
1.1 Introduction

When the Act of Union of 1800 took place, when Ireland joined the United Kingdom, the traffic across the Menai strait and Anglesey increased in the early 19th century. Travelers to the ferry port of Holyhead, where ships left for Ireland, had to make the dangerous crossing after a long and arduous journey from London. Soon plans were drawn up by Thomas Telford for ambitious improvements to the route from London to Holyhead, including a bridge over the Menai straits.

The bridge needed to have a clear span beneath it of 100 feet, beneath the main span, to allow the continual passage of tall sailing ships that used the Strait. This made the solution to the problem very clear, a suspension bridge design. The design of the bridge used sixteen huge chains in order to hold up a 579 foot length of road surface between the two towers. At the time, suspension bridges had been built previously, but never to such a scale as the proposed design.

1.2 Principle of suspension bridges

As with the name of the type of bridge, a suspension bridge is based upon the deck being suspended from a set of cables. These large cables go over the top of the supporting towers with hangars along the cable at regular intervals. These hangars reach down to the deck to provide support for it. The deck will provide a certain amount of stiffness in order to allow concentrated loads to be taken by a number of hangars. The beauty of a suspension bridge is that it allows for a relatively slender deck. The slenderness being made possible from the deck not carrying any compression. The deck is only designed to carry bending.

When designing a suspension bridge, one important piece of information that is required, is the shape that the cable will take when it is supporting a considerable uniform weight. In 1794 Fuss developed a famous formula for this situation

\[ H = \frac{wL^2}{8f} \]

Equation 1.1

Where \( w \) is the weight per unit length, \( L \) is the plan length of the cable, and \( f \) is the dip of the cable at midspan.

This equation shows the cable deforming into a parabola, where the horizontal component of force is constant throughout the cable.

In designing the suspension bridge another useful designing tool was theorized by Rankine. This work was based upon accurately analysing a bridge under vertical loading. The theory made the assumption that the deck would be sufficiently stiff to spread any concentrated load evenly across the entire span of the bridge.
2 Aesthetics

The aesthetics of a bridge are an important consideration in the design. If a bridge looked weak, people would have no confidence in using it and so the bridge would become useless. There are other general ‘rules’ that are adhered to also; with the odd exception. One such rule is that a suspension bridge across a valley would most likely look out of place.

There have been many guidelines written as to the aesthetics of bridges. Fritz Leonhardt believes there are ten distinct categories that a bridge should be assessed against for aesthetic purposes. Following, is an assessment of the Menai Bridge against these rules, which are relevant to this bridge.

2.1 Fulfillment of function

The Menai Bridge is a suspension bridge, one of the world’s first, and it shows its structure very clearly; cables going over the tops of piers in turn supporting the decking. The cable was originally made of large chains; and it is evident that these carry the weight of the decking, and transfers it down through the masonry towers. The arched piers continue further into the banks of the ‘river’, which seems a little excessive. This would however, give confidence to all users of the bridge, of its strength.

2.2 Proportions

The bridge is relatively well proportioned when viewed by the observer, there could however, be improvements. The deck of the bridge is a good depth to the eye in comparison to the cabling holding it up, and the towers between. The deck is also quite high above the water line, which looks right. The towers are of a low height which matches the profile of the surrounding land, and the span of the deck itself. In continuing from the towers to the banks of the water, are large piers that are masonry arches. These arches don’t quite appear right in that there is a large gap between the piers over the ‘river’, and then excessive stone under the decking to the banks.

2.3 Order

The order of the Menai Bridge is very good. There are no columns, just solid masonry blocks, and so there is no “mental disquiet” as there are no visual collisions, or too many visible lines. The eye flows nicely from one side of the bridge to the other, without being distracted by ‘breaks’ in the bridge.

2.4 Refinements

The towers of the bridge are tapered in both orientations, which you can see in elevation (Figure 1) of the bridge to the side. This however, is more of a structural feature than an aesthetic one. The fact that the secondary vertical supports are piers, this leads to the view between them being obscured at oblique angles. In some bridges this could be a bad thing, but with the Menai Bridge, it is not a particularly pressing problem though, as there is such a large view through the main span of the bridge.

2.5 Integration into the environment

The bridge sits well in its environment over the Menai straits. The most important thing being, it doesn’t look out of place. The heavy structure is placed toward the banks of the Strait, and the apparent lightweight proportion of the bridge makes up the main span above the Straits. The only other form of bridge that might look correct over this span is a cable stayed bridge. At the time of commission, this type of bridge would not have been an option however. Thus, a suspension bridge fits in very nicely.

