Placement Optimisation of Tidal Turbines

Team 4: Anna Young, Anton Souslov, Carmen C. van-de-l'Isle, Guannan Chen, Matthew Nunes, Sebastian Scott, Tristan Pryer, Wilfred Armfield, Zsófia Talyigás

ITT 15: Challenge for Bath Beacon

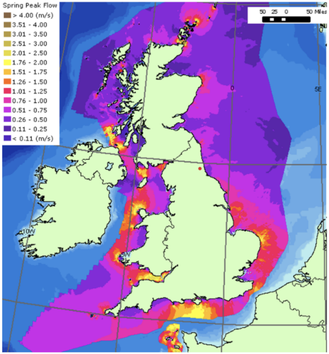
December 17, 2022

University of Bath

Problem Overview

Our problem has 2 Layers:

1. Placement of turbines across the UK
   - Can limit investigation to areas of high peak flow
2. Placement of turbines within a given channel
   - Fluid mechanics of individual turbines "sub-problem"



Peak flow heat map across the UK [1]

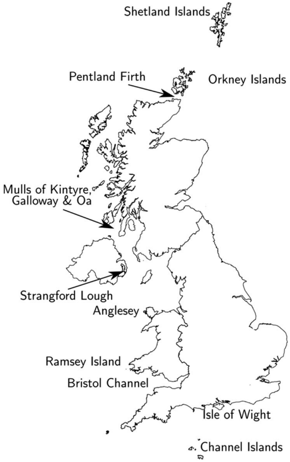
Toy problem: Sine of the Tides

Sine of the Tides Preliminaries

- Tide velocity U is approximately sinusoidal with a period of half a lunar day (12 hours 25 minutes)
- Power proportional to U^3: $P =$

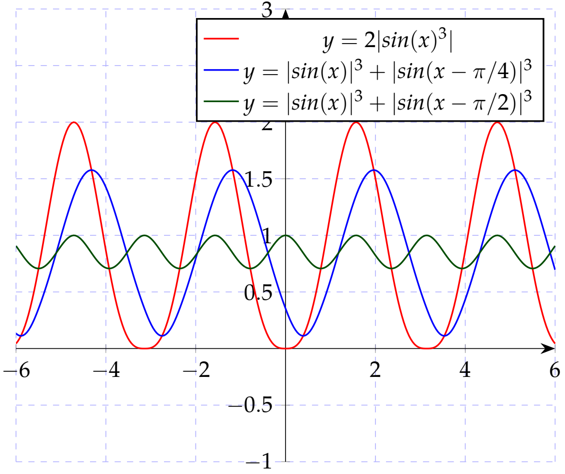
$$\frac{1}{2} \underbrace{\rho}_{\text{water density}} \overbrace{U^3}^{\text{velocity}} \pi \underbrace{R^2}_{\text{turbine radius}} \overbrace{C_{eff}}^{\text{efficiency}}$$

- Sinusoidal curves out of phase superpose destructively



Optimal Turbine Locations [4]

Motivational Plot



Plots of the Sum of $\sin^3$ Graphs in Varying Phase

Sine of the Tides Formulation

- Total Power at time t,

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \sin^3(\frac{2\pi}{T}(t-\theta_i)) \tag{1}$$

- We want to find $\boldsymbol{\alpha}^*$, where

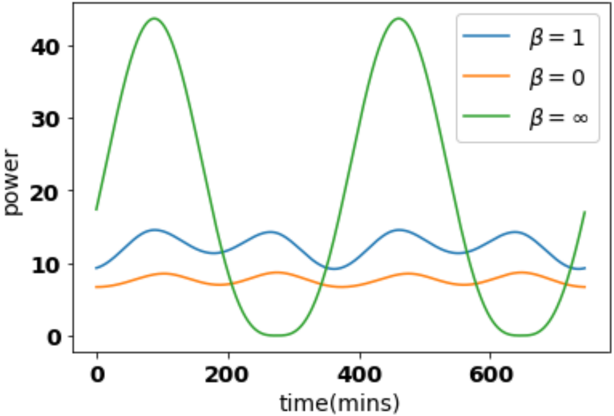
$$\boldsymbol{\alpha}^* := argmin_{\boldsymbol{\alpha}\in\Delta^S} \left\{Var(P(t|\boldsymbol{\alpha})) - \beta\mathbb{E}(P(t|\boldsymbol{\alpha}))\right\}, \tag{2}$$

