# Sequential Expert Consultation: On the Optimal Ordering of Product Reviews*

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**Abstract**

A decision maker consults product reviews sequentially on a platform. Reviews are either authentic or fake and each review is costly to read. In each round, the decision maker chooses in a sequentially rational way whether and which review to read. The platform's ordering rule determines the order of presentation of reviews as a function of their probability of being fake. We characterize the optimal ordering rule and show that it is stochastic. We find that removing reviews that are likely to be fake may worsen information transmission and that it is not true that reviews that are less likely to be fake should be consulted first.

**Keywords**: Cheap Talk; Strategic Information Transmission; Sequential Information Acquisition; Fake Reviews.

**JEL classification**: D81, D83.

## 1 Introduction

In many contexts, a decision maker acquires information from several sources before making a decision. A potential buyer on an online shopping platform might read several product reviews before making a purchase decision, a voter might read several online news articles before making up his mind on a political question. Consulting several sources can often

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be beneficial for decision makers because sources have uncertain goals or are imperfectly informed. In many instances, consultation occurs sequentially because each message that the decision maker reads requires time to process. In an online context, sources are furthermore typically made available by a platform (e.g., Google or Amazon) which determines the order in which sources are presented.

Though the above description matches a number of information search problems, our focus here is on online product reviews, where fake reviewers tied to firms coexist with authentic and benevolent reviewers. The phenomenon is widely acknowledged as economically significant. Competition authorities in the UK and US have since 2021 investigated Google and Amazon, alleging excessive leniency. A 2023 UK governmental report finds a prevalence of 10-15% of fake reviews across common goods and the rapid rise of AI language models could exacerbate the problem.[1] While focusing on reducing the number of fake reviews via legal and technological means is natural, we are witnessing an arms' race between producers and policers of fake reviews whose outcome is unpredictable.

Our approach is to optimize the design of review systems to minimize the harm caused by fake reviewers. We focus on the rule governing the presentation order of reviews, which we assume can condition on the platform's privately observed estimate of reviewers' trustworthiness.[2] Does the ordering rule affect the informativeness of reviewers and if so, how? What is the optimal rule, in terms of maximising information transmission? Is it deterministic or stochastic, and how exactly is it conditioned on reviewers' trustworthiness? And if an optimal rule is used, is it the case that removing reviews that are likely to be fakes improves consumer welfare?

To answer these questions, we analyse the following model. A decision maker (DM) must choose an action $a$ to match the underlying unobserved state (product quality) drawn from the unit interval. He faces $n$ perfectly informed and indistinguishable reviewers whom he consults sequentially, each at an arbitrarily small cost $c$. He consults in a sequentially rational way: at any point in time he chooses whether and whom to consult (where reviewers only differ in terms of their position in the presentation order) in a way that maximises his current expected payoff. Reviewers have two possible (privately observed) preference types: *unbiased* or *biased*. Unbiased reviewers share DM's objective to match the state. Biased reviewers wish to maximise DM's action. Each reviewer's probability of being unbiased (which parametrises his trustworthiness) is privately observed by the platform. DM only knows the aggregate profile (i.e., the empirical distribution) of trustworthiness levels. Reviewers know

[1]"Fake online reviews research: Investigating the prevalence and impact of fake online reviews", Department for Business and Trade, April 2023, Alma Economics

[2]In the case of online platforms, reviewers' trustworthiness can be estimated based on their account history (other reviews, account date creation, etc.)

the aggregate profile and their own trustworthiness. Reviewers communicate simultaneously via a cheap talk message while the ordering rule being commonly known.

We first show that under mild conditions every informative equilibrium in this class is partitional. The state space is partitioned into $N$ intervals corresponding to $N$ messages $t_1, .., t_N$. If reviewer $i$ is unbiased, he sends $t_r$ if the state lies in the $r$-th interval. If reviewer $i$ is biased, he always sends $t_N$. DM consults reviewers following the order of presentation. He continues to consult as long he has only received message $t_N$ and stops as soon as he observes $t_r < t_N$. For each ordering rule, we focus on the most informative equilibrium within this class, which is the equilibrium with the largest number of intervals.

Our focus is on identifying ordering rules that maximise DM's expected utility. If reviewers are equally trustworthy, intuition suggests that the rule should not matter. If reviewers are unequally trustworthy, consulting more trustworthy reviewers first would appear beneficial. We find that both of the above intuitions are violated. Whether or not reviewers are equally trustworthy, using a well chosen stochastic ordering rule improves DM's expected utility by improving individual reviewers' informativeness.

The fundamental driving force behind the result is a pre-emption motive. When the state is low, the earlier a biased reviewer's expected position in the presentation order and the higher the trustworthiness of reviewers located after him, the higher his incentive to deviate to the second highest message so as to pre-empt further consultation which might reveal the low state. In the example of an online store with reviews going from 1-5 starts, the pre-emption motive means that fake reviewers should only have incentives to post 5 start reviews.

We first show that the problem of finding an ordering rule that maximizes informativeness is a max-min problem, as in maximizing the minimum value from continued consultation when the state is very low. Ceteris paribus, increasing the likelihood that a very trustworthy reviewer is asked early improves the incentives of biased reviewers later in the order to send the highest message as opposed to the second highest one, because their message being read means that the very trustworthy reviewers all sent the highest message. The ordering rule that maximizes informativeness trades off these forces by making all reviewers' position uncertain and aligning expectations across reviewers. We then show that among rules that maximize informativeness, we can always find one such that it is incentive compatible for DM to consult reviewers following the presentation order

In terms of the application for online platforms, our results imply that it is not true that removing reviews that are likely sent by fake reviewers unequivocally improves information transmission. The reason is that there are two effects at play: fake reviews lower the quality of the reviews, but they improve the amount of information shared in equilibrium. We find that the net effect can go either way.

We then present a number of extensions. Our first extension focuses on deterministic rules. In our second extension, DM observes the trustworthiness of individual reviewers. It follows immediately that DM consults in a deterministic order, which in turn hurts DM by reducing informativeness. The third extension explores how DM's expected payoff is affected by the aggregate distribution of trustworthiness levels for a fixed number of reviewers. We find that for a fixed probability of the event that all reviewers are biased, the optimal distribution is obtained by maximising the trustworthiness of one reviewer while minimising others' trustworthiness.

**Literature review** Our paper contributes to the literature on cheap talk communication where reviewers are usually referred to as senders (or experts). Our setup builds on Morgan and Stocken (2001) who assume uncertainty about the reviewer's bias in the Crawford and Sobel (1982) canonical cheap talk model. Le Quement (2016) extends this problem to the case where the receiver consults several reviewers sequentially. Le Quement (2016) assumes that the aggregate distribution of trustworthiness levels is degenerate and exogenously imposes the fully random ordering rule. The issue of whether DM has an incentive to follow the presentation order is furthermore by definition trivial in such a homogeneous reviewers setup. This paper instead considers arbitrary distributions of trustworthiness levels and studies all possible ordering rules, focusing on the untouched question of the optimal ordering rule and taking into account the problem of ensuring that DM should be willing to follow the presentation order.

Ottaviani and Sørensen (2001) asks who should speark first in a context where several imperfectly informed reviewers are consulted, the ex ante competence of these reviewers being different and these being motivated by reputational concerns. Krishna and Morgan (2001a) re-examine Gilligan and Krehbiel (1989) and consider both heterogeneous and homogeneous reviewers who all observe the state. They find that some (not all) legislative rules lead to full revelation when combined with heterogeneous preferences.

In Austen-Smith (1993), the receiver faces two reviewers holding noisy information in a binary setup. Under some conditions, full revelation is possible with a single reviewer but not when two reviewers are consulted simultaneously. McGee and Yang (2013) study a setup where a decision maker's optimal decision is a (multiplicative) function of the uncorrelated types of two privately informed reviewers. In Li et al. (2016), a principal has to choose between two potential projects, information about returns being held separately by two reviewers who are each biased towards their own project. In both papers presented above, reviewers' informativeness levels are strategic complements (in contrast to our setup): informative communication by the other reviewer makes deviations from the truth more costly.

Alonso et al. (2008) consider information transmission in a multi-division organization, where each division's profits depend on how its decision matches its privately known local conditions and the other division's decisions. One possible decision protocol is centralization, whereby division managers report to central headquarters which decide for both divisions. They find that a stronger desire to coordinate decisions worsens headquarters' ability to retrieve information from divisions. In Rantakari (2016) or Moreno de Barreda (2010), the receiver is exogenously or endogenously also in possession of some information. The main effect is that information available to the receiver can crowd out the information transmission by the reviewer.

This paper also relates to the literature on Bayesian reputation building in games of information transmission (Sobel (1985), Benabou and Laroque (1992), Morris (2001) and Ely and Välimäki (2003)). Morris (2001) studies a two period advice game with a binary state space and uncertain reviewer preferences identical to ours. An unbiased reviewer has an incentive to lie in the early period in order to achieve a good reputation and be influential later. Our papers share the feature that biased reviewers' behavior exerts a negative externality on the informativeness of unbiased reviewers. In Morris (2001), an unbiased reviewer does not always truthfully announce a high signal because such an announcement hurts his reputation. In our paper, an unbiased reviewer communicates in a noisy way also when the state is not high, so as to discourage biased reviewers from deviating downwards.

The paper also connects to the literature on search and pricing on goods markets (see for example Baye et al (2006), Stahl (1989), Wolinsky (1986), Diamond (1971), Janssen and Parakhonyak (2014), Anderson and Renault (1999) and Baye and Morgan (2001)). In these models, firms also have an endogenous preference over the consumer's search decisions and typically wish to discourage further search. The recent strand on ordered search is of particular relevance. See for example Wright et al. (2019), Haan et al. (2018), Derakhshan et al. (2018), Armstrong (2017), Arbatskaya (2007), Armstrong et al. (2009), Wilson (2010).

Our paper is also related to Glazeret al. (2021), who study fake reviews in a dynamic setting and consider the platforms' problem of either sharing reviews or not based on their content, they find that in in terms of welfare the platform cannot do better than to show all reviews. Our paper considers a static setting where the platform chooses an ordering rule how to display the reviews. We find that not showing certain reviews can under some conditions improve welfare.

The paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the equilibrium for any given ordering rule. Section 4 studies welfare properties and derives the optimal ordering rule. Section 5 examines extensions.

## 2 The Model

The state of the world $\omega$ is drawn from the uniform distribution on $[0,1]$ and captures the underlying true product quality. An uninformed receiver (DM) faces a set of $n$ reviewers (reviewers) $\chi = \{A, B, ..\}$, each of whom privately observes the state and simultaneously sends a cheap talk message $m_i \in M = [0,1]$. In the first phase of the game, the receiver can sequentially consult the reviewers at a cost $c$ per reviewer, where $c$ is arbitrarily small but positive. Once he stops consulting, he picks an action $a \in \Re$ and his utility is $-(\omega - a)^2 - \widetilde{n}c$, where $\widetilde{n}$ is the number of reviewers consulted. The optimal action after information collection is simply the conditional expected value of $\omega$, which may correspond to the quantity of the product purchased. Our assumption on $c$ implies that DM will carry on consulting as long as he expects that more consultation can generate more information.