2.6 Texture

The texture of a bridge is an important feature of a bridge. It can help people be more comfortable in using the bridge though, in having the materials clearly defined through texture. The texture of the Menai Bridge is correct in that the piers and the towers have a rough finish to them from the material they are made of; limestone. The steel components are all smooth. This gives the bridge the right textural feel.

2.7 Character

Character is defined as the aggregate of features and traits that form the individual nature of some person or thing. The culmination of the features of this bridge has a lot of character. The masonry piers connecting with the decking and the cabling above, integrate different materials defining the bridge’s character nicely.

2.8 Complexity

The Menai Bridge doesn’t have any complexity or features that you can associate with nature. It follows with a lot of rules of aesthetics though, and is pleasing to the eye, making it easy for the viewers to have an understanding of how the bridge works.

2.9 Conclusion

The Menai Bridge has a lot of features that are in accordance with Leonhardt’s rules of aesthetics. The bridge sits well in its environment over the Menai Straits.

One improvement that could possibly be considered is the use of colour in the bridge. This is not necessarily something that might improve the aesthetic though given the surrounding environment.

Figure 2 Closer view of the piers
3 Construction

3.1 Foundations

The foundations of any bridge are often the most important and expensive part. This is especially the case with a suspension bridge. The towers of the bridge would be required to take all the vertical force of the bridge down into the ground. The more difficult part of the foundations in a suspension bridge is the anchorage of the cable, as this is a horizontal force, which is represented by equation 1.1. Figure 3 below shows how the foundations need to work in an ideal situation.

In the case of the Menai suspension bridge though, there are other important considerations in terms of foundations. The bridge is sited over water; the Menai straits. The ground conditions in such a site will usually consist of soft muds and or clays. This kind of material is prone to large settlements. The settlements in themselves might not be too much of a problem. The problem occurs when differential settlement occurs. If the two towers had vastly different heights after any settlement, this would be devastating to the bridge as it would affect the shape of the cable and thus the way the bridge would work. This leads to there needing to be a lot of investigation into the ground conditions of the area, in order to find two suitable locations of similar properties for the towers.

In having found a suitable location for the tower foundations, the next challenge would be in deciding what kind of foundations to use for the towers. This again would be dependant on the ground conditions, and where the bedrock, if any, was in relation to the surface. If it was found the bedrock was quite shallow, then it would probably be a good option to excavate down to the bedrock and build directly off of it. If the bedrock however, was found to be at a much greater depth, it would probably be a good idea to consider pile foundations that could be capped, and built upon.

The anchorage foundations need to resist lateral load, which is far more difficult to achieve, especially within the ground. One such method by which this is done is to excavate the ground and build massive blocks out of stone or concrete and anchor the cables within it. This needs to be done in such a way as to ensure there is minimum slip, and that no movement can be induced within the large mass. This is demonstrated below in figure 4. The block itself can be in connection to the ground through just shallow foundations if this is deemed sufficient, or piles if there is a requirement for this through soil conditions near the banks of the water.

3.2 The towers and piers

The towers of a suspension bridge are designed to effectively take all the vertical loading of the bridge. They can however be designed in order to take some moments as well. These moments can be bought into the columns depending upon the connection at the tower tops. This decision effects what kind of tower is required in the design. If it is decided that the towers were to be slender in design, one would design the tower to be pinned at the top and bottom. This would mean that the cable would need to be allowed to slip over the tops of the tower where required. If the cable were fixed at the top, any movement that the cable wanted to go through would be transferred into the towers, and thus generate a moment, giving rise to the need to much more substantial towers.

In the design of the Menai suspension bridge, one of the design requirements was to have the deck at least 100 feet above the water line to allow the continuing passage of tall sail ships of the period. This leads to a fairly significantly sized tower already; and this is just the base of it. When considering purely the horizontal force generated by bridge within the cable (equation 1.1), it leads to the fact that the force is reduced with a greater sag in the cable, and so, a taller tower, will allow for a potential greater sag and reduce the horizontal force. Thus, the maximum tower height is wanted without seeming too daunting against the rest of the bridge.

The towers themselves being made of locally mined limestone in the case of this bridge are designed with a gradual taper for structural purpose and ease of construction. In addition to these masonry towers that support the cabling, there are masonry piers that continue from the towers to the bank of the water. These piers are in the form of arches. Thus the deck of the bridge, from the banks of the water, up to the towers, is in fact sat on these piers, reducing the work the cables have to do. As this bridge was one of the first suspension bridges of such a scale, these additional piers were likely to be a precautious option. Rough dimensions are given below in figure 5.
3.3 Cabling and hangars

The cabling of a suspension bridge has to carry the weight of the deck along with the additional live loadings. It needs to transfer the vertical loads into the towers and the horizontal loads into the anchorages at the bridges extremities. The hangars of the bridge come off of the cable and as their name imply, hang down toward the deck in order to hold it up. The spacing of the hangars would be dependant upon the strength and stiffness of the deck itself. The less stiff the bridge is, the more hangars that would be required to prevent excessive deflection or load taken within the deck.

