Sequential Cheap Talk Consultation and the Optimal Ordering of Experts*

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Abstract

A decision maker consults informed senders of information sequentially, e.g. on an online platform. The senders are either biased or unbiased, and each of their messages is costly to read. The decision maker chooses in a sequentially rational way whether and which sender to consult, and takes an action when he stops consultation. The platform determines the presentation order of the senders in order to maximize the decision maker's welfare. We characterize the optimal ordering rule and show that it is stochastic, which indicates that more reliable senders are not necessarily placed earlier. We also find that removing senders who are likely to be biased may worsen information transmission.

Keywords: Cheap Talk; Strategic Information Transmission; Sequential Information Acquisition; Fake Reviews.

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1 Introduction

This paper studies sequential cheap talk communication where each of multiple senders observes a common true state and reports a message, and an uninformed receiver sequentially reads messages each at a small cost. The receiver chooses an action either after he decides

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to stop reading; or after he has read the messages from all senders. The senders are either biased or unbiased, but the receiver does not observe his type. Each sender has a probability that he is unbiased (i.e. his interest coincides with that of the receiver), which we interpret as the reliability of the sender. Our focus is on the optimal ordering of senders that induces information revelation to maximize the receiver's welfare, where the ordering depends on the levels of reliability.

This setup can be linked to the ordering of customer reviews on an online platform: the platform may decide which reviewers' comments should be presented first, given that some reviewers on some products may be biased and therefore "fake". The optimal ordering we derive in this paper indicates that i) it involves randomization; and ii) removing senders who are likely to be fake may reduce the quality of information the receiver is expected to obtain. This may explain why an online platform that aims to ensure the customers are well-informed may nonetheless keep reviewers who are likely to be fake, and why it may not necessarily present reviews from the most reliable reviewers first.[1]

In our model, the decision maker/receiver (DM) must choose an action a to match the underlying unobserved state (product quality) drawn from the unit interval. He faces n perfectly informed and senders whom he consults sequentially, each at an arbitrarily small cost c. DM consults in a sequentially rational way: at any point in time he chooses whether and whom to consult (where senders only differ in terms of their position in the presentation order) such that his current expected payoff is maximized. Senders have two possible (privately observed) types: unbiased or biased. Unbiased senders share DM's objective to match the state. Biased senders' objective is to maximize DM's action (such as sales) Each sender's probability of being unbiased (which parametrizes his reliability) is privately observed by the planner/platform that also determines the ordering rule of the senders according to their levels of reliability. DM knows the ordering rule and the probability distribution of the levels of reliability, but does not directly observe each sender's reliability. Each sender, who also knows the distribution and the ordering rule but also whether his is biased or unbiased, simultaneously sends a cheap talk message. The ordering rule can be deterministic, in which case each sender knows in which position his message will be when he sends it. The ordering rule can also be random, in which case each sender only knows the probability distribution of all senders' positions.

We first show that, under mild conditions, every informative equilibrium in the game is

[1]In this paper we study the ordering of reviewers according to their characteristics, but not the ordering of their reviews according to their contents. This is for simplicity and our model can be thought of as a first step towards the understanding of the optimal ordering of senders and their messages in the context of strategic communication. Needless to say the presentation algorithm used by a platform can incorporate reviewer characteristics, contents, and other factors.

partitional. The state space is partitioned into N intervals corresponding to N messages $t_1, .., t_N$. If sender i is unbiased, he sends t_r if the state lies in the r-th interval. If sender i is biased, he always sends the highest message t_N. For each ordering rule, we focus on the most informative equilibrium, which is the equilibrium with the largest number of intervals.

The planner determines the ordering rule in such as way that DM's expected utility is maximized. At a first sight, if the senders are equally reliable, the ordering rule may seem irrelevant, and if the they have different levels of reliability, presenting more reliable senders first may appear beneficial. We show that neither of these intuitions holds. Whether or not senders are equally reliable, using a well chosen stochastic ordering rule improves DM's expected utility by improving the informativeness of the messages.