Each reviewer has a privately observed type (1 or 2). Type 1, the unbiased type, has utility function $-(\omega - a)^2$. Type 2, the biased type, has utility function $a$. Type 1 is thus benevolent while type 2 wants to maximise DM's belief about $\omega$ in order to maximize the sales of the product. Reviewer $i$'s probability of being of type 1 is $p_i$ and thus parameterizes his ex ante trustworthiness. Each reviewer's type is independently drawn. DM knows the empirical distribution of $p_i$'s but does not observe the identity of the reviewer behind each message (the identifier $i$). reviewers know their own identity $(i)$ and the aggregate distribution of $p_i$'s. The platform observes each reviewer's $p_i$. Let $\eta = \prod_{i=1}^{n}(1 - p_i)$, so $\eta$ is the commonly known probability that all reviewers are biased.

The platform presents reviewers' messages in a presentation order which is generated by the commonly known ordering rule $\Gamma$. A presentation order dictates which reviewer's message is to be presented in which position in the sequence of the messages. The ordering rule is deterministic if one presentation order is assigned ex ante probability one. Denote by $D(\chi)$ the set of deterministic orders and denote by $d$ any element of this set. Denote by $\theta_d^\Gamma$ the probability assigned to $d$ under $\Gamma$. An ordering rule is given by $\Gamma = \{\theta_d^\Gamma\}_{d \in D(\chi)}$. Denote by $p^l$ the trustworthiness of the reviewer appearing in position $l$ of the presentation order. Denote by $m^l$ the message of the reviewer in position $l$ in the order. Denote by $p^{l,d}$ the trustworthiness of the reviewer appearing in position $l$ of order $d$.

A reviewer strategy pins down how he communicates for each preference type that he might be assigned and the given known ordering rule. A pure strategy for a reviewer $i \in \{A, B, ..\}$ is given by function $\mu_i^r$, for $r \in \{1, 2\}$, where $\mu_i^r : [0,1] \to [0,1]$ is such that $\mu_i^r(w)$ maps the state of nature $\omega \in [0,1]$ and the reviewer's type $r$ into a message in $M$. Note that we are omitting the ordering rule $\Gamma$ from the strategy simply for notational convenience. A communication strategy is monotone if $\mu_i^r(w)$, for $r \in \{1,2\}$, is weakly increasing in $\omega$. A profile of reviewer strategies induces monotonic beliefs if profiles of messages that are higher yield higher beliefs of DM. A precise definition is provided later.

A pure strategy of DM is composed of a sampling rule and an action rule. A pure sampling rule specifies, for any history of observed messages, whether or not DM continues to consult and which review he consults among the presented reviews. A pure action rule specifies the action $a$ chosen if DM stops consulting, for any history of observed messages.

Our equilibrium concept is Perfect Bayesian Equilibrium (PBE). Under a given ordering rule $\Gamma$, an equilibrium is given by a profile of strategies (one for each reviewer in $\chi$ and one for DM) as well as a system beliefs. A given profile of strategies and a system of beliefs constitute a PBE if players' strategies are sequentially rational given beliefs and other players' strategies, while beliefs are derived via Bayes' rule whenever possible. All the results stated in our analysis, whether positive or normative, are limit results in sense that there is some $\bar{c} > 0$ such that they hold true for any $c \in (0, \bar{c})$.

Note that given $c > 0$, there exists no fully revealing equilibrium in which all reviewers always truthtell and send $m = \omega$ whatever the state and their preference type. Such an equilibrium would be supported by out of equilibrium beliefs such that DM chooses a punishment action (say $a = 0$) whenever messages differ. However, the equilibrium breaks down because DM has a strict incentive to stop after one consultation given $c > 0$.

We focus on reviewer' pure strategies that are symmetric (strategies do not depend on reviewer's identity),[3] monotonic (i.e. weakly increasing in the state) and inducing monotonic beliefs (see the proof of Proposition 1 in Appendix B for a precise definition). We call such equilibria symmetric and monotone.

We show that all informative equilibria within this class **must** be partitional and thereby outcome equivalent to an equilibrium featuring the following simple strategy profile, which we shall focus on. There are thresholds $t_0 = 0 < t_1 < \ldots < t_{N-1} < t_N = 1$. An unbiased reviewer sends message $m = t_r$ if $\omega \in (t_{r-1}, t_r]$ $\forall r = 1, .., N$ and $t_1$ if $\omega = 0$. A biased reviewer always sends $m = t_N$. DM's sampling rule is a stopping rule. He stops consulting as soon as he encounters $t_r \neq t_N$. Indeed, after $t_r \neq t_N$, he acknowledges that he has now learned that $\omega \in (t_{r-1}, t_r]$ and will not learn more by consulting another review. On the other hand, he continues consulting as long as he has only encountered $t_N$ and has not consulted all reviewers. In this case, he remains uncertain about $\omega$ and might gain new information by consulting the next reviewer.

[3]However, we allow the reviewers' strategies to depend on their type $r$, the state of nature $w$, and the ordering rule $\Gamma$ (i.e. the reviewer's expected position in the consultation order).

For any out of equilibrium profile of messages $\mathbf{m}$ for which beliefs cannot be derived via Bayes' rule, denoting by $\underline{m}(\mathbf{m})$ the lowest message in this set, the induced belief of DM is assumed to be $E[\omega \,|\omega \in (t_{r-1}, t_r]]$ if $\underline{m}(\mathbf{m})$ is located in the $r$th interval. Furthermore, DM's (sequential) consultation follows the order of presentation. For this to be incentive compatible, it must be true that after consulting the first $r$ reviewers in the presentation order, the most informative reviewer (in expectation) is in position $r+1$, for any $r \in \{0, .., n-1\}$.[4] We call an equilibrium of the above type a simple partitional equilibrium of size $N$. Finally, we define an informative equilibrium as one in which it is not true that the action of DM is independent of observed messages.

We focus on ordering rules that are optimal from DM's perspective. It seems reasonable to assume that platforms aim at maximising the informativeness of reviews so that instances where a buyer returns a purchased items are minimised. The DM optimal ordering is also weakly or strictly preferred by all reviewer types. Unbiased reviewers share DM's preferences. Biased reviewers are indifferent among all ordering rules, as the expected value of DM's action is constant across all possible information generating experiments by the law of iterated expectation.

# 3 Positive Analysis

**Proposition 1.** *For any informative, symmetric and monotone equilibrium, there exists an outcome equivalent simple partitional equilibrium.*

Proof: See Appendix B.

The above Proposition establishes that under our mild assumptions on the strategies, it is without loss of generality to focus on simple partitional equilibria. In what follows, we characterise necessary and sufficient conditions for the existence of a simple partitional equilibrium featuring partition $\{t_r\}_{r=1}^{N-1}$ under ordering rule $\Gamma$. We analyse separately the incentives of reviewers and DM.

[4] Note that the equilibrium described above still exists under $c = 0$, as DM has no strict incentive to deviate from the assumed behaviour. From DM's perspective, the equilibrium is however trivially dominated by one in which he always consults all reviewers and all truthtell.

### 3.1 Reviewers' Incentives

We start by pinning down the beliefs of DM. Given $\{t_r\}_{r=1}^{N-1}$ and ordering rule $\Gamma$, the belief of DM when he first observes some $t_r \neq t_N$ is given by:

$$E[\omega \,|m = t_r\,] = \frac{t_{r-1} + t_r}{2}.$$

After he has consulted all $n$ reviewers and received message $t_N$ in total $n$ times, his belief is given by

$$\begin{aligned} & E[\omega \left|m^1 = .. = m^n = t_N\right.] \\ &= \frac{(1 - t_{N-1})\,(1 - \eta)}{(1 - t_{N-1})\,(1 - \eta) + \eta} \frac{t_{N-1} + 1}{2} + \frac{\eta}{(1 - t_{N-1})\,(1 - \eta) + \eta} \frac{1}{2}. \end{aligned}$$

The above expected value accounts for two possible events. Either at least one of the reviewers is unbiased, in which case $\omega \geq t_{N-1}$, or nothing has been learned about the state.

Given $\Gamma$ and $\{t_r\}_{r=1}^{N-1}$, an important quantity is the probability assigned by reviewer $i$ to the event that DM will observe $m^1 = .. = m^2 = t_N$ conditional on $\omega = 0$, reviewer $i$ sending $t_N$ and taking as given that $m_i$ will be observed. Denote this quantity by $\Psi_{i,\Gamma}$. We have

$$\Psi_{i,\Gamma} = P_{i,\Gamma}(m^1 = .. = m^n = t_N \,|\omega = 0, m_i = t_N\,).$$

In other words, $\Psi_{i,\Gamma}$ is the probability that the highest message (i.e. $m_i = t_N$) sent by reviewer $i$ is not contradicted when the state is the lowest it can be.

For any order $d \in D(\chi)$ and $i \in \chi$, denote by respectively $\chi_d^{i,-}$ and $\chi_d^{i,+}$ the set of reviewers who are presented before and after $i$. Let

$$Q_{i,\Gamma} := \sum_{d \in D(\chi)} \theta_d^\Gamma \left[\Pi_{j \in \chi_d^{i,-}} (1 - p_j)\right].$$

The above is the ex ante probability that reviewer $i$ will be consulted given $\omega = 0$, $\{t_r\}_{r=1}^{N-1}$ and ordering rule $\Gamma$. For every $d \in D(\chi)$, we have

$$P_{i,\Gamma}(d \left|\omega = 0, m^i = t_N\right.) = \frac{\theta_d^\Gamma \left[\Pi_{j \in \chi_d^{i,-}} (1 - p_j)\right]}{Q_{i,\Gamma}},$$

where by convention $\Pi_{j \in \chi_d^{i,-}} (1 - p_j) = 1$ if $\chi_d^{i,-} = \varnothing$. Next, we have

$$\begin{aligned} \Psi_{i,\Gamma} &= \sum_{d \in D(\chi)} \left(P_{i,\Gamma}(d \left|\omega = 0, m^i = t_N\right.) \left[\Pi_{j \in \chi_d^{i,+}} (1 - p_j)\right]\right) \\ &= \frac{\Pi_{j \neq i} (1 - p_j)}{Q_{i,\Gamma}}. \end{aligned}$$

That is, $\Psi_{i,\Gamma}$ equals the probability that all other reviewers are biased divided by the probability that $i$ is asked. Note the formula for $\Psi_{i,\Gamma}$ is independent of the assumed equilibrium partition $\{t_r\}_{r=1}^{N-1}$.

We now provide necessary and sufficient existence conditions for $\{t_r\}_{r=1}^{N-1}$ to be reviewer incentive compatible.

**Lemma 1.** *Fix* $\Gamma$. *Partition* $\{t_r\}_{r=1}^{N-1}$ *is reviewer incentive compatible if and only if,* $\forall r < N-1$ *and* $\forall i \in \chi$*:*

$$t_r - E[\omega \,|m = t_r] = E[\omega \,|m = t_{r+1}] - t_r, \tag{1}$$

$$t_{N-1} - E[\omega \,|m = t_{N-1}] = E[\omega \,\middle|m^1 = .. = m^n = t_N] - t_{N-1}, \tag{2}$$

$$\Psi_{i,\Gamma} E[\omega \,\middle|m^1 = .. = m^n = t_N] + (1 - \Psi_{i,\Gamma}) E[\omega \,|m = t_1] \geq E[\omega \,|m = t_{N-1}]. \tag{3}$$

Proof: See Appendix A.