When the bridge was first being constructed, the cabling consisted of chains as opposed to iron, as steel at the time was not deemed an acceptable option. This is an awkward option to choose, as the chain is only as strong as its weakest link and so each link had to be rigorously checked when being installed. These chains have since been replaced with steel cables in order to accommodate increasing loading upon the decking from modern traffic.

At the time of construction, each of these chains had to be hauled up onto the towers by teams of men and horses working to a drum and fife for motivation. If the bridge were to be constructed now, the cabling would be wound by machine over the towers.

3.4 The deck

The very first deck of the Menai suspension bridge was made out of timber. This would have been the cheapest available material, and at the time of construction, the loads the bridge was required to take would not have been great enough to warrant stronger materials.

Transportation during the life of the bridge will have evolved greatly since the commission of the bridge. At first the only vehicles to travel across the bridge will have been horse drawn cars. The dawn of the engine will have changed the loads vehicles can carry significantly, and without upgrading, the bridge would have become obsolete. Therefore, on several occasions, the deck has been re-built, in order to accommodate these changes.

At present, the deck of the Menai suspension bridge is a steel deck. It consists of several “beams” which run the length of the span, with secondary beams lying across them holding up the roadway, or “blacktop”. The reality is of course that the “beams” that run the length of the bridge, do not actually do this, as the deck consists of several sections that are suspended from the cabling above.

The deck would look as follows in figure 6.

Figure 6 Cross section of the deck

The construction of the deck back when it was originally being put up would have been a more difficult undertaking than today.

It is likely that the original construction would have taken place toward the edge of the water banks, or at the built towers, as these areas had masonry underneath supporting the decking. The deck would probably have been continued from this point being supported off of the hangers in order to avoid lifting large deck sections through great heights. This would also avoid blocking the shipping lanes beneath the bridge for any significant period of time.

The modern way of doing this would be to attach a gantry crane to the cabling of the bridge. The different sections of the bridge would then be transported by barge beneath the gantry which, would then pick up the sections and lift them into place to then attach to the hangers. The deck in its incomplete stage will most likely deform slightly in accordance with the shape of the main cables. This can be corrected out at a later stage however with alterations of the hangers.

3.5 Bearings and other joints

The deck of any bridge will need to be in contact with the ground at some stage through columns or piers. This contact has to be considered quite carefully in certain situations to ensure that movements of the bridge structures do not induce stresses that could lead to failure.

These considerations involve the use of bearings and other special joints, such as expansion joints. These joints will allow movement in the required planes so that there is no excess build up of stress.

In the case of the Menai suspension bridge, the deck in the incomplete stage will most likely deform due to extra thought. The deck is likely to expand due to temperature effects, and if the movement is not allowed for, the deck will be required to take the extra stress. This would mean in a suspension bridge that the deck would have to become a lot thicker which would ruin the appeal of the bridge.

It is best to avoid bearings where possible however due to the cost of them being installed with the bridge.

When the Menai suspension bridge was first constructed, it is unlikely that any such bearings would have been in the design, but figure 7 shows where they might be located in the design should they be required. The figure also shows the location of the required expansion joint in the deck.
4 Loads

The design of all bridges requires the bridge to be tested in analysis by imposing a series of different kinds of loads upon it. In analysing a bridge, ultimate limit state methods are used to check that failure will not occur. Once these checks are satisfied, serviceability limit state checks are applied to the bridge to ensure that it does not deflect or deform too much under these loads to keep the publics confidence in the bridge.

4.1 The different types of considered load

The loadings put onto a bridge are broken into five broad categories. These categories are dead load; the actually mass of the structure that enables the bridge to stand up. The superimposed dead load; the load on the bridge created by the actual road surface and the additional bits and pieces like bits of pipe work. The live traffic load; the load from the users of the bridge, mainly from lorries as cars and people are generally insignificant. The temperature; the temperature change over the period of a day can induce stresses into a bridge and so needs to be worked out to ensure it is not a problem. Then finally, the wind loading needs to be considered, this is especially important on a suspension type bridge due to the relatively un-stiff nature of the bridge type.

In using the above loading types, one then has to consider certain combinations of them in order to work out the worst effects the bridge may feel. The best way of doing this for both ULS and SLS is in 5 simple combinations.