The fundamental driving force behind this result is a preemption motive. Note that a biased sender can induce DM to stop consultation by sending the second highest message, since DM believes that it is from an unbiased sender and takes the action accordingly. Thus preemption of further consultation is detrimental to the information DM obtains. A biased sender's incentive to induce such preemption is higher as he is positioned earlier in the presentation order, because his lie is more likely to be exposed later by a sender who turns out to be unbiased. In other words, a biased sender's incentive to preempt decreases as his position becomes later, but needless to say it is impossible to put every sender in the last position. On the other hand, given the informativeness of messages, DM prefers to read messages from more reliable senders earlier. This trade-off gives rise to the optimality of a random presentation order.

Also, removing senders from a presentation order has two effects. Given the informativeness of the messages, then DM is worse off with a smaller number of senders no matter how (un)reliable they may be. However, removing senders also reduces a biased sender's incentive to preempt and thus increases the informativeness of each message, because his lie is less likely to be exposed later by other senders. The overall effect is thus ambiguous.

Literature review Our setup builds on Morgan and Stocken (2001) who assume uncertainty about the sender's bias in the Crawford and Sobel (1982) canonical cheap talk model. Le Quement (2016) extends this problem to the case where DM consults several senders sequentially. Le Quement (2016) assumes that the aggregate distribution of levels of reliability is degenerate and exogenously imposes the fully random ordering rule. As the senders are assumed to be homogeneous, whether DM has an incentive to follow the presentation order by definition trivial. This paper instead considers arbitrary distributions of reliability levels and studies all possible ordering rules, focusing on the untouched question of the optimal ordering rule, while ensuring that DM has incentive to follow the presentation order in equilibrium.

Miura (2014) and Kawai (2015) study sequential cheap talk with multidimensional state variable, where as Ottaviani and Sørensen (2001) asks who should speak first in a context where several imperfectly informed senders are consulted, the ex ante competence of these senders being different and these being motivated by reputational concerns. Krishna and Morgan (2001a) re-examine Gilligan and Krehbiel (1989) and consider both heterogeneous and homogeneous senders who all observe the state. They find that some (not all) legislative rules lead to full revelation when combined with heterogeneous preferences.

In Austen-Smith (1993), the receiver faces two senders holding noisy information in a binary setup. Under some conditions, full revelation is possible with a single sender but not when two senders are consulted simultaneously. McGee and Yang (2013) study a setup where a decision maker's optimal decision is a (multiplicative) function of the uncorrelated types of two privately informed senders. In Li et al. (2016), a decision maker has to choose between two potential projects, information about returns being held separately by two senders who are each biased towards their own project. In both papers presented above, senders' informativeness levels are strategic complements (in contrast to our setup): informative communication by the other sender makes deviations from the truth more costly.

Alonso et al. (2008) consider information transmission in a multi-division organization, where each division's profits depend on how its decision matches its privately known local conditions and the other division's decisions. One possible decision protocol is centralization, whereby division managers report to central headquarters which decide for both divisions. They find that a stronger incentive to coordinate decisions worsens headquarters' ability to retrieve information from divisions. Rantakari (2016) and Moreno de Barreda (2010) assume the receiver is exogenously or endogenously in possession of some information, and show that the receiver's information can crowd out information transmission by the sender.

This paper also relates to the literature on Bayesian reputation building in games of information transmission (Sobel (1985), Benabou and Laroque (1992), Morris (2001) and Ely and Välimäki (2003)). Morris (2001) studies a two-period advice game with a binary state space and uncertainty in sender preferences. An unbiased sender has an incentive to lie in the early period in order to achieve a good reputation and be influential later. Our paper and Morris (2001) share the feature that a biased sender's behavior imposes a negative externality on the informativeness of a message from an unbiased sender. In Morris (2001), an unbiased sender does not always truthfully announce a high signal because such an announcement hurts his reputation. In our paper, an unbiased sender communicates in a noisy way also when the state is not high, so as to prevent biased senders from deviating downwards to mimic an unbiased sender.