Conditions (1) and (2) ensure that unbiased reviewers have no deviation incentives, by requiring that at any threshold $\omega = t_r$, for DM$\in \{1, .., N-1\}$, an unbiased reviewer is indifferent between messages $t_r$ and $t_{r+1}$. Condition (1) implies that $t_r = \frac{r}{N-1} t_{N-1}$ for $r < N-1$. Using this and solving (2) for $t_{N-1}$ given $N$ and $\eta$ yields the unique solution

$$t^*_{N-1} = \frac{2N - \sqrt{4N\eta(-1+N)+1} + 1}{2N(1-\eta)}. \tag{4}$$

So (1) and (2) yield a unique admissible partition

$$\left\{ t^*_1 = \frac{t^*_{N-1}}{N-1}, .., t^*_r = \frac{r t^*_{N-1}}{N-1}, .., t^*_{N-1} \right\} \tag{5}$$

for any $N > 1$ and $\eta \in (0,1)$. Note in particular that the above partition is independent of the assumed ordering rule as the latter does not affect (1) nor (2). This will turn out to be a very useful property when we study optimal ordering rules. Note also that (1) and (2) rewrite as

$$\frac{E[\omega \,\middle|m^1 = .. = m^n = t_N]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}. \tag{6}$$

Condition (3) determines whether the partition pinned down by (1) and (2) is actually reviewer incentive compatible. A biased reviewer $i$ must prefer sending $m^i = t_N$ for any $\omega$. To ensure this, it is sufficient to ensure no deviation incentive at $\omega = 0$ (if there is no incentive to deviate at $\omega = 0$ then there is a weakly lower incentive to deviate at any $\omega \in [0,1]$). Now, consider incentives of a biased reviewer $i$ at $\omega = 0$. Sending $m_i = t_N$ is risky as DM will keep on sampling and with probability $(1 - \Psi_{i,\Gamma})$ may encounter an unbiased reviewer and learn that $\omega \leq t_1$. Sending $m_i = t_{N-1}$ is the best deviation because given the DM's belief and stopping rule it preempts any further sampling while it yields the second highest belief $E[\omega|t_{N-1}]$. Note that $\Psi_{i,\Gamma}$ is smaller the earlier $i$'s expected position in the presentation order and the higher the expected trustworthiness of the reviewers consulted after $i$. Using (1), condition (3) rewrites as:

$$\frac{E[\omega\,|m^1 = .. = m^n = t_N]}{t_{N-1}} \geq \frac{(1-\Psi_{i,\Gamma})+2}{2}. \tag{7}$$

Using (6) to replace the LHS in the above inequality (7), we may conclude that there exists a reviewer incentive compatible $N$-intervals partition if and only if

$$\Psi_{i,\Gamma} \geq \frac{N-2}{N-1}, \forall i \in \chi. \tag{8}$$

Furthermore, such a partition is unique if it exists. Note that $\frac{N-2}{N-1}$ is increasing in $N$, so an equilibrium of larger size (larger $N$) requires higher $\Psi_{i,\Gamma}$. The intuition is that a larger $N$ implies larger $E[\omega\,|t_{N-1}]$ and lower $E[\omega\,|t_1]$, so that a larger $N$ makes it more attractive to deviate to $m^i = t_{N-1}$ given $\omega = 0$. For a given order of consultation $\Gamma$, define

$$\Psi_{\Gamma}^{\min} = \min_{i\in\chi} \Psi_{i,\Gamma}, \tag{9}$$

which captures the incentive to send the highest message of the biased reviewer who has the largest incentive to deviate to the second highest message. Thus a partition of size $N$ is incentive compatible as long as this reviewer (given $\omega = 0$) does not deviate to $t_{N-1}$.

We summarise the above insights in the following Proposition.

**Proposition 2.** *a) Fix $\Gamma$. There exists a reviewer incentive compatible partition of size $N$ if and only if $\Psi_{\Gamma}^{\min} \geq \frac{N-2}{N-1}$. If it exists, it is unique and is given by the partition $\{t_r^*\}_{r=1}^{N-1}$ defined in (5).*

*b) Consider two ordering rules $\Gamma$ and $\Gamma'$. If a reviewer incentive compatible partition of size $N$ exists under both orders, then it features the same partition $\{t_r^*\}_{r=1}^{N-1}$.*

Comparing any two ordering rules $\Gamma$ and $\Gamma'$, we see that either the sets of reviewer incentive compatible partitions under $\Gamma$ and $\Gamma'$ are identical; or one is a superset of the other and contains equilibria of larger size, the only deciding factor being the size of $\Psi_{\Gamma}^{\min}$ and $\Psi_{\Gamma'}^{\min}$. A two-interval equilibrium always exists as $\frac{2-2}{2-1} = 0$, while equilibria of larger size require $\Psi_{\Gamma}^{\min} \geq \frac{1}{2}$.

### 3.2 DM's Incentives

We now analyse DM's incentive to consult following the presentation order. Given that all reviewers have identical type-dependent communication strategies, at any point in time the DM's optimal choice is to consult the reviewer (as pinned down by a position in the presentation order) whose expected trustworthiness is highest. The expected trustworthiness of the first reviewer in the presentation order is

$$E\left[p^1\right] = \sum_{d\in D(\chi)} P(\theta_d^\Gamma)p^{1,d}. \tag{10}$$

Given $\{t_r\}_{r=1}^{N-1}$ and $\Gamma$, assuming that DM has followed the presentation order in the first $k$ rounds of consultation and observed $m^1 = .. = m^k = t_N$, the expected value of $p^l$ for $l > k \geq 1$ is given by

$$E\left[p^l \left| m^1 = .. = m^k = t_N \right.\right] = \sum_{d\in D(\chi)} P(\theta_d^\Gamma \left| m^1 = .. = m^k = t_N \right.)p^{l,d}, \tag{11}$$

where

$$P(\theta_d^\Gamma, m^1 = .. = m^k = t_N) = \theta_d^\Gamma \left( t_{N-1}\prod_{i=1}^{k}(1-p^{i,d}) + 1 - t_{N-1} \right). \tag{12}$$

In words, conditional on $m^1 = .. = m^k = t_N$, DM updates his prior over the set of deterministic sequences assigned positive probability under $\Gamma$, each of which assigns a specific reviewer to position $l$. He uses this to derive the implied weighted average of $p^{l,d}$'s and to thus identify which reviewer to consult next.

**Lemma 2.** *Fix $\Gamma$ and $\{t_r\}_{r=1}^{N-1}$. Consulting following the order of presentation is DM incentive compatible iff :*

$$E\left[p^1\right] \geq E\left[p^l\right] \quad \forall l > 1, \tag{13}$$

$$E\left[p^{k+1} \left| m^1 = .. = m^k = t_N \right.\right] \geq E\left[p^l \left| m^1 = .. = m^k = t_N \right.\right]$$
$$\forall k, l \text{ such that } k \in \{1,..,n-1\} \text{ and } l > k+1. \tag{14}$$

The above condition ensures that DM always wants to follow the presentation order. The first inequality ensures that he wants to consult the first reviewer in the presentation order when consulting first. The second condition ensures that for any $k \in \{1,..,n-1\}$, after observing $m^1 = .. = m^k = t_N$ and thus deciding to consult again, the most informative reviewer is located in position $k+1$ of the presentation order.

# 4 Normative Analysis

## 4.1 Welfare Properties of Equilibria

In a partitional equilibrium featuring partition $\{t_r\}_{r=1}^{N-1}$, the expected payoff of DM is

$$\begin{aligned}
&\Pi_{DM}(N,\eta) \qquad (15)\\
&= -(1-\eta)\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(\frac{t_i+t_{i-1}}{2}-\omega\right)^2 d\omega\right]\\
&-\eta\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(E[\omega\,|m_A=m_B=t_N]-\omega\right)^2 d\omega\right]\\
&-\int_{t_{N-1}}^{1}\left(E[\omega\,|m_A=m_B=t_N]-\omega\right)^2 d\omega.
\end{aligned}$$

Above, the first line of the RHS expression corresponds to the scenario where $\omega \leq t_{N-1}$ and there is at least one unbiased reviewer. The second line is the scenario $\omega \leq t_{N-1}$ and there is no unbiased reviewer. The third line is the scenario $\omega > t_{N-1}$ so that all reviewers send $t_N$. We ignore sampling costs which are assumed arbitrarily small. We obtain the following results.

**Proposition 3.** *We have:*

*a) If an equilibrium of size $N$ exists under two ordering rules $\Gamma$ and $\Gamma'$, then DM achieves the same equilibrium expected payoff $\Pi_{DM}(N,\eta)$ under both ordering rules.*

*b) $\Pi_{DM}(N+1,\eta) > \Pi_{DM}(N,\eta)$ for any $N \geq 1$ and $\eta \in (0,1)$.*

*c) $\frac{\partial \Pi_{DM}(N,\eta)}{\partial \eta} < 0$ for any $N \geq 1$ and $\eta \in (0,1)$.*

Proof: See Appendix A.

The proof of point a) is as follows. Recall first that by point b) of Proposition 2, if an equilibrium of size $N$ exists under two ordering rules $\Gamma$ and $\Gamma'$, then it features the same partition $\{t_r^*\}_{r=1}^{N-1}$. Next, simply note that for a fixed partition, DM's expected utility depends only on one aspect, namely whether or not at least one of the reviewers is unbiased. If all reviewers are biased, he will end up consulting $n$ times and receive message $t_N$ $n$ times regardless of the consultation order. If at least one of the reviewers is unbiased, then given any state $\widetilde{\omega}$ he will end up with the same final belief under any consultation order. Specifically, if $\omega \leq t_{N-1}$, he will learn the interval in which $\widetilde{\omega}$ is located while if instead $\omega > t_{N-1}$, he will observe $n$ times message $t_N$.

Point b) states that among two partitional equilibria, the equilibrium with a larger number of intervals yields a higher expected utility of DM. This reflects the fact that a less coarse partition allows unbiased reviewers to communicate more informatively. Point c) captures the fact that a lower probability of all reviewers being biased implies a higher probability of learning $\omega$ accurately.

### 4.2 Optimal Ordering Rules

We now identify an optimal ordering rule, i.e. a rule that maximises the achievable expected payoff of DM.[5] By Proposition 3, an ordering rule $\Gamma$ is optimal if it yields the equilibrium partition of largest size among all ordering rules. By Proposition 2, an ordering rule $\hat{\Gamma}$ yields the largest achievable reviewer incentive compatible partition if it satisfies:

$$\hat{\Gamma} = \arg\max_{\Gamma} \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{16}$$

Denote by $N^{\max}$ the size of the largest achievable reviewer incentive compatible partition.

In principle, the incentive compatibility constraint of DM could complicate the search for an optimal ordering rule as some partitions that are reviewer incentive compatible under a given $\Gamma$ might not be part of an equilibrium as DM's incentive compatibility conditions are not satisfied. To account for this potential issue, we take a two steps approach in our search for an optimal ordering rule.