1. All permanent loads plus primary love loads
2. Combination 1, plus wind
3. Combination 1, plus temperature
4. All permanent loads plus secondary live loads (skidding and collision loads) and associated primary live loads
5. All permanent loads plus loads due to friction at supports

4.2 Dead and superimposed loads

The dead and superimposed loads of the bridge have factors placed on them as with the HA and HB loadings. These factors are γfl and γf3. According to which load is being calculated, dead and superimposed have different γfl due to the unpredictability of superimposed loads.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Unfactored</th>
<th>Factored</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>49.6 kN/m</td>
<td>57.28 kN/m</td>
</tr>
<tr>
<td>Superimposed</td>
<td>2.3 kN/m</td>
<td>4.43 kN/m</td>
</tr>
</tbody>
</table>

Table 4.2 Summary of the bridges dead loadings

4.3 Traffic and pedestrians

The live loads on a bridge as previously stated are from cars and pedestrians, but lorries are the main consideration, as they carry the heaviest of loads. These types of load are split into constituent parts. These parts being HA loading and HB loading.

HA loading consists of a uniformly distributed load acting over notional lanes of the bridge; notional lanes being strips that the bridge is divided into for analysis purposes. The uniformly distributed load is also acting along with a knife edge load to give the most adverse effect on the bridge. HA loading is the equivalent of heavy fast moving traffic.

HB loading represents an abnormal truck load on the bridge. The excessive weight of the truck lends to 112.5kN being taken through each wheel for analysis purposes.

The safety factors used in working out the final HA and HB loads are γfl and γf3. These safety factors are the load uncertainty factor and a further factor for types of bridge respectively.

The following table represents the loads on the bridge from HA and HB loading.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Unfactored</th>
<th>Factored</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA load</td>
<td>13.2 kN/m</td>
<td>15.25 kN/m</td>
</tr>
<tr>
<td>KEL</td>
<td>120 kN</td>
<td>138.6 kN</td>
</tr>
<tr>
<td>HB load</td>
<td>450 kN</td>
<td>519.75 kN</td>
</tr>
</tbody>
</table>

Table 4.3 HA and HB loadings

4.4 Wind loads

Analysis of how a structure reacts to wind load is of great importance, especially in a structure such as a suspension bridge where it is evidently possible for the deck in particular to move laterally.

The wind load can also act in many different directions or plane of the bridge. There are therefore several different values that are accounted for within wind analysis.

The first value that is required is the maximum wind gust, Vc, which is shown below.

\[ V_c = v K_1 S_1 S_2 \]  

Equation 4.41

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 )</td>
<td>31.8 m/s</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>1.56</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>1</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>1.21</td>
</tr>
<tr>
<td>( V_c )</td>
<td>60.03 m/s</td>
</tr>
</tbody>
</table>

Table 4.41 Calculation of \( V_c \)
The maximum wind gust value allows for loading values upon the bridge to then be calculated. The first wind loading that needs to be considered is the horizontal wind load, $P_t$. This is worked out as follows.

$$P_t = qA_1C_D$$

Equation 4.42

$q = 0.613 V_c^2$

$A_1$ = Solid horizontal projected area, 173.7 x 1.5 = 260.55 m²

$C_D$, drag co-efficient, 1.175

(Found from the b/d deck ratio)

$P_t = 676.28$ kN

Table 4.42 Calculation of $P_t$

The next wind loading is wind that affects the bridge in the vertical plane.

$$P_v = qA_3C_L$$

Equation 4.43

$q = 0.613 V_c^2$

$A_3$ = Plan area of the deck, 12 x 173.7 = 2084.4 m²

$C_L$, drag co-efficient, 0.375

(Found from the b/d ratio)

$P_v = 1726.67$ kN

Table 4.43 Calculation of $P_v$

There are two other wind loads that will affect a bridge. These loads are known as $P_{LS}$ and $P_{LL}$. These loads are the wind running down the length of the bridge, and wind hitting the vehicles travelling along the bridge and subsequently transferring to the bridge. The equations and values are as follows below.

$$P_{LS} = 0.25 qA_1C_D$$

Equation 4.44

$$P_{LL} = 0.5 qA_1C_D$$

Equation 4.45

$q = 0.613 V_c^2$

$A_1$ = 260.55 m²

$C_D = 1.175$

$P_{LS} = 169.07$ kN

Table 4.45 Calculation of $P_{LS}$

$q = 0.613 V_c^2$

$A_1$ = 260.55 m²

$C_D = 1.175$

$P_{LL} = 338.14$ kN

Table 4.46 Calculation of $P_{LL}$

4.5 Temperature induced loads

When any material is heated up, the size and shape of it will change slightly according to the material is it made of. If the material is held rigidly, as opposed to freely, stress will be induced into it. This is therefore an important consideration in bridge design, as a temperature increase across a day could increase the stress significantly if it is not catered for.