The paper also connects to the literature on search and pricing on goods markets (see for example Baye et al (2006), Stahl (1989), Wolinsky (1986), Diamond (1971), Janssen and Parakhonyak (2014), Anderson and Renault (1999) and Baye and Morgan (2001)). In these models, firms have an endogenous preference over the consumer's search decisions and typically wish to discourage further search. The recent strand on ordered search is of particular relevance. See for example Wright et al. (2019), Haan et al. (2018), Derakhshan et al. (2018), Armstrong (2017), Arbatskaya (2007), Armstrong et al. (2009), Wilson (2010).

Our paper is also related to Glazer et al. (2021), who study fake reviews in a dynamic setting and consider the platforms' problem of either sharing reviews or not based on their content. They find that in terms of welfare the platform cannot do better than to show all reviews. Our paper studies a static setting where the platform chooses an ordering rule of the reviewers. Our model indicates that removing some reviewers can under some conditions improve consumers' welfare.

The paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the equilibrium for any given ordering rule. Section 4 studies welfare properties and derives the optimal ordering rule. Section 5 examines extensions.

2 The Model

The state of the world ω is drawn from the uniform distribution on $[0,1]$. In the context of online customer reviews, the state captures the underlying true product quality. An uninformed receiver (DM) faces a set of n senders $\chi = \{A, B, ..\}$, each of whom privately observes the state and simultaneously sends a cheap talk message $m_i \in M = [0,1]$. In the first phase of the game, DM can sequentially consult the senders at a cost c per sender, where c is arbitrarily small but positive. Once he stops consulting, he chooses an action $a \in \Re$ and his payoff is given by $-(\omega - a)^2 - \widetilde{n}c$, where $\widetilde{n}$ is the number of senders consulted. The optimal action after information collection is simply the conditional expected value of ω, which may correspond to the quantity of the product purchased. Our assumption on c implies that DM will carry on consulting as long as he expects that more consultation can generate more information.

Each sender has a privately observed type (1 or 2). Type 1, the unbiased type, has payoff function $-(\omega - a)^2$. Type 2, the biased type, has payoff function a. Type 1 is thus benevolent while type 2's objective is to maximize DM's belief about ω. For example, as an online reviewer, a biased sender wants to maximize the sales of the product. Sender i's probability of being of type 1 is p_i and thus parameterizes his ex ante reliability. Each sender's type is independently drawn according to p_i. The profile of the levels of reliability $\{p_1, p_2, ..., p_i, ..., p_n\}$ among all n senders is common knowledge. However, DM does not observe the identity and thus the reliability of the sender behind each message. Each sender knows their own identity (i) and the distribution of p_i's, but does not directly observe any other sender's reliability p_{-i}. The planner, which can be considered as an online platform, observes each sender's p_i. Let $\eta = \prod_{i=1}^{n}(1 - p_i)$, where η is the commonly known probability that all senders are biased.

The planner presents senders' messages in a presentation order which is generated by the commonly known ordering rule Γ. A presentation order dictates which sender's message is to be presented in which position in the sequence of the messages. The ordering rule is deterministic if one presentation order is assigned ex ante probability one. Denote by $D(\chi)$ the set of deterministic orders and denote by d any element of this set. Denote by θ_d^Γ the probability assigned to d under Γ. An ordering rule is given by $\Gamma = \{\theta_d^\Gamma\}_{d \in D(\chi)}$. Denote by p^l the reliability of the sender appearing in position l of the presentation order. Denote by m^l the message of the sender in position l in the order. Denote by $p^{l,d}$ the reliability of the sender appearing in position l of order d.