- β is a tuning parameter to be determined

Sine of the Tides Data and Results, $\beta = 1$

Eight sites of tidal stream resource				
Site Name (i)	θ_i	U_i	Channel Width (km)	$\alpha_i(\%)$
Stroma	**647**	**3.52**	**12.49**	**33**
Kintyre	36	2.44	21.82	0
Drummore	13	2.45	39.33	0
Cemaes Bay	682	2.81	15.16	0
Ramsey Sound	**450**	**2.85**	**12.99**	**57**
Avonmouth	**479**	**2.28**	**19.31**	**10**
Ventnor	692	2.57	43.14	0
Maseline Pier	445	2.23	16.85	0

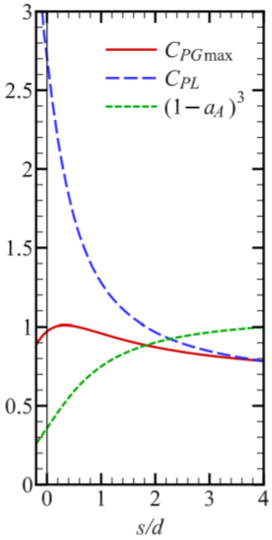
Sine of the Tides Plot Comparison



Comparison of Optimal plots for different values of β

Sine of the Tides Scaling Limit

- Linearity of turbine power goes away for large n
- Only scales up to 431MW output
  - Not enough!
- s = intra-turbine space
- d = diameter of turbine



Turbine Efficiency [3]

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \underbrace{\alpha_i}_{\text{1. extra constraints}} U_i^3 \sin^3\left(\frac{2\pi}{T}(t-\theta_i)\right) \qquad (3)$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i \underbrace{U_i^3}_{\text{2. not constant}} \sin^3\left(\frac{2\pi}{T}(t-\theta_i)\right) \tag{3}$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \underbrace{\sin^3}_{\text{3. no cutoff}} \left(\frac{2\pi}{T} (t - \theta_i) \right) \tag{3}$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \sin^3(\frac{2\pi}{T}(t-\theta_i)) \tag{3}$$

$$\boldsymbol{\alpha}^* := argmin_{\boldsymbol{\alpha}\in\Delta^S} \left\{ \underbrace{Var}_{\text{4. appropriate?}} (P(t|\boldsymbol{\alpha})) - \beta\mathbb{E}(P(t|\boldsymbol{\alpha})) \right\} \tag{4}$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \sin^3\left(\frac{2\pi}{T}(t-\theta_i)\right) \tag{3}$$

$$\boldsymbol{\alpha}^* := argmin_{\boldsymbol{\alpha}\in\Delta^S} \left\{ Var(P(t|\boldsymbol{\alpha})) - \underbrace{\beta}_{\text{5. which } \beta?} \mathbb{E}(P(t|\boldsymbol{\alpha})) \right\} \tag{4}$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \sin^3(\frac{2\pi}{T}(t-\theta_i)) \tag{3}$$

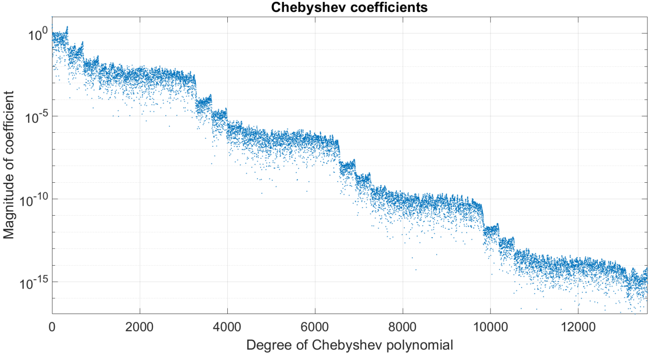
$$\boldsymbol{\alpha}^* := argmin_{\boldsymbol{\alpha}\in\Delta^S} \left\{ Var(P(t|\boldsymbol{\alpha})) - \beta \underbrace{\mathbb{E}(P(t|\boldsymbol{\alpha}))}_{\text{6. linear?}} \right\} \tag{4}$$

Sine of the Tides Future Work

$$P(t|\boldsymbol{\alpha}) := \sum_{i=1}^{S} \alpha_i U_i^3 \sin^3\left(\frac{2\pi}{T}(t-\theta_i)\right) \tag{3}$$

$$\boldsymbol{\alpha}^* := argmin_{\boldsymbol{\alpha}\in\Delta^S} \underbrace{\{Var(P(t|\boldsymbol{\alpha})) - \beta\mathbb{E}(P(t|\boldsymbol{\alpha}))\}}_{\text{7. right objective?}} \tag{4}$$

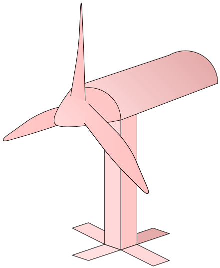
Sine of the Tides Chebyshev Expansion Coefficients