The first consideration with temperature is how much movement there will be due to temperature. The following equation works this out for us.

$$\Delta l = \alpha \Delta T l$$

Equation 4.51

$\alpha = 12 \times 10^{-6}$/Cº

Temperature co-efficient of steel

$l = 173.7$m

$\Delta T = 25$ Cº

$\Delta l = 0.052$m

Table 4.51 Calculation of change in length of bridge

The following equation is the stress as a result of the above result, if a movement of 0.052m is not allowed for within the bridge.

$$\sigma = \alpha \Delta T E$$

Equation 4.52

$\alpha = 12 \times 10^{-6}$/Cº

Temperature co-efficient of steel

$\Delta T = 25$ Cº

$E = 200,000$ N/mm$^2$

$\sigma = 60$ N/mm$^2$

Table 4.52 Calculation of additional stress in bridge due to temperature effects

In order for the additional stress of 60 kN/mm$^2$ to be avoided, an expansion joint is required. One possible expansion joint is shown in the figure below.

Figure 8 Expansion joint

5 Load carrying capacities

The suspension bridge follows relatively simple ideas of statics in the principles of how it works. The calculations are not particularly complex, and they can be done by hand quite effectively and efficiently in order to gauge relative strengths.
The figure below illustrates the basic dimensions and workings of a suspension cable, revealing how the forces are acting.

**Figure 9** Dimensions and forces on parabolic chain

In order to get an idea of the maximum weight that can be applied over the parabolic chain, first the size of the cables being used needs to be assumed or known. In the case of the Menai Bridge, it is a fair assumption to say that the cables are about 0.3m in diameter.

| Area of cabling \( \pi (0.3/2)^2 \) = 0.071 m² |
| Assume high yield steel \( \sigma_y = 460 \text{ N/mm}² \) |
| Maximum tension in cables before yield is therefore \( T_y = \sigma_y A \) |
| \( T_y = 32.5 \text{ MN} \) |

**Table 5.1** Tension that the cables can withstand

The tension that the cable can withstand having being worked out can then be used in order to work out the maximum possible load, \( w \) that the cable can withstand. This is done by considering the cable right at its end.

The following diagram shows how equation 5.2 is arrived at.

**Figure 10** The acting forces at end of parabolic chain

The forces at the end of the chain can be resolved using Pythagoras theorem. The value of tension has been worked out already. \( H \) is given in equation 1.1. The value of \( V \) can easily be worked out by inspecting figure 8.

\[ V = w l/2 \quad \text{Equation 5.1} \]

The values for \( V \) and \( H \) are kept in terms of \( w \) in the Pythagoras equation of:

\[ T = (H^2 + V^2)^{1/2} \quad \text{Equation 5.2} \]

When the formula is arranged and the maximum permissible tension put into the equation, the end result for a 0.3m diameter cable states that the maximum load can be 122 kN/m. Therefore with two cables supporting the Menai suspension, a total load of 244 kN/m can be put onto the deck of the bridge.

The value of 244 kN/m gives a rough indication as to the maximum load that the bridge could be subjected to. This value needs to be checked against the design loading of the bridge to ensure it is an acceptable design.

The first step in doing this, is understanding how the loads are applied to a bridge; traffic loads in particular to achieve the worst loading case. The following diagram illustrates how the traffic loads are spread along the bridge.

**Figure 11** Distribution of traffic loads

In 1858 a powerful design tool for vertical loads on a suspension bridge was developed by Rankine. The theory he developed made the assumption that the deck was sufficiently stiff, so that a concentrated load could be spread evenly across the entire span of the bridge. This is how the capacity calculation shall be completed here.

| Dead load, \( W_{DL} = 57.28 \text{ kN/m} \) |
| Superimposed dead load, \( W_{SIDL} = 4.43 \text{ kN/m} \) |
| HA loading = 15.25 kN/m |
| Or 5.08 kN/m² in a notional lane |
| HB loading = 519.75 kN per axle |
| There are 4 axles and so total load of 2079 kN |
| Area one lane of Full HA load, \( 173.7 \times 3 = 521.1 \text{ m}² \) |
| Area of one 1/3 HA load strip, \( 46.85 \times 3 = 140.55 \text{ m}² \) |
| Total full HA area, \( 521.1 \times 2 = 1042.2 \text{ m}² \) |
| Total 1/3 HA area, \( 140.55 \times 4 = 562.2 \text{ m}² \) |
| Total HA load, \( 5.08 \times 1042.2 + 5.08 \times 1/3 \times 562.2 = 6246.368 \text{ kN} \) |
| Total traffic load from HA and HB loads = 8325.368 kN |
| Using Rankines’ theory, traffic gives, 47.93 kN/m |
| Dead loads and traffic loads gives, 109.63 kN/m |
| This design load gives the bridge a safety factor of \( 244 / 109.63 = 2.2 \) |