A sender strategy pins down how he communicates for each preference type that he might be assigned and the given known ordering rule. A pure strategy for a sender $i \in \{A, B, ..\}$ is given by function μ_i^r, for $r \in \{1, 2\}$, where $\mu_i^r : [0, 1] \to [0, 1]$ is such that $\mu_i^r(w)$ maps the state of nature $\omega \in [0, 1]$ and the sender's type r into a message in M. Note that we are omitting the ordering rule Γ from the strategy simply for notational convenience. A communication strategy is monotone if $\mu_i^r(w)$, for $r \in \{1, 2\}$, is weakly increasing in ω. A profile of sender strategies induces monotonic beliefs if profiles of messages that are higher yield higher beliefs of DM. A precise definition is provided later.

A pure strategy of DM is composed of a consultation rule and an action rule. A consultation rule specifies, for any history of observed messages, whether or not DM continues to consult and which review he consults among the presented reviews. An action rule specifies the action a chosen if DM stops consulting, for any history of observed messages.

Our equilibrium concept is Perfect Bayesian Equilibrium (PBE). Under a given ordering rule Γ, an equilibrium is given by a profile of strategies (one for each sender in χ and one for DM) as well as a system beliefs. A given profile of strategies and a system of beliefs constitute a PBE if the players' strategies are sequentially rational given each player's belief and the other players' strategies, while the beliefs are derived via Bayes' rule whenever possible. All the results stated in our analysis, whether positive or normative, are limit results in sense that there is some $\bar{c} > 0$ such that they hold true for any $c \in (0, \bar{c})$.

Note that given $c > 0$, there exists no fully revealing equilibrium in which all senders always report truthfully and send $m = \omega$ whatever the state and their preference type. Such an equilibrium would be supported by out of equilibrium beliefs such that DM chooses a punishment action (say $a = 0$) for the senders whenever messages contradict. However, this equilibrium breaks down because DM has a strict incentive to stop after one consultation, given $c > 0$.

We focus on senders' pure strategies that are symmetric (strategies do not depend on a sender's identity),[2] monotonic (i.e. weakly increasing in the state) and inducing monotonic beliefs (see the proof of Proposition 1 in Appendix B for a precise definition). We call such equilibria symmetric and monotone.

We show that all informative equilibria within this class **must** be partitional and thereby outcome equivalent to an equilibrium featuring the following simple strategy profile, which we focus on. There are thresholds $t_0 = 0 < t_1 < \ldots < t_{N-1} < t_N = 1$. An unbiased sender reports $m = t_r$ if $\omega \in (t_{r-1}, t_r]$ $\forall r = 1, .., N$ and t_1 if $\omega = 0$. A biased sender always sends $m = t_N$. DM's consultation rule is a stopping rule. He stops consulting as soon as he encounters $t_r \neq t_N$. Indeed, after $t_r \neq t_N$, he acknowledges that he has now learned that $\omega \in (t_{r-1}, t_r]$ and will not learn any more by consulting another sender. On the other hand, he continues consultation as long as he has only encountered t_N's and has not consulted all senders yet since c is very small. In this case, he might gain new information by consulting the next sender.

For any out of equilibrium profile of messages $\mathbf{m}$ for which beliefs cannot be derived via Bayes' rule, denoting by $\underline{m}(\mathbf{m})$ the lowest message in this set, the induced belief of DM is assumed to be $E[\omega \,|\omega \in (t_{r-1}, t_r]]$ if $\underline{m}(\mathbf{m})$ is located in the rth interval. Furthermore, DM's (sequential) consultation follows the order of presentation. For this to be incentive compatible, it must be true that after consulting the first r senders in the presentation order, the most informative sender (in expectation) is in position $r + 1$, for any $r \in \{0, .., n - 1\}$.[3] We call an equilibrium of the above type a simple partitional equilibrium of size N. Finally, we define an informative equilibrium as one in which DM's action is influenced by observed messages with positive probability.

