The Extreme Team

ITT 17

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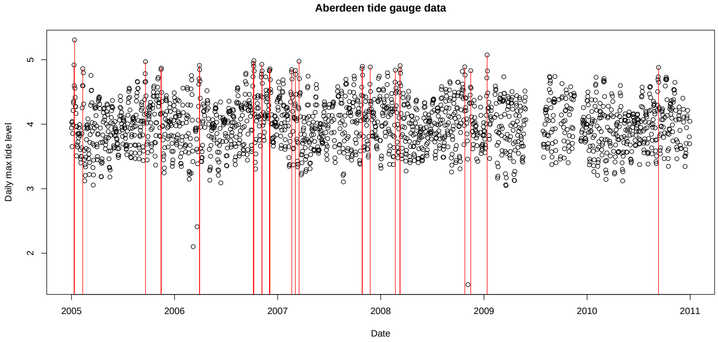
Overview

- We are investigating extreme values in tidal levels
- Data: British Oceanographic Data Centre (BODC) coastal tidal gauge data
- What are we looking at
  - GEV (Generalised Extreme Value Distribution)
  - Detecting change-point, trend, and seasonality in monthly maxima
  - Spatial correlation between tide gauges

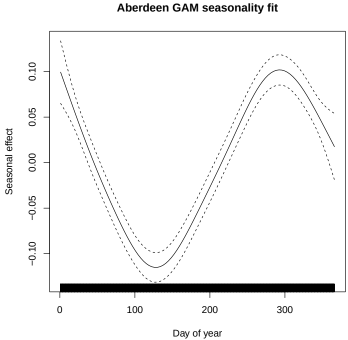
BODC coastal tide gauge data

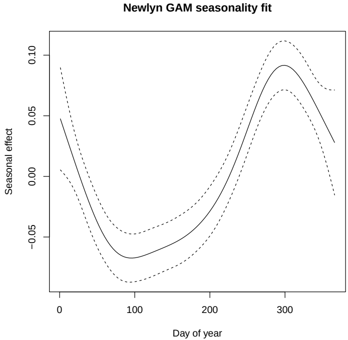
- The raw data from BODC contains data points marked with "T" (interpolated) or "M" (unlikely)
- We remove these data points and consider the daily maxima of the remaining data (to remove daily tidal effects)



Aberdeen daily maximum tide level, with values exceeding the 99% quantile marked in red.

Removing seasonality using GAM



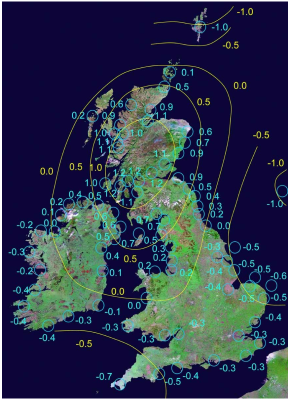


Fitting a GAM to the seasonal component of daily max tide gauge data.

- There is an effect on the mean daily maxima due to day of the year!

Adjusting for land movement

- Land movement (rising/sinking) will affect tide gauge measurements since the tide level is measured relative to land
- For Aberdeen: 0.6mm/year (land rising!)
- For Newlyn: -0.6mm/year (land sinking!)



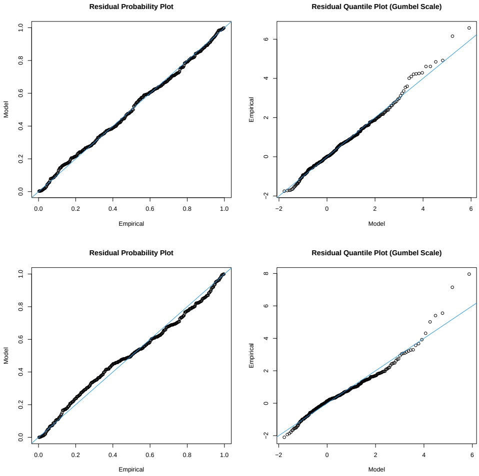
Land movement data [a]

[a] See Shennan, Milne, and Bradley (2011) https://doi.org/10.1002/jqs.1532

Fitting GEV model

- We remove the seasonal effect as fitted by the GAM model, and also adjust for land movement
- We calculate the monthly maxima of the daily data
- Then we can fit a Generalised Extreme Value (GEV) model:
  - Without trend: $y_i \sim GEV(\mu, \sigma, \xi)$
  - With trend only in μ: $y_i \sim GEV(\mu_0 + \mu_1 x_i, \sigma, \xi)$
  - With trend only in σ: $y_i \sim GEV(\mu_0, \sigma_0 + \sigma_1 x_i, \xi)$
  - With trend in both μ and σ: $y_i \sim GEV(\mu_0 + \mu_1 x_i, \sigma_0 + \sigma_1 x_i, \xi)$
- In the above, x is the time of an observation, expressed as the number of years after 1990

Fitting GEV model



Diagnostic plots from fitting the best version of the GEV model in terms of AIC. **Top**: Newlyn - model with drift only in μ. **Bottom**: Aberdeen - model with drift in both μ and σ.

Let's deal with the BEAST

BEAST: Bayesian Estimator of Abrupt change, Seasonal change, and Trend.