We first ignore DM's incentive compatibility condition and find a necessary and sufficient condition for an ordering rule to solve (16). As we will show in Lemma 3 shortly, all of these ordering rules yield the same value of $\Psi_{\Gamma}^{\min}$ and the same largest reviewer incentive compatible partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$. Next, we show that among these ordering rules, there exist at least one such that under this largest partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive compatibility constraint is also satisfied. This second step is accomplished in two substeps, by first identifying a simple class of rules that satisfy (16) and then searching within this class.

We next show that optimal rules are necessarily random in a way that balances the reviewers' likelihood to be biased with their expected order in the sequence. To understand why the optimal order needs to be random, imagine an example with just two reviewers $A$ and $B$. Consider the incentives of $A$ when it is biased and when the state is low ($w = 0$). If $A$ is last in the consultation order with probability 1 then its message is only observed if the message sent by $B$ was $t_N$ (as otherwise DM stops consultation after the first message). Thus, $A$'s best response is to say $t_N$ as that induces the highest possible action of DM with probability 1.

[5] Recall that a rule typically yields a set of simple partitional equilibria, and we focus, for each rule, on the equilibrium that leads to the maximum number of partitions.

If, however, $A$ is first in the consultation order, sending message $t_N$ induces DM to consult again, which runs the risk of $B$ being unbiased and thus sending message $t_1$, which induces DM to play the lowest possible equilibrium action. If instead $A$ sends message $t_{N-1}$ it induces DM to stop consultation and play the second highest equilibrium action. $A$ thus has a trade-off between inducing the second highest equilibrium action and risking either the highest or lowest equilibrium action.

Therefore, the optimal order should put $A$ second in the consultation order. However, the same reasoning applies to reviewer $B$. This means that that it would be optimal for both reviewers to be second. The way to implement this is to choose an order that assigns positive probability to both the event where $A$ is the last and the event where $B$ is the last.

The optimal order is random in a way that balances the probabilities of every deterministic order with the probabilities of being biased of each reviewer. If in the example above $A$ is very likely to be biased and $B$ is very unlikely to be biased, the optimal orders assigns a higher probability to the order where $A$ is the last. This is achieved by equating all reviewers' beliefs, conditional on its message being observed, about the likelihood that DM observes only the highest message after the reviewer's own message, i.e. by equating $\Psi_{i,\Gamma}$ for all reviewers $i$.

**Lemma 3.** *An order $\Gamma$ satisfies (16) if and only if*

$$\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta} \sum_k \frac{p_k}{1-p_k}$$

*for all $i, j \in \chi$.*

Proof: See Appendix A.

The above Lemma has two important features. First, in all rules satisfying (16), we have $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for any $i, j$, and therefore the largest reviewer incentive compatible partition is identical. To see this, assume there exists exists exactly one reviewer $k$ such that $\Psi_{k,\Gamma} \leq \Psi_{j,\Gamma}$ for all $j$, with at least one strict inequality say for reviewer $l$. Then by continuity of the functions $\{\Psi_{i,\Gamma}\}_i$, which are linear equations on the probabilities of all deterministic orders with coefficients that are polynomials in $\{p_i\}_i$, it is possible to find an order $\Gamma'$ where $\Psi_{k,\Gamma'} > \Psi_{k,\Gamma}$ and $\Psi_{k,\Gamma'} < \Psi_{l,\Gamma'} < \Psi_{l,\Gamma}$. This means that $\min_{i\in\chi}\Psi_{i,\Gamma'}$ is larger than $\min_{i\in\chi}\Psi_{i,\Gamma}$, which leads to a contradiction.

The second insight from Lemma 3 is that if the reviewers' beliefs are such that $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for all $i, j$, these beliefs pin down a unique admissible set of beliefs for all reviewers, namely $\Psi_{i,\Gamma} = \frac{\eta}{1-\eta} \sum_k \frac{p_k}{1-p_k}$ for all $i$. This means that given any ordering rule, we can easily check whether or not it satisfies (16).

While Lemma 3 gives necessary and suffient conditions for an presentation order to satisfy (16), one faces an issue of dimensionality when explicitly constructing ordering rules that satisfy (16). Lemma 3 yields $n$ equations whereas an ordering rule is pinned down by $n! - 1$ unkowns, as there are $n!$ possible deterministic orders while the probabilities of all deterministic orders must add up to 1. We take a constructive approach and identify a class of ordering rules that satisfy (16). The class builds on the concept of latin squares, first studied in the 18th century by Korean and Swiss mathematicians Choi Seok-Jeong and Leonhard Euler.

**Definition** *a) A latin square ordering rule is an ordering rule such that exactly $n$ deterministic orders $\{d_1, \ldots, d_n\}$ have strictly positive probability and for every $i \in \chi$ and $l \in \{1, .., n\}$, there is a unique $d \in \{d_1, \ldots, d_n\}$ for which reviewer $i$ occupies position $l$. b) A proportional latin square ordering rule is a latin square ordering rule such that for any $d \in \{d_1, \ldots, d_n\}$, $\theta_d = \frac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)}$, where $i$ is the reviewer who occupies position 1 in $d$.*

For example, for $\chi = \{A, B, C\}$ there are two possible latin square ordering rules. One is such that only $\{\theta_{ABC}, \theta_{BCA}, \theta_{CAB}\}$ are positive and the other is such that only $\{\theta_{ACB}, \theta_{BAC}, \theta_{CBA}\}$ are positive. One can represent each of these as a square, where each row corresponds to a different deterministic order assigned positive probability. Each of the obtained squares is a latin square. The first of these latin square rules yields the following proportional latin square ordering rule:

$$\begin{aligned}
\theta_{ABC} &= \frac{p_A/(1-p_A)}{\sum_{i\in\chi} p_i/(1-p_i)}, \\
\theta_{BCA} &= \frac{p_B/(1-p_B)}{\sum_{i\in\chi} p_i/(1-p_i)}, \\
\theta_{CAB} &= \frac{p_C/(1-p_C)}{\sum_{i\in\chi} p_i/(1-p_i)}.
\end{aligned}$$

Note that in any proportional latin square ordering rule, the probability that a reviewer appears first in the presentation order is increasing in the reviewer's own trustworthiness and decreasing in other reviewers' trustworthiness.

**Lemma 4.** *All proportional latin square ordering rules satisfy (16).*

Proof: See Appendix A.

The above Lemma establishes that proportional latin square rules achieve the maximal partition size if we ignore DM's incentive constraint. There is no known way to characterize all latin squares of a general order $n$. Moreover, it is not known how many latin squares of a particular order exist, although this number is exponentially increasing in $n$. Thus, since proportional latin squares are a subset of all possible optimal ordering rules, there is no known way to characterize all optimal rules.

The next question is whether we can find any proportional latin square ordering rule that is, furthermore, incentive compatible for DM. The answer is positive.

**Lemma 5.** *Among the set of proportional latin square rules, there is one such that given the rule and maximum partition it induces, consulting according to the order of presentation is incentive compatible for DM.*

The proof of the above Lemma is as follows. There is a simple $(n-1)$-step algorithm for identifying a proportional latin square rule $\Gamma^*$ such that given $\Gamma^*$ and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive conditions are satisfied. To see this, start from an arbitrary proportional latin square ordering rule $\Gamma$ and assume that the equilibrium partition is $\{t_r^*\}_{r=1}^{N^{\max}-1}$. It is easy to show using the rearrangement inequality that in the first consultation by the DM, the reviewer appearing in position 1 of the presentation order is the most trustworthy reviewer in expectation.

Consider now the second consultation assuming that DM observed message $t_N^*$ in the first consultation. Assess the relative trustworthiness of reviewers located in positions 2 to $n$ of the presentation order. If reviewer 2 is the most trustworthy reviewer, then keep the ordering rule $\Gamma$ and proceed to reviewer 3. If instead some $r > 2$ is the most trustworthy reviewer, then construct a new ordering rule $\Gamma'$ by permutating reviewers 2 and $r$ in all of the $n$ sequences that have positive probability under $\Gamma$. Note that $\Gamma'$ is also a proportional latin square rule and it is such that in the second round, reviewer 2 is the most trustworthy reviewer. The reason is that in the second round, the expected trustworthiness of reviewers 2 and $r$ have now been interchanged, as is immediately clear from (11) and (12).

Repeat the procedure for the third consultation, by checking who is the most trustworthy reviewer after $t_N^*$ has been observed in the first and second rounds. Iterate the procedure until consultation in the $n$-th round is reached. At this point, one has constructed a proportional latin square rule $\Gamma^*$ such that given $\Gamma^*$ and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive conditions are satisfied.

# 5 Extensions

## 5.1 Optimal Deterministic Ordering Rules

Note first that any deterministic ordering rule is trivially suboptimal. For a deterministic ordering rule $\Gamma$ pinned down by $d \in D(\chi)$, recalling that $\chi_d^{i,+}$ denotes the set of reviewers who are consulted after reviewer $i$, we have $\Psi_{i,d} = \Pi_{j \in \chi_d^{i,+}}(1-p_j)$ and $\Psi_d^i \neq \Psi_d^j$ for any $i, j$, which violates a necessary condition for optimality.

**Remark 1.** *Assume only deterministic orderings are allowed, the only deterministic ordering rule that can be part of an equilibrium is such that $i$ appears before $j$ if $p_i > p_j$.*

The proof is as follows. A reviewer incentive compatible partition allows for a finer partition in equilibrium if the other reviewers are more likely to be biased. Thus the first reviewer should be the most trustworthy one, because that way the probability that all of the rest is biased is maximized. The same reasoning applies to all reviewers who follow.

This ordering rule yields the highest $\min_{i\in\chi}\{\Psi_{A,d}, \Psi_{B,d}, ..\}$ and thus the largest equilibrium size among all deterministic ordering rules. For any deterministic order pinned down by $d$, it is immediate that

$$\min_{i\in\chi}\{\Psi_{A,d}, \Psi_{B,d}, ..\} = \Psi_{i,\Gamma}$$

if $i$ is the first reviewer consulted. It follows immediately that the most attractive deterministic ordering rule, in terms of inducing the equilibrium with the largest number of partitions, is such that the first reviewer consulted is the reviewer with the highest $p_i$. Indeed, for $i, i' \in \chi$, it holds true that $\Pi_{j\in\chi_{-i}}(1-p_j) > \Pi_{j\in\chi_{-i'}}(1-p_j)$ if and only if $p_i > p_{i'}$.

### 5.2 Observable Trustworthiness Levels

Consider the case where the platform shares its estimate of reviewers' likelihood of being biased with consumers.[6] Our model indicates that this is not beneficial to consumers.

Suppose that DM now knows the identity $i$ of each reviewer and thus observes $p_i$ directly for each reviewer. Note that the DM's equilibrium beliefs are as in the main model. $E[\omega \,|m^1 = .. = m^n = t_N]$ is affected by reviewers' trustworthiness levels only via $\eta$, which is independent of how exactly the entries in $\{p_A, .., p_Z\}$ are allocated among individual reviewers.

Clearly, in any partitional equilibrium, DM consults more trustworthy reviewers first (as such DM's consultation strategy induces the optimal deterministic ordering rule discussed previously), which means that in equilibrium we must have $\Psi_\Gamma^{\min} = \Pi_{j\in\chi_{-i}}(1-p_j)$, where $i$ is the most trustworthy reviewer. This is strictly less than the value of $\Psi_{\Gamma'}^{\min}$ achieved by a proportional latin square ordering rule $\Gamma'$.