We focus on ordering rules that are optimal from DM's perspective. For example, it seems reasonable to assume that an online platform aims at maximizing the informativeness of reviews, so as to maximize overall sales on the platform while minimizing the volume of returns. The DM optimal ordering is also weakly or strictly preferred by all sender types. Unbiased senders share DM's preferences. We will see that biased senders are indifferent

[2]However, we allow the senders' strategies to depend on their type r, the state of nature w, and the ordering rule Γ (i.e., the sender's expected position in the consultation order).

[3]Note that the equilibrium described above still exists under $c = 0$, as DM has no strict incentive to deviate from the assumed behaviour. From DM's perspective, the equilibrium is however trivially dominated by one in which he always consults all senders and all report truthfully.

among all ordering rules, as the expected value of DM's action is constant across all possible information generating experiments by the law of iterated expectation.

3 Equilibrium

Proposition 1. *For any informative, symmetric and monotone equilibrium, there exists an outcome equivalent simple partitional equilibrium.*

Proof: See Appendix B.

The Proposition establishes that under our mild assumptions on the strategies, it is without loss of generality to focus on simple partitional equilibria. In what follows, we characterize necessary and sufficient conditions for the existence of a simple partitional equilibrium featuring partition $\{t_r\}_{r=1}^{N-1}$ under ordering rule Γ. We look at the incentives of the senders and those of DM separately.

3.1 Senders' Incentives

Let us pin down the beliefs of DM. Given $\{t_r\}_{r=1}^{N-1}$ and ordering rule Γ, the DM's belief when he first observes some $t_r \neq t_N$ is given by

$$E[\omega\,|m = t_r] = \frac{t_{r-1} + t_r}{2}.$$

After he has consulted all n senders and received message t_N in total n times, his belief is given by

$$\begin{aligned}&E[\omega\,|m^1 = .. = m^n = t_N] \\ &= \frac{(1 - t_{N-1})(1-\eta)}{(1 - t_{N-1})(1-\eta) + \eta}\frac{t_{N-1} + 1}{2} + \frac{\eta}{(1 - t_{N-1})(1-\eta) + \eta}\frac{1}{2}.\end{aligned}$$

The expected value above accounts for two possible events. Either at least one of the senders is unbiased, in which case $\omega \geq t_{N-1}$, or nothing has been learned about the state as all senders are biased.

For what follows, given Γ and $\{t_r\}_{r=1}^{N-1}$, it is convenient to define $\Psi_{i,\Gamma}$ as the probability assigned by sender i to the event that DM will observe $m^1 = .. = m^2 = t_N$, conditional on $\omega = 0$ and sender i reporting t_N. We have

$$\Psi_{i,\Gamma} := P_{i,\Gamma}(m^1 = .. = m^n = t_N\,|\omega = 0, m_i = t_N).$$

In other words, $\Psi_{i,\Gamma}$ is the probability that the highest message from sender i (i.e. $m_i = t_N$) is not contradicted when the state is the lowest.

For any order $d \in D(\chi)$ and $i \in \chi$, denote by respectively $\chi_d^{i,-}$ and $\chi_d^{i,+}$ the set of sebders who are presented before and after i. Let

$$Q_{i,\Gamma} := \sum_{d \in D(\chi)} \theta_d^{\Gamma} \left[\Pi_{j \in \chi_d^{i,-}} (1 - p_j) \right].$$

The above is the ex ante probability that sender i will be consulted given $\omega = 0$, $\{t_r\}_{r=1}^{N-1}$ and ordering rule Γ. For every $d \in D(\chi)$, we have

$$P_{i,\Gamma}(d \left| \omega = 0, m^i = t_N \right.) = \frac{\theta_d^{\Gamma} \left[\Pi_{j \in \chi_d^{i,-}} (1 - p_j) \right]}{Q_{i,\Gamma}},$$

where by convention $\Pi_{j \in \chi_d^{i,-}} (1 - p_j) = 1$ if $\chi_d^{i,-} = \varnothing$.