Chebyshev Expansion Coefficients for EMEC Tidal flow data

Single Site Scenario

- Individual turbine design
- Optimal turbine layout
  - Wake effects



Turbine bolted to the sea bed

Single Site Scenario

- Individual turbine design
- Optimal turbine layout
  - Wake effects

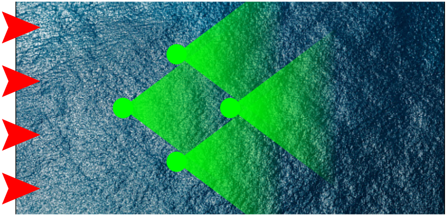


Diagram of a turbine array

Turbine layout optimisation

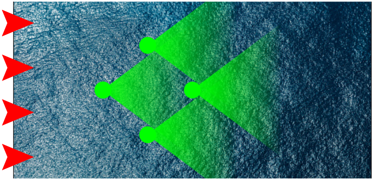
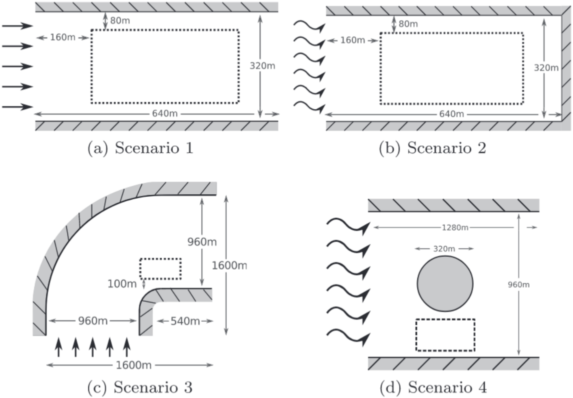


Diagram of a turbine array

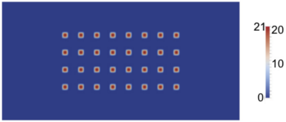
$$\begin{aligned} \max \quad & P(\boldsymbol{\tau}) && \text{power} \\ \text{subject to} \quad & \mathbf{F}(\mathbf{u}, \boldsymbol{\tau}) = \mathbf{0} && \text{governing equation} \\ & \mathbf{r} \leq \mathbf{0} && \text{additional restrictions} \end{aligned}$$

Turbine layout optimisation

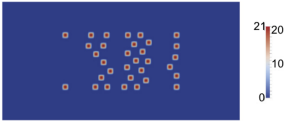


Four idealised scenario examples (1D)[2]

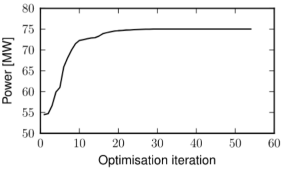
Turbine layout optimisation



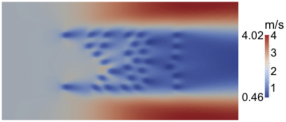
(a) Initial turbine positions



(b) Optimised turbine positions



(c) Optimisation convergence

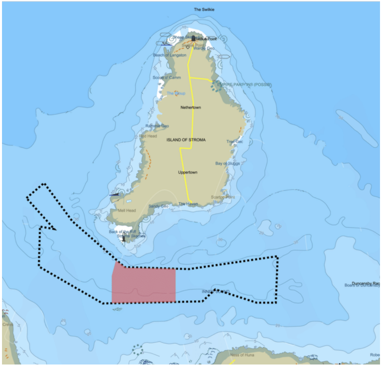


(d) Velocity magnitude

Results of scenario 1 with minimum distance constraints[2]

Turbine layout future Work

- Realistic tidal forcing and geometry
  - Penalizing the use of certain positions
- The impact of large arrays on the free-stream velocity.
- Account for bathymetric effects.



A potential site map

[1] https://www.renewables-atlas.info/explore-the-atlas/.

[2] Simon W Funke, Patrick E Farrell, and MD Piggott. "Tidal turbine array optimisation using the adjoint approach". In: *Renewable Energy* 63 (2014), pp. 658–673.

[3] Takafumi Nishino and Richard HJ Willden. "The efficiency of an array of tidal turbines partially blocking a wide channel". In: *Journal of Fluid Mechanics* 708 (2012), pp. 596–606.

[4] C. R. Vogel et al. "Prospects for Tidal Stream Energy in the UK and South America: A Review of Challenges and Opportunities". In: *Polytechnica* 2.1 (2019), pp. 97–109. doi: 10.1007/s41050-019-00017-y. url: https://doi.org/10.1007/s41050-019-00017-y.

Thank you