**Table 5.2** Calculation of design load for comparison with bridge capacity

The calculation in table 5.2 merely represents the loads on the bridge that may occur with heavy traffic and an oversized vehicle traveling upon it. This is not the most critical case though. That particular load case could occur in bad weather conditions, such that the wind is also loading the bridge deck vertically as shown in equation 4.43.

| \( P_v = 1726.67 \text{ kN} \) |
| Total vertical load = 10,052 kN |
| Rankines’ theory gives 57.87 kN/m |
| Total distributed load is therefore 119.57 kN/m |

**Table 5.3** Vertical loads plus wind

The cabling according to the calculations completed above has strength in excess to deal with the vertical loads that the bridge could be subjected to. These loads however need to be transferred into the cabling efficiently so that the deck is not subject to carrying the loads. This
depends upon the hangers from the cables, and their spacing.

There are two things to consider when deciding upon the spacing of the hangers. The first is the deflection of the deck. The hangers will effectively act as supports to the deck and so the space between them will govern how much the deck deflects. This is not the only problem though, as the deck of a suspension bridge is going to be relatively thin, and so will not be capable of carrying large moments, and so the moment capacity of the deck needs to be calculated, which again is dependant on the length between hangers from the main cables.

The first consideration for the cable spacing shall be in terms of deflection. If a reasonable limit for maximum deflection is set, the length of unsupported deck can be determined. The following is the equation for deflection when a uniform load is applied over a ‘simply’ supported section.

\[ \delta = \frac{5wl^4}{384EI} \]  
Equation 5.3

In this equation, \( E \) is the young’s modulus which, in steel is 200,000 N/mm². The value \( I \) is the second moment of area of the deck which is illustrated in figure 6. The part of the deck that is expected to do any structural work is just the steel, and so only the \( I \)-value of this shall be calculated.

\[
I = \sum((l_{i}^{3} + y_{i}^{2}A))
\]

Where \( y \) is the distance from neutral axis and \( A \) is area of part considered.

Neutral axis = \[
(\frac{(2000 \times 700 \times 305) + (12,000 \times 500 \times 950))}{(2000 \times 500) + (12,000 \times 500)) = \]
836.4 mm from base of deck

\[
I = \frac{(12,000 \times 500^{3})}{12} + \frac{(12,000 \times 500 \times 113.6^{2})}{12} + \frac{(500 \times 700^{2})}{12} + \frac{(500 \times 700 \times 463.6^{2})}{12}
\]

= \[5.6 \times 10^{11} \text{ mm}^4\]

Table 5.4 Calculation of the decks I-value

The spacing of the hangers according to deflection can now be calculated with a limiting deflection of 50mm with the I-value of the deck known.

\[
\delta = 50 \text{mm}
\]

Worst load, \( w = 120 \text{kN/m} \)

\[ E = 200,000 \text{ N/mm}^2 \]

\[ I = 5.6 \times 10^{11} \text{ mm}^4 \]

Therefore the acceptable hanger spacing according the equation 5.3 is –

43,500mm

Table 5.5 Calculation of hanger spacing according to deflection

The spacing of the hangers according to the moment capacity of the deck can also be calculated with the \( I \)-value using the following basic formula.

\[ \sigma = \frac{My}{I} \]  
Equation 5.4

The maximum moment the deck can in theory withstand is given by this equation. The value obtained can then be substituted into the equation for maximum moment due to a uniformly distributed load in order to work out the acceptable spacing of the hangers.

\[
M_{\text{max}} = \frac{wl^2}{8}
\]  
Equation 5.5

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>275 N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>836.4 mm</td>
</tr>
<tr>
<td>( I )</td>
<td>5.6 \times 10^{11} \text{ mm}^4</td>
</tr>
</tbody>
</table>

Therefore deck has 184.12 MNm capacity

\( W = 120 \text{kN/m} \)

Therefore the acceptable hanger spacing according the equation 5.4 and 5.5 is –

96,500mm

Table 5.6 Calculation of hanger spacing according to moment capacity

These spacing calculations suggest that the spacing of the hangers could be a maximum of 43m. This might be the case purely for the purposes of theory within the deck, but this value could not possibly be the value used for several reasons.

The first reason is related to transfer of load; spacing of 43m along the deck would mean that there were a total number of four hangers from the cables. This would not transfer the loads from the deck efficiently to the cables, and so the spacing would need to be reduced to increase this efficiency of load transfer.

The second reason that the spacing could not be this small is due to torsion. If the bridge were loaded heavily in-between the hangers on one side of the bridge and a strong horizontal wind (equation 4.42) began acting, the deck would be subject to twisting.

This twisting action would be counteracted by the hangers simply being subjected to additional tension. This prevents the deck moving too much. In order for this to be in effect, the spacing of the hangers needs to be much closer.