Next, we have

$$\begin{aligned} \Psi_{i,\Gamma} &= \sum_{d \in D(\chi)} \left(P_{i,\Gamma}(d \left| \omega = 0, m^i = t_N \right.) \left[\Pi_{j \in \chi_d^{i,+}} (1 - p_j) \right] \right) \\ &= \frac{\Pi_{j \neq i} (1 - p_j)}{Q_{i,\Gamma}}. \end{aligned}$$

That is, $\Psi_{i,\Gamma}$ equals the probability that all other senders are biased divided by the probability that i is asked. Note the formula for $\Psi_{i,\Gamma}$ is independent of the assumed equilibrium partition $\{t_r\}_{r=1}^{N-1}$.

We now provide necessary and sufficient existence conditions for $\{t_r\}_{r=1}^{N-1}$ to be sender incentive compatible.

Lemma 1. *Fix Γ. Partition $\{t_r\}_{r=1}^{N-1}$ is sender incentive compatible if and only if, $\forall r < N-1$ and $\forall i \in \chi$*

$$t_r - E[\omega \left| m = t_r \right.] = E[\omega \left| m = t_{r+1} \right.] - t_r, \tag{1}$$

$$t_{N-1} - E[\omega \left| m = t_{N-1} \right.] = E[\omega \left| m^1 = .. = m^n = t_N \right.] - t_{N-1}, \tag{2}$$

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right.] + (1 - \Psi_{i,\Gamma}) E[\omega \left| m = t_1 \right.] \geq E[\omega \left| m = t_{N-1} \right.]. \tag{3}$$

Proof: See Appendix A.

Conditions (1) and (2) ensure that no unbiased senders have deviation incentives, by requiring that at any threshold $\omega = t_r$, for DM$\in \{1, .., N-1\}$, an unbiased sender is indifferent between messages t_r and t_{r+1}. Condition (1) implies that $t_r = \frac{r}{N-1} t_{N-1}$ for $r < N-1$. Using this and solving (2) for t_{N-1} given N and η, we have the unique solution

$$t_{N-1}^* = \frac{2N - \sqrt{4N\eta(-1+N)+1} + 1}{2N(1-\eta)}. \tag{4}$$

Therefore (1) and (2) yield a unique admissible partition

$$\left\{t_1^* = \frac{t_{N-1}^*}{N-1}, .., t_r^* = \frac{rt_{N-1}^*}{N-1}, .., t_{N-1}^*\right\} \tag{5}$$

for any $N > 1$ and $\eta \in (0,1)$. Note in particular that the partition described above is independent of the assumed ordering rule as the latter does not affect (1) or (2). This will turn out to be a very useful property when we study optimal ordering rules. Note also that (1) and (2) rewrite as

$$\frac{E[\omega \,| m^1 = .. = m^n = t_N]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}. \tag{6}$$

Condition (3) determines whether the partition pinned down by (1) and (2) is actually sender incentive compatible. A biased sender i must send $m^i = t_N$ for any ω. To ensure this, it is sufficient to ensure no deviation incentive at $\omega = 0$ (if there is no incentive to deviate at $\omega = 0$ then there is a weakly lower incentive to deviate at any $\omega \in [0,1]$). Now, consider incentives of a biased sender i at $\omega = 0$. Sending $m_i = t_N$ is "risky", as DM will keep on sampling and with probability $(1 - \Psi_{i,\Gamma})$ may encounter an unbiased sender and learn that $\omega \leq t_1$. Sending $m_i = t_{N-1}$ is the best deviation because given the DM's belief and stopping rule it preempts any further sampling while it yields the second highest belief $E[\omega|t_{N-1}]$. Note that $\Psi_{i,\Gamma}$ is smaller the earlier i's expected position in the presentation order and the higher the expected reliability of the senders consulted after sender i. Using (1), (3) rewrites as:

$$\frac{E[\omega \,| m^1 = .. = m^n = t_N]}{t_{N-1}} \geq \frac{(1-\Psi_{i,\Gamma})+2}{2}. \tag{7}$$

