

BOOK REVIEW

Patterns and Interfaces in Dissipative Dynamics. By L. M. PISMEN. Springer, 2006.
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This substantial monograph summarizes a broad selection of results concerning the dynamics of model nonlinear dissipative partial differential equations. Well-known cases that are treated include the FitzHugh–Nagumo model for activator-inhibitor dynamics, and the Cahn–Hilliard equation describing coarsening. Throughout, the presentation is from a physicist’s viewpoint: the author provides helpful descriptive commentary illuminating the theory, although, in some places, model-independent mathematical details that would help set the theory in a broader context are omitted.

The preface and ‘historical note’ at the end of the introduction might lead the reader to expect an exposition narrowly related to the author’s own many contributions to the field. On closer inspection this is not the case: the net is cast appreciably wider. It is also clear that the exposition takes account of much recent work.

The book comprises a short introduction followed by five chapters of almost equal lengths. The first chapter contains an introduction to bifurcation theory, chaotic dynamics, multiple-scales asymptotics, and Turing instabilities. As this list of topics suggests, the treatment here is rapid and feels a little superficial, even given the physical perspective. To my mind, this is the least satisfactory part of the book. A reader unfamiliar with these topics has better expositions to turn to, for example the classic textbooks by Guckenheimer & Holmes (1986) or Wiggins (2003). The recent books by Hoyle (2006) and Golubitsky & Stewart (2000) provide a much more careful explanation of the mathematical framework for understanding pattern formation.

Chapter 2 explores the dynamics of interfaces between homogeneous states, concentrating first on the canonical PDE $u_t = D\nabla^2 u + f(u)$ for a single scalar variable $u(\mathbf{x}, t)$ with cubic polynomial nonlinearity $f(u)$, and then moving on to the generalized Cahn–Hilliard equation $u_t = -\nabla^2[K\nabla^2 u + f(u)]$. Curvature effects in two spatial dimensions are discussed in detail. Chapter 3 revisits these ideas for two-variable ‘activator-inhibitor’ systems: asymptotic results are derived in the limits in which the inhibitor varies only on long lengthscales, relaxes quickly to equilibrium, and is only weakly coupled to the activator. Full detail is given for most calculations but the presentation nevertheless remains clear and coherent. Chapter 3 closes with a discussion of spiral wave dynamics and instabilities; in my opinion it is a pity that model-independent aspects of the dynamics (for example tip meandering) are lost in the details of the presentation.

Chapters 4 and 5 cover topics in, respectively, steady-state and oscillatory pattern formation; there is substantial (although by no means complete) overlap with the recent book by Hoyle 3, although the author covers more ground here, while providing enough detail for the reader to obtain a very good feel for the current state of research. Almost all of the discussion of chapter 5 is couched in terms of the complex Ginzburg–Landau equation $u_t = u + (1 + i\eta)\nabla^2 u - (1 + i\nu)u|u|^2$. Spiral waves make a second appearance: this discussion could perhaps have been linked better with that presented in chapter 3. The book concludes with a brief look at the effect of external time-periodic forcing on oscillatory systems.

The layout and figures are generally very clear and the material is well-organised. There are a few serious typographical errors, indicating that the book would have benefited from more careful proofreading. Overall I recommend this book as a very useful reference and guide to the literature. It will undoubtedly be of use to those working in very different areas of nonlinear science.

REFERENCES

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- HOYLE, R. B. 2006 *Pattern Formation: An Introduction to Methods*. Cambridge University Press.
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