The capacity of the hangers themselves will have a large bearing on the spacing apart that they can be. The diameter of a hanger shall be assumed to be 50mm and the spacing of them derived from the load that acts upon the deck of the bridge.

\[
T = \sigma A
\]

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>275 N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of hanger = 50mm</td>
<td></td>
</tr>
<tr>
<td>Area of steel = (50/2)² \times \pi</td>
<td></td>
</tr>
<tr>
<td>= 1963.5 \text{ mm}²</td>
<td></td>
</tr>
<tr>
<td>( T = 275 \times 1963.5 )</td>
<td></td>
</tr>
<tr>
<td>= 539.9 \text{kN}</td>
<td></td>
</tr>
</tbody>
</table>

Greatest load to act on bridge is, \( w = 120 \text{kN/m} \)

Total load on bridge then is, 20,844 \text{kN}

This is 10,422 \text{kN} per main cable

In order to achieve spacing, ratio of forces required

\[ \frac{10422}{539.9} = 19.3 \]

Spacing, \( l = 173.7 \div 20 \)

= 8.68 m

In order to impose some factor of safety in the hangers, a spacing of 6.9m shall be used which gives a total of 25 hangers on the main span

Table 5.6 Calculation of spacing of hangers

6 Vibrations
The Menai suspension bridge is primarily for the use of road vehicles, as it was built as a portal to Anglesey so that cargo could get the ferry to Ireland with greater ease. This does not prevent or prohibit pedestrians crossing the bridge on foot however, and as pedestrians can use the bridge, there needs to be a check on the vibrations of the bridge so that there is no discomfort to the pedestrian strolling over.

The way to make this check is to work out the natural frequency of the bridge. The natural frequency is an important number as it dictates the bridges safety. The general rule is that if a bridge has a frequency of above 5 Hz, it is safe for use. The reason this is such an important consideration is to do with oscillations, if the bridge oscillated at the same frequency as its natural frequency, the intensity of the oscillations can grow and the bridge could possibly tear itself apart. A frequency of 1 Hz is particularly dangerous as it is close to the frequency of the wind for example.

The formula to work out the rough natural frequency of an object like a beam, or deck is as follows.

$$W_n = (\beta_w)^2 \frac{(EI/ml)}{l}$$  \hspace{1cm} \text{Equation 6.1}

The first part of this formula is totally dependant upon the end conditions of the beam or deck. The second moment of area unusually has to be given in m$^4$, and the m value needs to be given in kg/m.

The following table calculates the natural frequency of the bridge.

<table>
<thead>
<tr>
<th>Type of End Conditions</th>
<th>$W_n$</th>
<th>$f_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-fixed ends, ($\beta_w$)$^2$</td>
<td>22.3733</td>
<td>7.81 Hz</td>
</tr>
<tr>
<td>Fixed-pinned ends, ($\beta_w$)$^2$</td>
<td>15.4182</td>
<td>5.38 Hz</td>
</tr>
<tr>
<td>Fixed-free ends, ($\beta_w$)$^2$</td>
<td>3.5160</td>
<td>4.8 Hz</td>
</tr>
</tbody>
</table>

The actual frequency of the bridge probably lies between these values, so an average would be a reasonable assumption to make as the main span is in-fact a pinned-pinned connection.

Therefore, $f_n = 4.8$ Hz

Table 6.1 Calculation of natural frequency

A frequency of 5 Hz is a cut off point in terms of pedestrians crossing the bridge unless it is proven that the acceleration of the vibration would not affect the individual crossing. There are two formulas that require checking. The first is a limiting value of acceleration, and the second is the actual acceleration of the bridge due to the frequency.

$$a < 0.5 (f_n)^{1/2}$$  \hspace{1cm} \text{Equation 6.2}

$$a = 4 \pi^2 f_n^2 y_k \psi$$  \hspace{1cm} \text{Equation 6.3}

The equation 6.3, gives the actual acceleration, and the value $y$ is the deflection from a static load of 0.7 kN in the centre of a span, and $\psi$ is the dynamic response factor, which is the dampening ability of the bridges materials.

The following table works out the acceleration to see if it is acceptable.

<table>
<thead>
<tr>
<th>$f_n$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>0.5 x (4.8)$^{1/2}$</td>
</tr>
<tr>
<td></td>
<td>1.096 m/s$^2$</td>
</tr>
<tr>
<td>$y_k$</td>
<td>($700 \times 6900^3$) / ($48 \times 200,000 \times 5.6 \times 10^{11}$)</td>
</tr>
<tr>
<td></td>
<td>4.2 x 10$^{-6}$ mm</td>
</tr>
<tr>
<td>$K$</td>
<td>1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>15</td>
</tr>
<tr>
<td>$a$</td>
<td>4 x $\pi^2$ x 4.8$^2$ x 4.2 x 10$^{-6}$ x 1 x 15</td>
</tr>
<tr>
<td></td>
<td>0.057 m/s$^2$</td>
</tr>
</tbody>
</table>

Table 6.2 Calculation of acceleration of vibration

The value of acceleration attained is much less than the limiting value. This shows that the bridge is safe for pedestrians to cross without any problems.