Using (6) to replace the LHS in the above inequality (7), we may conclude that there exists a sender incentive compatible N-interval partition if and only if

$$\Psi_{i,\Gamma} \geq \frac{N-2}{N-1}, \forall i \in \chi. \tag{8}$$

Furthermore, such a partition is unique if it exists. Note that $\frac{N-2}{N-1}$ is increasing in N, so an equilibrium of larger size (larger N) requires higher $\Psi_{i,\Gamma}$. The intuition is that a larger N implies larger $E[\omega \,|t_{N-1}]$ and lower $E[\omega \,|t_1]$, so that a larger N makes it more attractive to deviate to $m^i = t_{N-1}$ given $\omega = 0$. For a given order of consultation Γ, define

$$\Psi_\Gamma^{\min} := \min_{i \in \chi} \Psi_{i,\Gamma}, \tag{9}$$

which captures the incentive to send the highest message of the biased sender who has the largest incentive to deviate to the second highest message. Thus a partition of size N is incentive compatible as long as this sender (given $\omega = 0$) does not deviate to t_{N-1}.

We summarize the insights so far in the following Proposition:

Proposition 2. *a) Fix* Γ*. There exists a sender incentive compatible partition of size* N *if and only if* $\Psi_\Gamma^{\min} \geq \frac{N-2}{N-1}$*. If it exists, it is unique and is given by the partition* $\{t_r^*\}_{r=1}^{N-1}$ *defined in (5).*

b) Consider two ordering rules Γ *and* Γ'*. If a sender incentive compatible partition of size* N *exists under both orders, then they feature the same partition* $\{t_r^*\}_{r=1}^{N-1}$*.*

Comparing any two ordering rules Γ and Γ', we see that either the sets of sender incentive compatible partitions under Γ and Γ' are identical; or one is a superset of the other and contains equilibria of larger size, which is determined by the size of $\Psi_\Gamma^{\min}$ and that of $\Psi_{\Gamma'}^{\min}$. A two-interval equilibrium always exists as $\frac{2-2}{2-1} = 0$, while equilibria of larger size require $\Psi_\Gamma^{\min} \geq \frac{1}{2}$.

3.2 DM's Incentives

We now analyse DM's incentive to consult following the presentation order. Given that all senders have identical type-dependent communication strategies, at any point in time the DM's optimal choice is to consult the sender (as pinned down by a position in the presentation order) whose expected reliability is highest. The expected reliability of the first sender in the presentation order is

$$E\left[p^1\right] = \sum_{d \in D(\chi)} P(\theta_d^\Gamma) p^{1,d}. \tag{10}$$

Given $\{t_r\}_{r=1}^{N-1}$ and Γ, assuming that DM has followed the presentation order in the first k rounds of consultation and observed $m^1 = .. = m^k = t_N$, the expected value of p^l for $l > k \geq 1$ is given by

$$E\left[p^l \,\middle|\, m^1 = .. = m^k = t_N\right] = \sum_{d \in D(\chi)} P(\theta_d^\Gamma \,\middle|\, m^1 = .. = m^k = t_N) p^{l,d}, \tag{11}$$

where

$$P(\theta_d^\Gamma, m^1 = .. = m^k = t_N) = \theta_d^\Gamma \left(t_{N-1} \prod_{i=1}^{k} (1 - p^{i,d}) + 1 - t_{N-1} \right). \tag{12}$$

In words, conditional on $m^1 = .. = m^k = t_N$, DM updates his prior over the set of deterministic sequences assigned positive probability under Γ, each of which assigns a specific sender to position l. He uses this to derive the implied weighted average of $p^{l,d}$'s and to thus identify which sender to consult next.

