Suspension Modelling of Drilling Fluid

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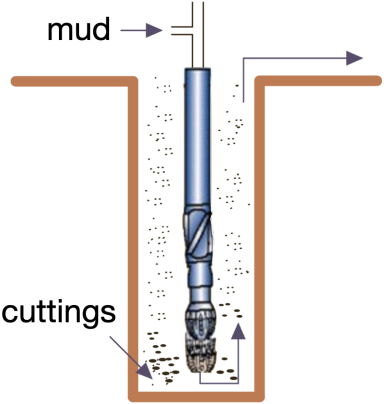
Schlumberger SAMBa

What's the problem?



- Operation time
  - Cuttings blockage
  - Mud velocity
- Liquid loss
  - Cost issue
  - Safety issue

What's the problem?

Table 1 – Principal and constitutive equations of the Eulerian–Eulerian (EE) model.

Principal governing equations and hydrodynamic forces	
Continuity equation	$\frac{1}{\rho rs}\left(\frac{\partial}{\partial t}(\alpha_s\rho_s)+\nabla\cdot(\alpha_s\rho_s\vec{v}_s)=\sum_{l=1}^{n}(\dot{m}_{ls}-\dot{m}_{sl})\right)$
Fluid-Fluid Momentum conservation equation	$\frac{\partial}{\partial t}(\alpha_l\rho_l\vec{v}_l)+\nabla\cdot(\alpha_l\rho_l\vec{v}_l\vec{v}_l)=-\alpha_l\nabla p+\nabla\cdot\tau_l+\alpha_l\rho_l\vec{g}+\sum_{s=1}^{N}[K_{ls}(\vec{v}_l-\vec{v}_s)+\dot{m}_{ls}\vec{v}_{ls}-\dot{m}_{sl}\vec{v}_{sl}]+(\vec{F}_q+\vec{F}_{lift,q}+\vec{F}_{wl,q}+\vec{F}_{vm,q}+\vec{F}_{td,q})$
Fluid-Solid Momentum conservation equation	$\frac{\partial}{\partial t}(\alpha_s\rho_s\vec{v}_s)+\nabla\cdot(\alpha_s\rho_s\vec{v}_s\vec{v}_s)=-\alpha_s\nabla p-\nabla p_s+\nabla\cdot\tau_s+\alpha_s\rho_s\vec{g}+\sum_{s=1}^{N}[K_{ls}(\vec{v}_l-\vec{v}_s)+\dot{m}_{ls}\vec{v}_{ls}-\dot{m}_{sl}\vec{v}_{sl}]+(\vec{F}_q+\vec{F}_{lift,q}+\vec{F}_{wl,q}+\vec{F}_{vm,q}+\vec{F}_{td,q})$
Fluid-Solid exchange coefficient (Syamlal and O'Brien, 1987)	$K'_{sl}=\frac{3\alpha_S\alpha_l\rho_l}{4v_{r,s}^2\psi d_S}C_D\left(\frac{Re_S}{v_{r,s}}\right)\left\|\vec{v}_s-\vec{v}_l\right\|$
Drag coefficient (Dalla Valle, 1943)	$C_D=\left(0.63+\frac{4.8}{\sqrt{\frac{Re_S}{v_{r,s}}}}\right)^2$
Particle Reynolds number	$Re_s=\frac{\rho_l d_S\|\vec{v}_S-\vec{v}_l\|}{\mu_l}$
Particle terminal velocity and coefficients	$v_{r,s}=0.5\left(A-0.06Re_s+\sqrt{(0.06Re_s)^2+0.12Re_s(2B-A)+A^2}\right)$ $A={\alpha_l}^{4.14}$ $B=0.8{\alpha_l}^{1.28}\ for\ \alpha_l\leq 0.85\ and\ B=0.8{\alpha_l}^{2.65}\ for\ \alpha_l>0.85$
Constitutive equations for solid–liquid multiphase flow	
Granular viscosity — Syamlal et al., 1993	$\mu_s=\frac{\alpha_S d_S\rho_S\sqrt{\Theta_S\pi}}{6(3-e_{SS})}\left[1+\frac{2}{5}g_{0,SS}\alpha_S(1+e_{SS})(3e_{SS}-1)\right]$
Granular bulk viscosity — Lun et al., 1984	$\lambda_S=\frac{4}{3}\alpha_S^2\rho_S d_S g_{0,SS}(1+e_{SS})\left[\frac{\Theta_S}{\pi}\right]^{1/2}$
Collisional dissipation of energy — Lun et al., 1984	$\gamma_{\Theta_S}=\frac{12(1-e_{SS}^2)g_{0,SS}}{d_S\sqrt{\pi}}\rho_S\alpha_s^2{\Theta_S}^{3/2}$
Solids Pressure — Lun et al., 1984	$p_s=\alpha_s\rho_S\Theta_S+2\rho_S(1+e_{SS})\alpha_S^2 g_{0,SS}\Theta_S$
Granular temperature	$\Theta_S=\frac{1}{3}(v_{S,i}\cdot v_{S,i})$
Radial Distribution — Lun et al., 1984	$g_{0,SS}=\left[1-\left(\frac{\alpha_S}{\alpha_{S,max}}\right)^{\frac{1}{3}}\right]^{-1}$
Frictional Pressure — Johnson and Jackson, 1987	$P_{friction}=Fr\frac{(\alpha_S-\alpha_{S,min})^n}{(\alpha_{S,max}-\alpha_S)^p}$