### 5.3 Varying the Pool of Reviewers

We here investigate the role of the distribution of trustworthiness levels assuming that an optimal ordering rule is used by the platform ($\Psi_{i,\Gamma} = \frac{\eta}{1-\eta}\sum_{k\in\chi}\frac{p_k}{1-p_k}$ for all $i$). We restrict

[6]For instance, Yelp.com shares information about reviewers with customers and Amazon used to do so.

ourselves to comparing distributions that yield the same value of $\eta$, the probability that all reviewers are biased. We look for the optimal profile of $p_i$'s conditional on this constraint and, furthermore, assuming a potential lower bound on the trustworthiness of any individual $\varepsilon \in [0, 1-\eta^{\frac{1}{n}}]$. We thus solve

$$\max_{\{p_i\}_{i\in\chi}} \quad \frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} \tag{17}$$

$$\text{s.t.} \ \prod_i (1-p_i) = \eta, \tag{18}$$

$$\min_{i\in\chi} p_i \geq \varepsilon, \ \text{ for } \varepsilon \in [0, 1-\eta^{1/n}). \tag{19}$$

Define in what follows $\mathbf{p} = \{p_i\}_{i\in\chi}$ and define $\eta(\mathbf{p})$ as the corresponding value of $\prod_i(1-p_i)$. Let $\Lambda(\eta, n)$ be the set of all distributions involving $n$ reviewers and that yield the same value of $\eta$.

**Proposition 4.** *a) The solution to problem (17)-(19) is given by* $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ *for some* $i \in \{1, \ldots, n\}$ *and* $p_j = \varepsilon$ *for all* $j \neq i$.

*b) Consider two profiles* $\mathbf{p}, \mathbf{p}' \in \Lambda(\eta, n)$ *such that for some* $i, j$ *we have* $p'_i > p_i$ *and* $p'_j < p_j$ *while* $p_k = p'_k$ *for all* $k \notin \{i, j\}$. *If* $p_i > p_j$, *then* $\mathbf{p}'$ *yields a weakly higher expected payoff of DM and vice versa if instead* $p_i < p_j$.

Proof: See Appendix A.

Concerning Point a). If $\varepsilon = 0$, then the solution to the problem is trivial. The objective function can always be made equal to 1 by setting $p_i = 1 - \eta$ for any one $i \in \{1, \ldots, n\}$ and $p_j = 0$ for all $j \neq i$. In this case $\prod_i(1-p_i) = \eta$ and $\frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} = \frac{\eta}{1-\eta}\frac{1-\eta}{\eta} = 1$. This is enough to guarantee the existence of an equilibrium of any size (and recall that larger equilibria yield a higher DM expected payoff). In general, assuming a lower bound $\varepsilon > 0$, the optimal distribution is one where all probabilities take the lowest possible value but one of them, which takes the highest. Point b) compares pools of reviewers in which we shift trustworthiness levels between two reviewers by making the more trustworthy reviewer even more trustworthy and the less trustworthy reviewer even less trustworthy, in a way that keeps $\eta$ fixed. We see that such an polarising shift is beneficial to DM, in a way that echoes point a).

## 5.4 Adding or Removing Reviewers

Form our analysis it is easy to see that removing reviewers who are likely to be biased is not necessarily beneficial (as advocated by some online platforms).[7]

[7] See for instance https://www.aboutamazon.com/news/policy-news-views/.

Removing a reviewer, regardless of its probability of being biased, has two effects. A direct effect is that the probability of learning the state $1-\eta$ decreases and this is detrimental for DM. Intuitively, removing a reviewer reduces the amount of information available from senders; if the sender removed is biased there is no change to this pool while if the sender removed is honest then there is a strict decrease in the information available.

An indirect effect is that, since the optimal number of partitions depends positively on $\frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i}$, removing a reviewer increases $\frac{\eta}{1-\eta}$ but decreases $\sum_i \frac{p_i}{1-p_i}$. Thus, removing a reviewer can potentially increase the number of partitions in equilibrium, which in turn increases the amount of information transmitted in equilibrium to DM. Intuitively, fewer (potentially) biased reviewers may reduce every reviewer's belief that all reviewers after itself will send the highest message. That is, the incentive for a biased reviewer to deviate to the second highest message ($m_i = t_{N-1}$) is higher when there are fewer fake reviewers.

Therefore, removing a reviewer reduces information available but may increase the information transmitted in equilibrium. The net result of these two effects is ambiguous. We can find numerical examples were removing those reviewers that are most likely to be biased is negative or positive. Similarly, there are also numerical examples were adding reviewers, even if they are likely to be honest, is positive or negative.

# 6 Conclusion

We have presented a model of information transmission between reviewers, some of whom can be biased, and a receiver. The model maps into the setting of online reviews. We have shown that the optimal way to display product reviews is random in such a way that it equates the beliefs that the consumer will learn the truth about the product after reading each review, and that removing reviews that are likely to be fake is not necessarily beneficial.

The model that we present in this paper can also be applied to the case of user generated commenting on general issues, such as Quora or newspaper comment sections. The pool of reviewers providing opinions is a mixture of honest citizens and agenda driven partisans possibly tied to organisations. Reviewers are as such largely ex-ante identical from readers' perspective, but the platform may have access to data that allows it to estimate the trustworthiness of individuals. The platform is free to decide in which order responses are shown and might condition the order on these estimates. Google's search page offers another instance of the ordering problem. For any given search query, the PageRank algorithm provides an ordered set of results. In this particular case, however, different sources typically have different trustworthiness levels in the eyes of readers.

The main result of this paper is to identify how to optimally order reviewers in a sequential consultation problem. We find that that the order should be stochastic, which implies that less trustworthy reviewers might sometimes be asked earlier. Experimental work would be called upon to qualitatively test our predictions, in order to see whether reviewers' behaviour is driven by the pre-emption motive that drives our findings.

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# 8 Appendix A

## 8.1 Proof of Lemma 1

**Step 1** From the incentives of unbiased reviewers, it must be true that

$$E[\omega \left| m^1 = .. = m^n = t_N \right.] - t_{N-1} = t_{N-1} - \frac{t_{N-1} + t_{N-2}}{2} \tag{20}$$

and it must also be true that all thresholds between $t_0 = 0$ and $t_{N-1}$ are equally spaced, which means that for any $K < N-1$, we have $t_r = \left(\frac{K}{N-1}\right)t_{N-1}$. Using $t_{N-2} = \left(\frac{N-2}{N-1}\right)t_{N-1}$, (20) is equivalent to:

$$\frac{E[\omega \left| m^1 = .. = m^n = t_N \right.]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}.$$

Inserting the closed form expression for $E[\omega \left| m^1 = .. = m^n = t_N \right.]$, we obtain for any given $N$ and $\eta$, the unique solution value of $t_{N-1}$ which is given by (4).

**Step 2** In an equilibrium featuring the partition $\{t_r\}_{r=1}^{N-1}$, let $m(\omega^*)$ denote the message sent by an unbiased reviewer if the state is $\omega^*$ and $\omega^* < t_{N-1}$. Denote by $E[\omega \left| m(\omega^*) \right.]K$'s expected value of the state if he encounters the equilibrium message $m(\omega^*)$. From the incentives of biased reviewers, we need that for every reviewer $i \in \chi$ and for every $\omega^* \leq t_{N-1}$, it holds true that:

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right.] + (1 - \Psi_{i,\Gamma}) E[\omega \left| m(\omega^*) \right.] \geq \frac{t_{N-1} + t_{N-2}}{2}. \tag{21}$$

This condition ensures that any biased reviewer is willing to send $m_N$ rather than deviate to $m_{N-1}$, whatever the realised state. Clearly, $E[\omega \left| m(\omega^*) \right.]$ is increasing in $\omega^*$, so the condition is most difficult to satisfy for $\omega^* = 0$. Thus, (21) is satisfied if and only if it is satisfied at $\omega^* = 0$. We thus need that for every reviewer $i \in \chi$ it holds true that:

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right.] + (1 - \Psi_{i,\Gamma}) \frac{t_1}{2} \geq \frac{t_{N-1} + t_{N-2}}{2}. \tag{22}$$

Recall that the size of every interval to the left of $t_{N-1}$ is identical and given by:

$$2\left(E[\omega \left| m^1 = .. = m^n = t_N \right.] - t_{N-1}\right).$$

We may thus rewrite the constraint (22) as

$$\Psi_{i,\Gamma}E[\omega\,|m^1 = .. = m^n = t_N] + (1-\Psi_{i,\Gamma})\left[E[\omega\,|m^1 = .. = m^n = t_N] - t_{N-1}\right]$$
$$\geq 2t_{N-1} - E[\omega\,|m^1 = .. = m^n = t_N]$$

which is equivalent to

$$\frac{E[\omega\,|m^1 = .. = m^n = t_N]}{t_{N-1}} \geq \frac{(1-\Psi_{i,\Gamma})+2}{2}.$$

Now, bringing together the two conditions derived from the incentives of biased and unbiased reviewers, an equilibrium with $N$ intervals exists if and only if:

$$\frac{2(N-1)+1}{2(N-1)} \geq \frac{(1-\Psi_{i,\Gamma})+2}{2}.$$

## 8.2 Proof of Proposition 3

The proof analyses the general case of $n \geq 2$ reviewers.

### 8.2.1 Point b): Effect of N

**Step 1** Recall that in equilibrium, we have:

$$t_{N-1}\left(\frac{2(N-1)+1}{2(N-1)}\right) = E[\omega\,|m^1 = .. = m^n = t_N].$$

In what follows, define $f(N,\eta) = t^*_{N-1}$, where $t^*_{N-1}$ is given as in (4).

**Step 2** $\Pi_{DM}(N,\eta)$ is given by minus the following sum:

$$\begin{aligned}&(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \\ &+(1-\eta)\int_{f(N,\eta)}^{1}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\ &+\eta\int_0^1\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.\end{aligned} \tag{23}$$

This can be further decomposed into the following elements:

$$\begin{aligned}&(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \\ &+\eta\int_0^{f(N,\eta)}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\ &+\int_{f(N,\eta)}^{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\ &+\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.\end{aligned} \tag{24}$$

Consider the four lines that constitute expression (24) above. The expression in the last line is decreasing in $N$, as we shall show in next step. In the subsequent step, we prove that the sum of the three expressions appearing in the first, second and third line is also decreasing in $N$. This proves point a).

**Step 3** Consider:

$$\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.$$

Note first that $\int_t^1(\omega-t)^2\,d\omega=-\frac{1}{3}(t-1)^3$ is trivially decreasing in $t$. Now, we need to show that $f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)$ is increasing in $N$. Note that:

$$\begin{aligned}&\frac{\partial\left(\left(\frac{2(N-1)+1}{2(N-1)}\right)f(N,\eta)\right)}{\partial N}\\&=\frac{1}{4N^2(1-\eta)(N-1)^2\sqrt{4\eta N^2-4\eta N+1}}G_0(\eta,N),\end{aligned}$$

where

$$\begin{aligned}G_0(N,\eta)=&\sqrt{4\eta N^2-4\eta N+1}-2N\eta-2N\\&-2N\sqrt{4\eta N^2-4\eta N+1}+2N^2\eta+2N^2+1.\end{aligned}$$

We simply need to show that $G_0(N,\eta)>0$. Simple algebraic manipulation shows that this is equivalent to proving that $-4N^2(\eta-1)^2(N-1)^2<0$, which is always true.

