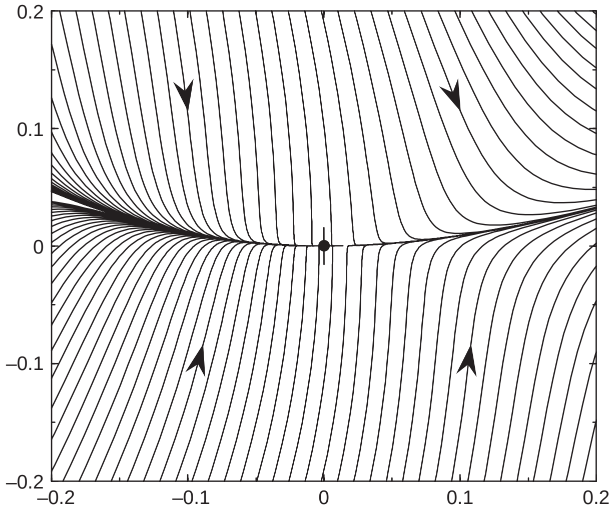
5.3 Numerical Example of Evolution on a Centre Manifold

Consider the two-dimensional system

$$\begin{aligned} \dot{x} &= \mu + x^2 + xy + y^2 \\ \dot{y} &= 2\mu - y + x^2 + xy \end{aligned}$$

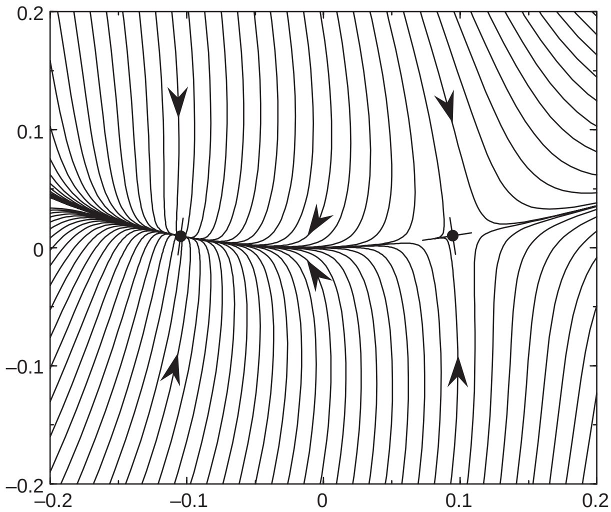
See notes for the algebraic details of the bifurcation at $x = y = \mu = 0$.

For $\mu = 0$ trajectories decay exponentially fast towards a centre manifold $y = x^2 + O(3)$ on which the dynamics is given by $\dot{x} = x^2 + O(3)$. The fixed point at $x = 0$ is nonhyperbolic.

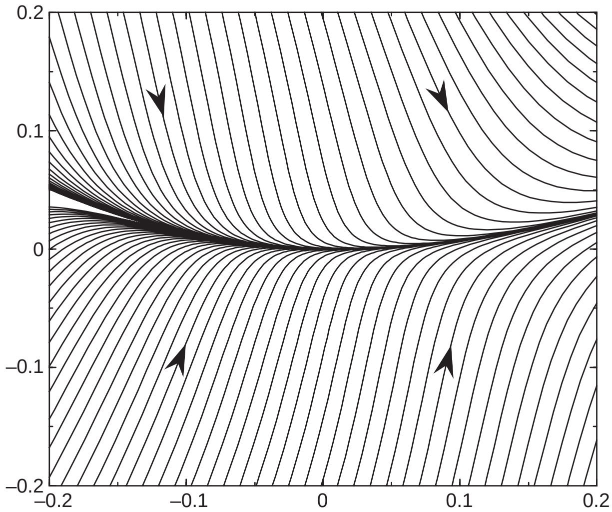


$\mu = 0$

For $|\mu| \ll 1$ trajectories decay exponentially fast towards the extended centre manifold $y = 2\mu + x^2 + O(3)$ on which the dynamics is given by $\dot{x} = \mu + x^2 + 2\mu x + 4\mu^2 + O(3)$.



$\mu = -0.01$



$\mu = 0.01$

So there is a saddle-node bifurcation which creates new fixed points in $\mu < 0$. For $\mu < 0$ there is a saddle at $x = \sqrt{-\mu} + O(\mu)$ and a node at $x = -\sqrt{-\mu} + O(\mu)$; for $\mu > 0$ there are no fixed points near $x = 0$.