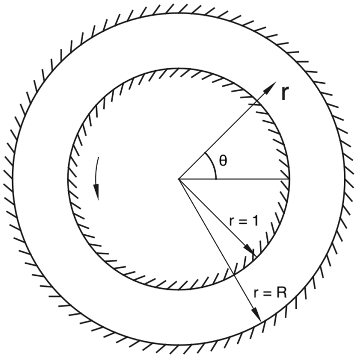
No real-time results:

Simulation for seconds of the process requires hours of running time.

What do we want?

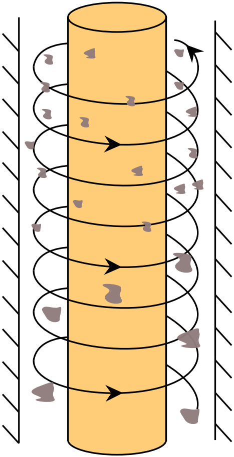
Simple fast math models which involves dominant physics.

What's the geometry?



- The spinning pipe
- The mud
- The cuttings
- The surrounding rock

What's the geometry?



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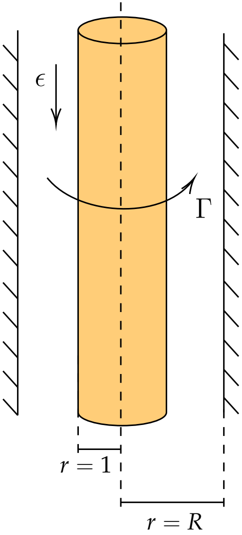
Mud Modelling — Setup



- Flow up an annular pipe.
- Drill moves downwards with small speed ϵ, rotational speed Γ.
- Use cylindrical coordinates: $r, \theta, x = \epsilon z$.
- Flow with velocity components $u_r = \epsilon v_r, u_\theta, u_z$.
- BCs:

$$v_r(1) = 0, \qquad v_r(R) = V_R(x),$$

$$u_z(1) = -\epsilon, \quad u_z(R) = 0,$$

$$u_\theta(1) = \Gamma, \quad u_\theta(R) = 0.$$

Mud — Governing Equations

We obtain:

Conservation of mass: $$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial u_z}{\partial x} = O(\epsilon),$$

Radial momentum: $$\nu\frac{\partial q}{\partial r} + \frac{u_\theta^2}{r} = O(\epsilon),$$

Azimuthal momentum: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} = O(\epsilon),$$

Axial momentum: $$c(x) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = O(\epsilon).$$

Here, pressure has been decomposed as:

$$p = -\rho\nu\left[\epsilon^{-1}\int^x c(\hat{x})d\hat{x} + q(x,r)\right].$$

Mud Fluid Profile

- Radial momentum determines q.
- Azimuthal momentum: Taylor-Couette flow

$$u_\theta = \frac{\Gamma}{R^2 - 1}\left[\frac{R^2}{r} - r\right] + O(\epsilon).$$

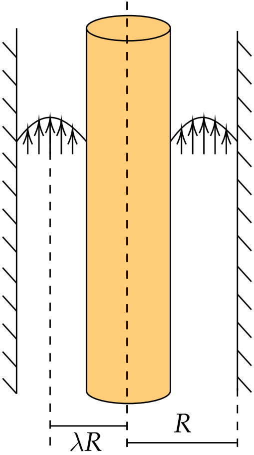
- Axial momentum: Poiseuille flow

$$u_z = \frac{c(x)}{4}\left[1 + (R^2 - 1)\log_R(r) - r^2\right] + O(\epsilon)$$

- Conservation of mass finds v_r and c:

$$c(x) = f(R)\int^x v_R(\hat{x})\, d\hat{x}.$$

What about non-Newtonian flow?



- General form of conservation of momentum:

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = -\nabla p + \nabla \cdot \boldsymbol{T}.$$

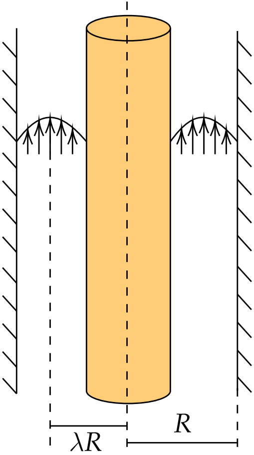
- Try considering a *power law* for the extra stress $\boldsymbol{T}$:

$$\boldsymbol{T} = \mu(\dot{\gamma})(\nabla \mathbf{u} + \nabla \mathbf{u}^T),$$

with

$$\mu(\dot{\gamma}) = K \dot{\gamma}^{n-1}.$$

What about non-Newtonian flow?