**Step 4** Consider the three expressions appearing in the first, second and third line of (24). The sum of these equals:

$$\begin{aligned}T(N,\eta)=\frac{1}{192N^3(\eta-1)^3}\frac{2N-1}{(N-1)^3}&\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^3\\&\left(4\eta N^2-4\eta N+1\right).\end{aligned}$$

We want to prove that $T(N,\eta)$ is always decreasing in $N$. Note that:

$$\begin{gathered}\frac{\partial T(N,\eta)}{\partial N}=\frac{1}{(\eta-1)^3}\frac{1}{192N^4(N-1)^4}\\\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^2G_1(\eta,N),\end{gathered}$$

where

$$\begin{gathered}G_1(N,\eta)=\\10N-10N^2\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}+8N\eta-3\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}\\+10N\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}-24N^2\eta+16N^3\eta-12N^2+8N^3\\+50N^2\eta\sqrt{4\eta N^2-4\eta N+1}-80N^3\eta\sqrt{4\eta N^2-4\eta N+1}\\+40N^4\eta\sqrt{4\eta N^2-4\eta N+1}-10N\eta\sqrt{4\eta N^2-4\eta N+1}-3.\end{gathered}$$

To show that $\frac{\partial T(N,\eta)}{\partial N} < 0$, we simply need to show that $G_1(N,\eta) > 0$. Simple algebraic manipulation shows that this in turn equivalent to proving that:

$$-4N^2(\eta-1)^2(N-1)^2\left(4N\eta-16N-4N^2\eta+16N^2+3\right)<0,$$

which is always true.

### 8.2.2 Point c): Effect of $\eta$

**Step 1** Consider expression (24). The expression appearing in the last line is trivially increasing in $\eta$, as proved now. We have:

$$\begin{aligned}&\frac{\partial\left(f(N,\eta)\right)}{\partial\eta}\\&=-\frac{1}{2N(\eta-1)^2\sqrt{4\eta N^2-4\eta N+1}}G_0(\eta,N),\end{aligned}$$

where $G_0(N,\eta)$ was defined earlier in our analysis of comparative statics with respect to $N$. We wish to prove that the above is negative. This is equivalent to showing that $G_0(N,\eta)>0$, which we already proved is true.

**Step 2** Consider the three expressions appearing in the first, second and third line of (24). We now show that the sum of these three expressions (denoted $T(N,\eta)$) is increasing in $\eta$. Note that:

$$\begin{aligned}\frac{\partial T(N,\eta)}{\partial\eta}&=\frac{1}{192N^3}\frac{2N-1}{(N-1)^3}\frac{1}{(\eta-1)^4}\\&\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^2G_2(\eta,N),\end{aligned}$$

where

$$\begin{gathered}G_2(N,\eta)=\\10N-10N^2\sqrt{4\eta N^2-4\eta N+1}+8N\eta-3\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}\\+10N\sqrt{4\eta N^2-4\eta N+1}-24N^2\eta+16N^3\eta-12N^2+8N^3\\+10N^2\eta\sqrt{4\eta N^2-4\eta N+1}-10N\eta\sqrt{4\eta N^2-4\eta N+1}-3.\end{gathered}$$

To show that $\frac{\partial T(\eta,N)}{\partial\eta}>0$, we simply need to show that $G_2(\eta,N)>0$. Simple algebraic manipulation shows that this in turn equivalent to proving that

$$\left(4N\eta-16N-4N^2\eta+16N^2+3\right)>0,$$

which is always true.

### 8.3 Proof of Lemma 3

The proof is decomposed into the statement of four Lemmas which together yield the result. Recall that we focus on ordering rules that satisfy:

$$\Gamma = \arg\max_{\Gamma} \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{25}$$

The first Lemma below shows that any ordering rule $\Gamma$ that solves (25) is such that $\Psi_{i,\Gamma}$ is constant across reviewers. We then show that the achieved value of $\Psi_{i,\Gamma}$ is the same across all ordering rules that solve (25). Finally, we explicitly pin down the achieved value of (25).

**Lemma 6.** *An ordering rule $\Gamma$ solves (25) if and only if $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for all $i, j$ and consequently $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$for all $i, j$.*

*Proof.* Notice that an ordering rule solves $\max_\Gamma \min_i \Psi_{i,\Gamma}$ if and only if it solves $\min_\Gamma \max_i(1-p_i)Q_{i,\Gamma}$

We proceed by contradiction. Let $\widetilde{\chi}$ be the set of reviewers such that $\widetilde{\chi} = \arg\min_i \Psi_{i,\Gamma} = \arg\min_i \frac{\eta}{(1-p_i)Q_{i,\Gamma}} = \arg\max_i(1-p_i)Q_{i,\Gamma}$. If the statment of the Lemma is not true then there exists a reviewer $k \notin \widetilde{\chi}$. Assume first that $\widetilde{\chi}$ contains only one reviewer, say $A$. Then all reviewers who are not $A$ do not belong to $\widetilde{\chi}$.

Take any deterministic order assigned positive probability in the ordering rule $\Gamma$ such that $A$ acts immediately before a reviewer not in $\widetilde{\chi}$, call the deterministic order $d$ and that reviewer $k$. Such order always exists as otherwise $A$ is last in all deterministic orders with positive probability, which implies that $A \notin \widetilde{\chi}$. Create a new ordering rule $\widehat{\Gamma}$ identical to $\Gamma$ but such that order $d$ has probability $\widehat{\theta}_d = \theta_d - \varepsilon$ for some small $\varepsilon > 0$, and order $d'$, which is the same as $d$ but where the positions of $a$ and $k$ are swapped, has probability $\widehat{\theta}_{d'} = \theta_{d'} + \varepsilon$.

Notice that $\Gamma$ and $\widehat{\Gamma}$ generate $\{Q_{j,\Gamma}\}_j$ and $\{Q_{j,\widehat{\Gamma}}\}_j$ respectively such that $Q_{j,\Gamma} = Q_{j,\widehat{\Gamma}}$ for all $j \neq a, k$ and $Q_{k,\widehat{\Gamma}} > Q_{k,\Gamma}$ and $Q_{a,\widehat{\Gamma}} < Q_{a,\Gamma}$. Since $Q_{i,\Gamma}$ for all reviewer $i$ is linear in the probabilities of ordering rule $\Gamma$, by continuity $\varepsilon$ can be chosen such that $(1-p_a)Q_{a,\widehat{\Gamma}} > (1-p_k)Q_{k,\widehat{\Gamma}}$.

We have just proven that there exists an ordering rule $\widehat{\Gamma}$ with $\max_i(1-p_i)Q_{i,\widehat{\Gamma}} < \max_i(1-p_i)Q_{i,\Gamma}$, which implies that $\Gamma$ does not solve $\min_\Gamma \max_i(1-p_i)Q_{i,\Gamma}$, a contradiction.

If $\widetilde{\chi}$ instead has more than one reviewer, repeat the reasoning in this proof to arrive at a contradiction. Thus proving the Lemma. □

An ordering rule solving (25) thus requires $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for all $i, j$ which implies $\Psi_{i,\Gamma} = \Psi^j_\Gamma$ for all reviewers $i, j$. Next, we show that all ordering rules solving (25) lead to the same value for $\Psi_{i,\Gamma}$ for all $i$.

**Lemma 7.** *Take any two ordering rules* $\Gamma$ *and* $\Gamma'$ *that solve (25), with their respective* $Q_\Gamma = \{Q_{j,\Gamma}\}_j$ *and* $Q_{\Gamma'} = \{Q_{j,\Gamma}\}_j$*. Then,* $(1-p_j)Q_{j,\Gamma} = (1-p_j)Q_{j,\Gamma}$ *and consequently* $\Psi^j_\Gamma = \Psi^j_{\Gamma'}$ *for all* $j$*.*

*Proof.* Notice first that for any ordering $\Gamma$ with its respective $\{Q_{j,\Gamma}\}_j$ we have

$$\sum_j p_j Q_{j,\Gamma} = 1 - \eta. \tag{26}$$

The left hand side is the probability that the receiver learns the truth; the sum for every reviewer of the probability that this reviewer is asked and tells the truth (notice that when $w = 0$ it is not possible for two reviewers to be asked and both tell the truth, as when one does so consultation stops). The right hand side is the same but expressed differently; it is the probability that at least one reviewer is honest (i.e. not true that all reviewers are biased). Algebraically, if $d$ is any deterministic order and $d_i$ is the reviewer who occupies the $i$-th position in this order then

$$\begin{aligned}
&\sum_j p_j Q_{j,\Gamma} \\
&= \sum_d \theta_d (p_{d_1} + p_{d_2}(1-p_{d_1}) + p_{d_3}(1-p_{d_2})(1-p_{d_1}) + \dots \\
&+ p_{d_n}(1-p_{d_{n-1}})\dots(1-p_{d_1})) \\
&= \sum_d \theta_d (1-\eta) = (1-\eta)\sum_d \theta_d = 1-\eta.
\end{aligned}$$

Since (25) implies $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for any $i, j$, we have $Q_{j,\Gamma} = Q_{i,\Gamma}\frac{1-p_i}{1-p_j}$. This leads to

$$\sum_j Q_{j,\Gamma} = (1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j}. \tag{27}$$

On top of that, (25) implies $\sum_j (1-p_j)Q_{j,\Gamma} = n(1-p_i)Q_{i,\Gamma}$ for any $i$. If we combine (26) and (27) with this observation we obtain

$$\begin{aligned}
(1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j} - (1-\eta) &= n(1-p_i)Q_{i,\Gamma} \\
(1-p_i)Q_{i,\Gamma} &= \frac{1-\eta}{\sum_j \dfrac{p_j}{1-p_j}}.
\end{aligned}$$

That is, the value of $(1-p_i)Q_{i,\Gamma}$ and consequently of $\Psi_{i,\Gamma}$ is the same under all odering rules solving (25) and for all $i$. □

**Lemma 8.** *Let* $\Gamma$ *be an ordering rule that solves (25). Then* $\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta}\sum_j \frac{p_j}{1-p_j}$ *for all* $i, j$*.*

*Proof.* In every ordering rule $\Gamma$ solving (25), since for any $i,j$ we have $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ and $\Psi_{i,\Gamma} = \frac{\prod_{j\neq i}(1-p_j)}{Q_{i,\Gamma}} = \frac{\eta}{Q_{i,\Gamma}(1-p_i)}$, it follows immediately that for every reviewer $i$

$$\Psi_{i,\Gamma} = \frac{\eta}{1-\eta}\sum_{k\in\chi}\frac{p_k}{1-p_k}.$$

□

## 8.4 Proof of Lemma 4

For a given latin square odering rule, given reviewer $i$ and position $l$, let $d_l$ be the unique deterministic order such that $\theta_{d_l} > 0$ and reviewer $i$ occupies position $l$. Furthermore, let $\{1_{d_l}, 2_{d_l}, \ldots, (l-1)_{d_l}\}$ be the reviewers who occupy positions $\{1, 2, \ldots, l-1\}$ respectively in deterministic order $d_l$. We have,

$$\begin{aligned}
&(1-p_i)Q_{i,\Gamma}\\
&= (1-p_i)\left(\theta_{d_1} + \theta_{d_2}(1-p_{1_{d_2}}) + \ldots + \theta_{d_n}\prod_{j\neq i}(1-p_j)\right)\\
&= (1-p_i)\left(\begin{array}{c}\dfrac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)} + \dfrac{p_{1_{d_2}}/(1-p_{1_{d_2}})}{\sum_j p_j/(1-p_j)}(1-p_{1_{d_2}})\\ + \ldots + \dfrac{p_{1_{d_n}}/(1-p_{1_{d_n}})}{\sum_j p_j/(1-p_j)}\prod_{j\neq i}(1-p_j)\end{array}\right)\\
&= \frac{1}{\sum_j \dfrac{p_j}{1-p_j}}\left(p_i + p_{1_{d_2}}(1-p_i) + \ldots + p_{1_{d_n}}\prod_{j\neq 1_{d_n}}(1-p_j)\right)\\
&= \frac{1-\eta}{\sum_j \dfrac{p_j}{1-p_j}}.
\end{aligned}$$

That is,

$$(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma} = \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}$$

for all $i,j$, which is the requirement for an optimal random order.