7 Maintenance and inspection

All structures throughout there lifetimes are going to have different parts become weaker due to accidents and fatiguing. This means the inspection to detect these weaknesses is an essential part of the structures lifetime. This is no different with the Menai suspension Bridge.

The Menai Bridge rests over the Menai straits, which consists of sea water. Sea water can be a very corrosive agent to structural members, whittling them away to reduce their capacities. The Menai Bridge has motor vehicles passing over it; these vibrations over long periods of time may begin to loosen some of the bolts upon the structure. This is of obvious concern, especially within the deck of the bridge, where a loss of stiffness from loosening of bolts may prevent the loads being transferred to the main cables; in the suspension bridge case.

The Menai suspension bridge would be thoroughly checked by inspectors regularly. They would focus their attentions mainly upon the cables of the bridge at specific locations. These locations being at the cable anchorage, over the tops of the towers, and the connections between the hangers and the main cable.

The Menai suspension bridge does not have exceptionally terrific height and so there is no walkway by the side of the cables for inspectors to walk along. It is far easier for them to be hoisted by a cherry picker to complete the inspection at the required height. If any member appears to be damaged in any manner, it should be tested and replaced as necessary without any disruption to the bridges running where possible.

8 Susceptibility to intentional damage

The Menai suspension bridge, at the time of commission, would not have had much thought to its future in terms of anyone wishing to intentionally damage or destroy the bridge. Thus the thought to design against any such actions to be taken against the bridge would not have been considered.

These days however, acts of terrorism upon all kinds of different targets, lends the need for structures to be protected against such actions of destruction.
On a suspension bridge there are several key points which would render collapse, should they be damaged. These areas are the anchorages for the cables, the towers that support the cables and the cables themselves. In the case of the Menai suspension bridge, the destruction of the towers themselves would probably be a difficult thing to achieve as they are made from solid stone. Thus the anchorage and cable itself would be of more concern. If these were to completely fail, it is likely that the entirety of the deck could collapse. This would likely be the case even if with the factor of safety in the cables being high, as without a cable on the one side of the deck, the whole thing would want to twist. This twisting is simply not designed for and the deck would not be able to deal with it; and so collapse would occur.

The deck is not mentioned as a risk for intentional damage, as although it would be easy enough for someone determined to ruin the deck of the bridge, it would only render the bridge unusable until the deck had been repaired. It would not be a critical failure to the overall structure.

On the whole, it is unlikely that the Menai suspension bridge would be the target of any kind of terrorist attack, but in theory there could be accidents upon the afore mentioned bridge areas and so it is best to have at least considered the possibility of the event occurring.

Conclusions

The Menai suspension bridge is a very old suspension bridge; one of the first ever constructed. The age of the bridge and the development of the world have lead to many changes having to be required. The first clear change was the change of steel cables from the chains that were originally used. There has also been a change in the deck of the bridge so that it can cope with the demands of modern loads placed upon it.

The bridge has therefore been subject to a number of changes, but throughout all of these changes, it has maintained its aesthetic appeal as a fine example of a suspension bridge. The combination of stone piers with a steel deck works well together, and the heights of the towers relative to the span of the bridge give it a slim feel that fits in with the surrounding environment that is very pleasing to the eye. This can be seen in the images below.

References

[1]. Figure 1, Picture of the Menai bridge http://www.anglesey-history.co.uk/places/bridges/
[3]. Figure 2, Picture of Menai bridge http://www.anglesey-history.co.uk/places/bridges/
[4]. Calculations assisted with British standards BS 5400
[5]. Information of Thomas Telfords design proposal including bearings and dimensions Civil Engineering 160 May 2007, Pages 26-30 Paper 15012 Doi: 10.1680/cien.2007.160.5.26
[6]. Figure 12, Picture of the Menai bridge Civil Engineering 160 May 2007, Pages 26-30 Paper 15012 Doi: 10.1680/cien.2007.160.5.26
[7]. Figure 13, Picture of the Menai bridge Civil Engineering 160 May 2007, Pages 26-30 Paper 15012 Doi: 10.1680/cien.2007.160.5.26
[8]. Figure 14, Picture of the Menai bridge http://en.wikipedia.org/wiki/Menai_Suspension_Bridge