- What happens to axial momentum?
- If $1 \leq r \leq \lambda R$

$$u_z = -G|G|^{1/n-1}\left|\frac{1}{2K}\right|^{1/n}\int_1^r\left(\frac{\lambda^2R^2}{\zeta}-\zeta\right)^{1/n}d\zeta + c_1$$

- If $\lambda R \leq r \leq R$

$$u_z = -G|G|^{1/n-1}\left|\frac{1}{2K}\right|^{1/n}\int_r^R\left(\zeta-\frac{\lambda^2R^2}{\zeta}\right)^{1/n}d\zeta + c_2$$

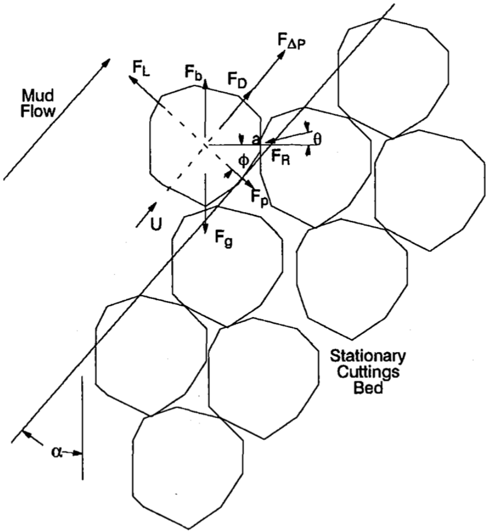
- Much more complicated!

Modelling Cutting transport

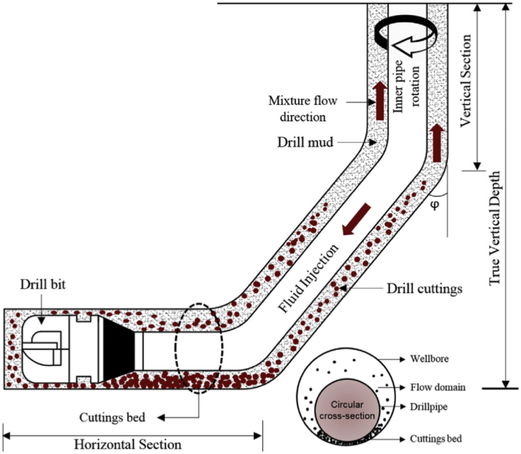
Two phase flow model or Kinetics model?

Modelling Cutting transport

Two phase flow model or Kinetics model?

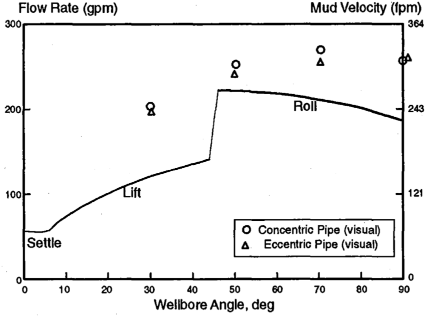


Different regimes



Different regimes

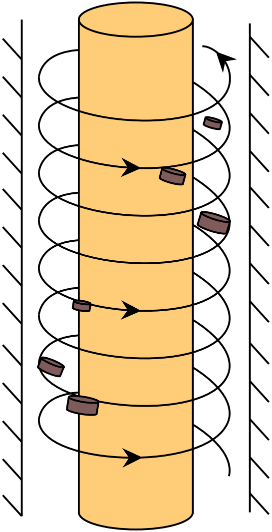
Kinetic models have been explored by Clark et al, with a comparison to experimental results.



Cuttings transport to the surface depends on wellbore angle:

- Near-horizontal angles: rolling behaviour
- Intermediate angles: lifting behaviour
- Near-vertical angles: settling behaviour

Drag-gravity balance



In a steady state where the drag of mud balances the gravity of cuttings:

$$F = \frac{\rho\pi}{2} r^2 u^2 - \rho\pi r^3 g = 0$$

$$\Rightarrow u_{rel} \approx \sqrt{2gr}$$

The velocity difference between the mud and the cutting is proportional to the radius of the cutting discs.

What's next?

Write mixture velocity in terms of the respective flow rates of the cuttings and mud, i.e.

$$U_{mix} = \frac{Q_c + Q_m}{A}.$$

This lends itself well to modelling the cuttings and mud separately to then ascertain the overall behaviour of the mixture, i.e. taking the approach for modelling the mud mentioned before, and then plugging this in alongside a kinetic approach for the cuttings.

What's next?

- Modelling mixture velocity.
- Coupling the non-Newtonian characteristics with the Newtonian mud-loss model.
- Developing simple fast math models for sediment transport.