## 8.5 Proof of Proposition 4

### 8.5.1 Point a)

Consider the constrained optimisation problem defined in (17), (18) and (19). We use Kuhn-Tucker:

$$L(\{p_i\},\lambda,\{\mu_i\}) = \frac{\eta}{1-\eta}\sum_i\frac{p_i}{1-p_i} + \lambda\Big(\prod_i(1-p_i)-\eta\Big) - \sum_i \mu_i(p_i-\varepsilon)$$

with $\lambda \geq 0$ and $\mu_i \geq 0$ for all $i$. Since the problem is symmetric for $\{p_i\}$ we can assume without loss of generality that the first $k \in \{1, 2, \ldots, n\}$ probabilities are stritly greater than $\varepsilon$ and the last $n-k$ probabilities are equal to $\varepsilon$. In other words, $\{\mu_i\}_{i=1}^k = 0$ and $\{\mu_i\}_{i=k+1}^n > 0$ for some $k$. The problem is then to solve the K-T conditions for any $k$, and then choose the $k$ that maximizes the objective function. Note we cannot have $k = 0$ as this would mean $\prod_i(1-p) = (1-\varepsilon)^n$, which is not in general equal to $\eta$. We have for all $i$

$$\begin{aligned} \frac{\partial L}{\partial p_i} &= \frac{\eta}{1-\eta}\frac{1}{(1-p)^2} - \lambda\frac{\eta}{1-p_i} - \mu_i \\ &= 0 \\ \frac{\partial^2 L}{\partial^2 p_i} &= \frac{\eta}{1-\eta}\frac{-2}{(1-p)^3} - \lambda\frac{\eta}{(1-p_i)^2} \\ &< 0. \end{aligned}$$

Note that for those $i$ for which $\mu_i = 0$ we have $\frac{\eta}{1-\eta} - \lambda\eta(1-p_i) = 0$, which implies $p_i = 1 - \frac{1}{\lambda(1-\eta)}$. That is, at the optimum those $p_i$ which are not $\varepsilon$ all are equal to some value $p$ given by $p = 1 - \frac{1}{\lambda(1-\eta)}$.

Therefore, we have $\eta = (1-p)^k(1-\varepsilon)^{n-k}$. This means

$$p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}.$$

Thus, at the optimum we have that the first $k$ probabilities are equal to $p$ and the rest are equal to $\varepsilon$. We are left to calculate what is the optimal $k$. For given $k$ we have that the objective function is equal to

$$\psi = \frac{\eta}{1-\eta}\left[k\left(\left(\frac{(1-\varepsilon)^{n-k}}{\eta}\right)^{\frac{1}{k}} - 1\right) + (n-k)\frac{\varepsilon}{1-\varepsilon}\right]$$

Let us study the behaviour of this expression as a function of $k$. Taking the derivative with respect to $k$ and using the fact that $p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}$ we obtain

$$\frac{\partial \psi}{\partial k} \propto \frac{p}{1-p} - \frac{\varepsilon}{1-\varepsilon} - \frac{1}{1-p}\log\frac{1-\varepsilon}{1-p},$$

where $p$ depends on $k$.

Notice that $\frac{\partial \psi}{\partial k}_{p=\varepsilon} = 0$ and that $\frac{\partial^2\psi}{\partial k \partial p} = -\frac{1}{(1-p)^2}\log\frac{1-\varepsilon}{1-p} < 0$. Hence, we have that $\frac{\partial \psi}{\partial k}$ is decreasing in $p$ and equal to 0 at the lowest possible value for $p$. Therefore, it is negative. This means that the $k$ that maximizes $\Psi$ is the minimum possible. That is, $k = 1$. Therefore, the optimal solution is $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ for any one $i \in \{1, \ldots, n\}$ and $p_j = \varepsilon$ for all $j \neq i$.

#### 8.5.2 Point b)

Assume that for some $\varepsilon > 0$ we have that $p_i$ increases to $p_i' = p_i + \varepsilon$ and that $p_j$ decreases to $p_j' = p_j - \rho(\varepsilon)$, where $\rho(\varepsilon)$ solves

$$(1 - p_i - \varepsilon)(1 - p_j + \rho(\varepsilon)) = (1 - p_i)(1 - p_j).$$

which is equivalent to

$$\rho(\varepsilon) = \left( \frac{(1 - p_i)}{(1 - p_i - \varepsilon)} - 1 \right) (1 - p_j)$$

It is easy to show that

$$\frac{p_i'}{1 - p_i'} + \frac{p_j'}{1 - p_j'} > \frac{p_i}{1 - p_i} + \frac{p_j}{1 - p_j}.$$

Therefore, $\Psi^{*'} > \Psi^*$.

# 9 Appendix B

## 9.1 Proof of Proposition 1

### 9.1.1 Preliminary definitions

Recall that $m^l$ denotes the message appearing in position $l$ of the presentation order. Consider an observed history $h$ in which DM consulted $k$ reviewers, first consulting the reviewer located in position $l$, then the reviewer in position $l'$, then the reviewer in position $l''$, etc. We denote such a history by the k-entries vector $h = \{m^l, m^{l'}, m^{l''}, ..\}$. We denote the $r$th entry of $h$ by $h_r$. We say that a history has length $k$ if DM consulted $k$ times. We say that two histories $h$ and $h'$ are *comparable* if, across these two histories, DM faced the same presentation order, followed the same order of consultation, and has consulted the same number of times (so the histories have the same length). The action rule $\alpha$ pins down the action $\alpha(h)$ taken by DM if he stops consulting and chooses an action after $h$.

**Definition 1.** *reviewers' strategiesinduce monotonic beliefs when for any two comparable histories $h$ and $h'$of lengh $k \in \{1, .., n\}$, if it holds true that there is some $i \in \{1, \ldots k\}$ such that $h_j = h_j'$ for all $j \neq i$ and $h_i > h_i'$ then it holds true that $E[\omega|h] \geq E[\omega|h']$, assuming that DM's beliefs are formed via Bayes rule and the reviewers' strategy profile.*

The following examples illustrate the above definition.

**Example 1.** *Given* $n = 5$*, if* $h = \{m, m', m''\}$ *and* $h' = \{m, \widetilde{m}', m''\}$ *with* $m' > \widetilde{m}'$ *then if reviewers' strategies induce monotonic beliefs it must be that* $E[\omega|h] \geq E[\omega|h']$.

**Example 2.** *Given* $n = 5$*, if* $h = \{m, m', m''\}$ *and* $h' = \{m, m', m'', m'''\}$ *then even if reviewers' strategies induce monotonic beliefs we cannot establish an ordinal relation between* $E[\omega|h]$ *and* $E[\omega|h']$.

**Example 3.** *Given* $n = 5$*, if* $h = \{m, m', m''\}$ *and* $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ *with* $m' > \widetilde{m}'$ *and* $m'' > \widetilde{m}''$ *then if reviewers' strategies induce monotonic beliefs it must be that* $E[\omega|h] \geq E[\omega|h']$.

**Example 4.** *Given* $n = 5$*, if* $h = \{m, m', m''\}$ *and* $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ *with* $m' < \widetilde{m}'$ *and* $m'' > \widetilde{m}''$ *then even if reviewers' strategies induce monotonic beliefs we cannot establish an ordinal relation between* $E[\omega|h]$ *and* $E[\omega|h']$.

**Definition 2.** *An equilibrium is monotone if reviewer strategies are monotonic and induce monotonic beliefs.*

**Definition 3.** *An equilibrium is partitional if it satisfies the following description. There is a sequence of strictly increasing thresholds* $\{t_0, t_1, \ldots, t_N\}$ *with* $N > 1$*,* $t_0 = 0$ *and* $t_N = 1$ *such that the following holds true. For any two comparable histories* $h$ *and* $h'$*of lengh* $k$*, if it holds true that there is some* $i \in \{1, \ldots k\}$ *such that* $h_i = h'_i$ *for* $i \neq j$ *and* $h_j > h'_j$ *where either* $h_j, h'_j \in [t_k, t_{k+1})$ *for some* $k \in \{0, \ldots, N-1\}$ *or* $h_j, h'_j \in [t_{N-1}, 1]$*, then we have* $\alpha(h) = \alpha(h')$.

### 9.1.2 Proof

In what follows, as stated in the main text, we restrict ourselves to symmetric and monotone equilibria. The proof is decomposed into three Lemmas. The first Lemma establishes that there can never be a subset of the state space for which DM learns the state perfectly if he meets an unbiased reviewer. The second Lemma uses this property to show that any equilibrium must be partitional. The third Lemma, building on this, shows that for any informative equilibrium that satisfies our restrictions there exist an an outcome equivalent simple partitional equilibrium.

**Lemma 9.** *(No Perfect Communication on an Interval) There exists no symmetric and monotone equilibrium where there is a non-degenerate interval* $\widetilde{A}$ *such that if* $\omega \in \widetilde{A}$ *then if the receiver consults an unbiased reviewer he stops consultation and plays* $\alpha = \omega$.

*Proof.* **Step 1 (assume the contrary and define** $\sup \widetilde{A}$ **as the supremum of the state for which perfect communication is possible)** Assume the contrary, then there is a possibly uncountable collection of disjoint non-degenerate sets $\{\widetilde{A}_i\}_i$ such that if $\omega \in \widetilde{A}_i$

for some $i$ then the receiver stops consultation and plays $m$. For all $i$ let $\sup \widetilde{A}_i$ be the supremum of set $\widetilde{A}_i$. Create an increasing sequence in $[0, 1]$ by ordering increasingly the set of all suprema $\{\sup \widetilde{A}_i\}_i$. Since such sequence is bounded by 1, by the monotone convergence theorem it converges to its supremum. Let $\sup \widetilde{A}$ be such supremum.