We have data $\mathcal{D} = \{(y_i, t_i)\}_{i=1}^{n}$. Proposed model:

$$\hat{y}(t_i) = \underbrace{S(t_i)}_{\text{piecewise harmonic model}} + \underbrace{T(t_i)}_{\text{piecewise linear model}}$$

$$S(t_i) = \sum_{l=1}^{L_k} \left[a_{k,l} \cdot \sin\left(\frac{2\pi l t_i}{P}\right) + b_{k,l} \cdot \cos\left(\frac{2\pi l t_i}{P}\right)\right]$$

$$T(t_i) = a_j + b_j t_i$$

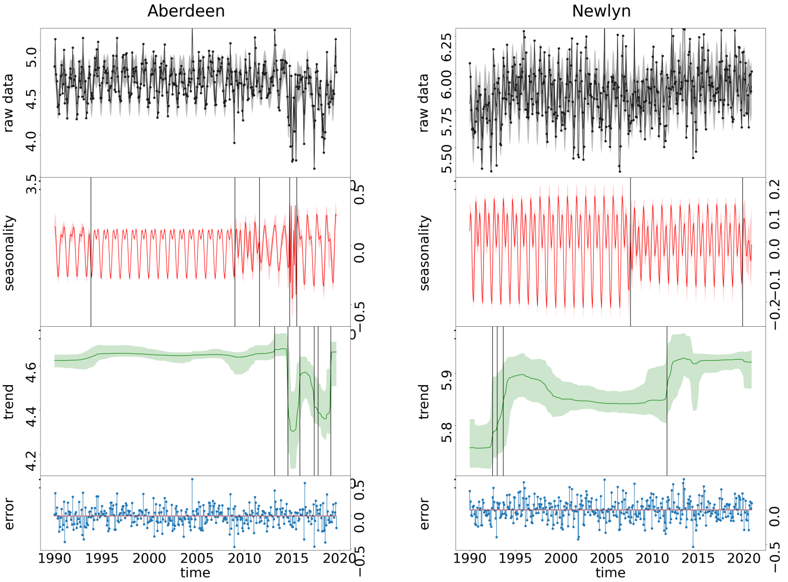
$$y_i = \hat{y}(t_i) + \underbrace{\epsilon_i}_{\mathcal{N}(0,\sigma^2)}$$

The posterior looks like this:

$$P(\Theta|\mathcal{D}) = P(\mathcal{D}|\Theta) \cdot \pi(\Theta) = \Pi_{i=1}^{n} \mathcal{N}(y_i; \hat{y}(t_i;\Theta), \sigma^2) \cdot \pi(\Theta)$$

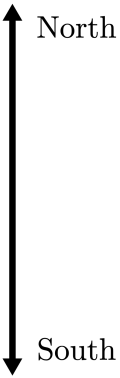
$\xrightarrow{\text{Bayesian model}}$ best values of parameters

Let's deal with the BEAST



Seasonal Mann Kendall test - Trends for different cities in the UK

City	Trend
Wick	-
Aberdeen	-
North Shields	-
Immingham	$\downarrow$
Liverpool	-
Lowestoft	-
Portsmouth	$\uparrow$
Bournemouth	-
Weymouth	-
Devonport	-
Newlyn	$\uparrow$

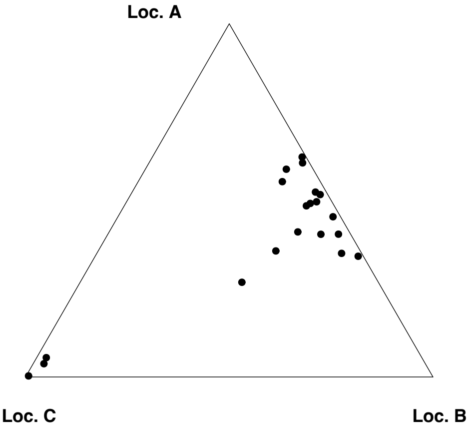


Detecting changes in extremal dependence

> "A changing climate leads to changes in frequency, severity [and] **spatial extent** . . . of climate extremes and can result in unprecedented extremes."
>
> — IPCC 2018 report

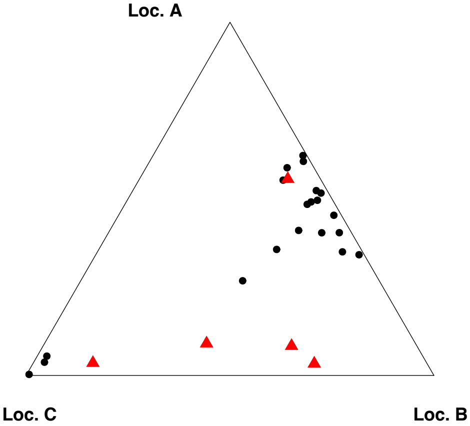
- How are spatial patterns in UK flooding changing over time?
- What are the implications for flood risk mitigation?

What is a change in extremal dependence?



Historical data shows extremes hit A and B jointly, or C singly.

What is a change in extremal dependence?



Newly observed events hit B and C jointly.

What exists and what doesn't

- Hyp. test for non-constant dependence with $d = 2$ locations (Drees 2022).
- DAMEX algorithm detects abnormal extremes, even for large d (Goix 2017).

Develop an algorithm for modelling (high-dimensional) extremes with non-constant dependence.

Directions for future work

- Further extend fitting of Generalised Extreme Value distribution – include additional covariates to improve model fit
- How to extrapolate to improve predictions on extreme events.
- How to include the Generalised Extreme Value distribution in the BEAST framework.
- Make predictions of e.g. Value at Risk and Expected Shortfall/Conditional Value at Risk using the GEV model, and test quality of predictions
- Spatial dependence - develop models that permit changing dependence structure; apply to UK coastal/river flooding data.

Thanks!
Any Questions?