**Step 2 (if state is $\omega$=$\sup \widetilde{A}$ then unbiased reviewer believes action must be $\sup \widetilde{A}$)** If the state is $\omega = \sup \widetilde{A}$ then an unbiased reviewer believes with probability 1 in equilibrium that the action of the receiver must be $\sup \widetilde{A}$ once he stops consultation. To see this notice first that by monotonicity this action is greater or equal than $\sup \widetilde{A} - \varepsilon$ for all small enough $\varepsilon > 0$. If $\sup \widetilde{A} = 1$ then the action is $\sup \widetilde{A}$ with certainty, and if $\sup \widetilde{A} < 1$ but the action played is not $\sup \widetilde{A}$ with some probability then the expected action must by strictly higher than $\sup \widetilde{A}$. That is, there exists an $\varepsilon > 0$ such that the action played is $\sup \widetilde{A} + \varepsilon$ with some probability $p$, in which case the unbiased reviewer's best response is in $m \in ((1-p)\sup \widetilde{A} + p(\sup \widetilde{A} - \varepsilon), \sup \widetilde{A}) \subset \widetilde{A}$. Thus, there is a deviation incentive of unbiased reviewers, a contradiction.

**Step 3 (define $m_{\widetilde{A}}$ as the message that induces action arbitrarily close to $\sup \widetilde{A}$)** For small $\varepsilon > 0$ there exist a message $m_{\widetilde{A}}(\varepsilon)$ such that if sent by a reviewer the receiver stops consultation and plays $\sup \widetilde{A}(\varepsilon) - \varepsilon$. By monotonicity $m_{\widetilde{A}}(\varepsilon)$ is increasing in $\varepsilon$ and, furthermore, it is bounded above by 1. Thus, it converges to its supremum. Let $m_{\widetilde{A}}$ be such supremum.

**Step 4 (action is strictly higher than $\sup \widetilde{A}$ when all messages are greater or equal to $\sup \widetilde{A}$)** By monotonicity, if the state is $\omega = \sup \widetilde{A}$ then unbiased reviewers send message $m \geq m_{\widetilde{A}}$. On top of that, for any state by monotonicity biased reviewers always send message $m \geq m_{\widetilde{A}}$. A biased reviewer believes that if consulted then for any state the expected action played by the receiver in equilibrium, say $\alpha$, is at least $\sup \widetilde{A}$, as otherwise he can guarantee any action arbitrarily close to $\sup \widetilde{A}$ by sending message $m_{\widetilde{A}}$. In equilibrium, by monotonicity, if the state is $\omega \in [0, \sup \widetilde{A})$ the receiver only plays an action of at least $\sup \widetilde{A}$ if he consults biased reviewers only, and less than $\sup \widetilde{A}$ otherwise, say $\hat{\alpha} < \sup \widetilde{A}$ at most. Using incentive compatibility for biased reviewers we must have $\delta\alpha + (1-\delta)\hat{\alpha} \geq \sup \widetilde{A}$, which since $\delta \in (0, 1)$ and $\hat{\alpha} < \sup \widetilde{A}$ means $\alpha > \sup \widetilde{A}$. That is, the equilibrium action of the receiver must be strictly higher than $\sup \widetilde{A}$ when all messages he receives are $m_{\widetilde{A}}$ or above regardless of the state of nature.

**Step 5 (if $\omega = \sup \widetilde{A}$ all messages are greater or equal to $\sup \widetilde{A}$)** Assume $\omega = \sup \widetilde{A}$, then the receiver receives all messages equal to or above $\sup \widetilde{A}$, which by the paragraph above means he plays an action strictly greater than $\sup T$, but this is incompatible with unbiased reviewers' equilibrium beliefs. As we showed previously if the state is $\omega = \sup \widetilde{A}$ then an unbiased reviewer believes with probability 1 that the action played by the receiver is $\sup \widetilde{A}$. $\square$

**Lemma 10.** *All informative, symmetric and monotone equilibria are partitional.*

*Proof.* **Step 1 (by contradiction, exists two histories with increasing actions in some interval $\widetilde{A}$ for some reviewer $j$)** Assume there is an equilibrium that is not partitional. This means that there exists a non-degenerate interval $\widetilde{A}$ and a pair of comparable equilibrium histories $h_{-j}, h'_{-j}$ for all reviewers but $j$ with $h_i = h'_i$ for reviewers $i \neq j$, such that for all $h_j, h'_j \in \widetilde{A}$ we have $h_j \neq h'_j$ implies $\alpha(h) \neq \alpha(h')$.

By monotonicity for all $h_j, h'_j \in \widetilde{A}$ with $h_j > h'_j$ we have $\alpha(h) > \alpha(h')$. Assume henceforth without loss of generality that $h_j > h'_j$.

**Step 2 (biased reviewers always send at least $\sup \widetilde{A}$)** By monotonicity in equilibrium for any state biased reviewers always send message of at least $\sup \widetilde{A}$. This is because in at least one equilibrium history (the one given in step one of the proof) it leads to a strictly higher action than anything below $\sup \widetilde{A}$, and for any other equilibrium history it leads, by monotonicity, to an action at least as high as any other message below $\sup \widetilde{A}$.

**Step 3 (define function $\hat{\alpha}(h_j)$ as the increasing action in history $h_{-j} \times h_j$ as a function of $h_j \in \widetilde{A}$)** For any history $h_{-j}$ where some reviewer $j$ is not consulted, define the function $\hat{\alpha} : \widetilde{A} \to I$ as a strictly increasing map between $h_j$ and the action played in history $h = h_{-j} \times h_j$. We have that $\hat{\alpha}$ is increasing and that $I \subseteq [\hat{\alpha}(\inf \widetilde{A}), \hat{\alpha}(\sup \widetilde{A})]$.

**Step 4 (if the image set of $\hat{\alpha}$, i.e. $I$, contains an interval, we contradict Lemma 9)** If there exists an $x \in \mathring{I}$ and $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$ then $I$ contains intervals. That is, there exists an $x \in \mathring{I}$ and an $\varepsilon > 0$ such that for all state $\omega \in (x, x + \varepsilon)$ there exists a message $m \in \widetilde{A}$ such that $\hat{\alpha}(m) = \omega$. Since biased reviewers always send message $\sup \widetilde{A}$, after observing $m$ the receiver learns the that reviewer is unbiased and his strict best response is to stop consultation and play $\hat{\alpha}(m)$. This contradicts Theorem 1.

**Step 5 (if the image set of $\hat{\alpha}$, i.e.$I$, does not contain any interval, we still contradict Lemma 9 because $I$ must be dense in some subset of $I \cap R$)** Assume instead that for all $x \in \mathring{I}$ there exists no $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$. That is, $I$ contains no intervals.

**Step 5.1 ($I$ contains no intervals but for at least one point in $I$ there is aonther one in $I$infinitesimally close by)** Note that for all $\delta > 0$ there exists an $x, x' \in I$ with $x < x' < x + \delta$. This is because otherwise there exists a $\delta > 0$ such that for all $x \in I$ we have $(x, x + \delta) \not\subset I$. This means that $I$ has at most $\frac{\sup I - \inf I}{\delta}$ elements. This is a contradiction as

$\hat{\alpha}$ is a strictly increasing mapping from a set with infinitely many elements so its domain $I$ must also have infinitely many elements.

**Step 5.2 (for all error $\varepsilon > 0$ and for some states not in$I$ reviewer can induce an action$\varepsilon$-close to the state)** We have that for all $\varepsilon > 0$ if we take $\delta \in (0, 2\varepsilon)$ and $x, x' \in I$ such that $x < x' < x + \delta$ then for all $\hat{x} \in (x, x')$ with $\hat{x} \notin I$ either $|x - \hat{x}| < \frac{\delta}{2} < \varepsilon$ or $|x' - \hat{x}| < \frac{\delta}{2} < \varepsilon$.

**Step 5.3 (there is then an interval with full communication, a contradiction)** Notice that since $x, x' \in I$ there exists $m, m' \in \widetilde{A}$ respectively such that $\hat{\alpha}(m) = x$ and $\hat{\alpha}(m') = x'$. That is, we have found that for any $\varepsilon > 0$ and any state $\omega$ in the interval $[x, x']$ we can find a message that induces an action at least $\varepsilon$-close to $\omega$. This means that again we have found an interval where there is full communication of the state, a contradiction to theorem 1.

□

**Lemma 11.** *In all informative, symmetric and monotone equilibria, there exists a sequence of strictly increasing thresholds* $\{t_0, t_1, \ldots, t_{m-1}, t_m\}$ *with* $t_0 = 0$ *and* $t_m = 1$ *such that:*

1. *Biased reviewers always send a message in* $[t_{N-1}, t_N]$,
2. *unbiased reviewers all send the same message,*
3. *DM keeps consulting as long as he has received messages in* $[t_{N-1}, t_N]$*, and stops consulting either once he has received a message not in* $[t_{N-1}, t_N]$*, or when he has consulted all reviewers,*
4. *if DM observes a message not in* $[t_{N-1}, t_N]$*, say it belongs to* $[t_{k-1}, t_k]$ *with* $k \in \{1, \ldots, N-1\}$*, he then plays an action* $\alpha(k)$ *that is strictly increasing in* $k$*. If DM only observes messages in* $[t_{N-1}, t_N]$*, he then plays an action* $\alpha(N) > \alpha(k)$ *for all* $k \in \{1, \ldots, N-1\}$.

*Proof.* **Step 1 (Eliminate redundant partitions)** Take any informative partitional equilibria. If for any two comparable equilibrium histories $h$ and $h'$ such that there is some reviewer $i$ where for all $i \neq j$ and $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ (or $h_i = h'_i = t_N$) and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$ and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) = \alpha(h')$, then we can redefine the partitions as $\{t_0, \ldots, t_r, t_s, t_{s+1}, t_N\}$ without-loss of generality. If for any two comparable equilibrium histories $\hat{h}$ and $\hat{h'}$ where again all messages but one are in the same interval and the action is not increasing, we can again redefine the partitions eliminating the cut-offs where the action of the receiver is non-increasing.

Continuing in this fashion we get to a partition $t_0, t_1, \ldots, t_{m-1}, t_m$ with $t_0 = 0$ and $t_m = 1$ such that there exists two comprable equilibrium histories $h$ and $h'$ where there is some reviewer $i$ if for all $i \neq j$ we have $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$ and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) > \alpha(h')$.

**Step 2 (Only two different messages are ever observed in equilibrium)** By monotonicty, we have then that biased reviewers always send message $m \in [t_{N-1}, t_N]$ for a given state. Therefore, in equilibrium, any message that is not in $[t_{N-1}, t_N]$ was sent by an unbiased reviewer with probability 1. Moreover, since strategies are symmetric and reviewers do not observe other reviewers' messages, i.e. the history of observed messages, all unbiased reviewers send the same message for given state of the world. This means that in a partitional equilibrium there are only two messages ever observed by the receiver, the one sent by biased reviewers, and the one sent by unbiased reviewers.

**Step 3 (Perfect information if message received is not in the top partition)** Therefore, since messages not in $[t_{N-1}, t_N]$ are only ever sent by unbiased reviewers, once the receiver observes a message not in $[t_{N-1}, t_N]$ he does not have incentives to keep consulting reviewers, as he has learned as much as can be learnt in equilibrium. Thus, he stops consultation.

**Step 4 (Summing up)** Altoghether steps 1-3 in this proof lead to the result in the Lemma. Note finally that it is immediate that for any given equilibrium of the form described in the above Lemma, there exists a unique outcome equivalent simple partitional equilibrium.

□