Lemma 2. *Fix* Γ *and* $\{t_r\}_{r=1}^{N-1}$*. Consulting following the order of presentation is DM incen-*

tive compatible iff :

$$E\left[p^1\right] \geq E\left[p^l\right] \ \forall l > 1, \quad (13)$$

$$E\left[p^{k+1}\left|m^1 = .. = m^k = t_N\right.\right] \geq E\left[p^l\left|m^1 = .. = m^k = t_N\right.\right]$$
$$\forall k, l \text{ such that } k \in \{1, .., n-1\} \text{ and } l > k+1. \quad (14)$$

The above condition ensures that DM always wants to follow the presentation order. The first inequality ensures that he wants to consult the first sender in the presentation order when consulting first. The second condition ensures that for any $k \in \{1, .., n-1\}$, after observing $m^1 = .. = m^k = t_N$ and thus deciding to consult again, the most informative sender is located in position $k+1$ of the presentation order.

4 Optimal Ordering

4.1 Welfare Properties of Equilibria

In a partitional equilibrium featuring $\{t_r\}_{r=1}^{N-1}$, the expected payoff of DM is given by

$$\begin{aligned}
&\Pi_{DM}(N, \eta) \quad (15)\\
&= -(1-\eta)\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(\frac{t_i + t_{i-1}}{2} - \omega\right)^2 d\omega\right]\\
&- \eta\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(E[\omega\,|m_A = m_B = t_N] - \omega\right)^2 d\omega\right]\\
&- \int_{t_{N-1}}^{1}\left(E[\omega\,|m_A = m_B = t_N] - \omega\right)^2 d\omega.
\end{aligned}$$

In the above, the first line of the RHS expression corresponds to the scenario where $\omega \leq t_{N-1}$ and there is at least one unbiased sender. The second line refers to the scenario where $\omega \leq t_{N-1}$ and there is no unbiased sender. The third line is the scenario $\omega > t_{N-1}$ so that all senders report t_N. We ignore sampling costs which are assumed arbitrarily small in the calculation of the expected payoff. We obtain the following results:

Proposition 3. *We have*

a) If an equilibrium of size N exists under two ordering rules Γ and Γ', then DM achieves the same equilibrium expected payoff $\Pi_{DM}(N, \eta)$ under both ordering rules.

b) $\Pi_{DM}(N+1, \eta) > \Pi_{DM}(N, \eta)$ for any $N \geq 1$ and $\eta \in (0, 1)$.

c) $\frac{\partial \Pi_{DM}(N,\eta)}{\partial \eta} < 0$ for any $N \geq 1$ and $\eta \in (0, 1)$.

Proof: See Appendix A.

The proof of point a) is as follows. Recall first that by point b) of Proposition 2, if an equilibrium of size N exists under two ordering rules Γ and Γ', then it features the same partition $\{t_r^*\}_{r=1}^{N-1}$. Next, simply note that for a fixed partition, DM's expected utility depends only on one aspect, namely whether or not at least one of the senders is unbiased. If all senders are biased, DM will end up consulting n times and keep receiving the same message t_N, regardless of the consultation order. If at least one of the senders is unbiased, then given any state $\widetilde{\omega}$ he will end up with the same final belief under any consultation order. Specifically, if $\omega \leq t_{N-1}$, he will learn the interval in which $\widetilde{\omega}$ is located, while if instead $\omega > t_{N-1}$, he will consult all n senders observe the same message t_N.

Point b) states that among any two partitional equilibria, the equilibrium with a larger number of intervals yields a higher expected utility of DM. This reflects the fact that a less coarse partition allows unbiased senders to communicate more informatively. Point c) captures the fact that a lower probability of all senders being biased implies a higher probability of learning ω accurately.

4.2 Optimal Ordering Rules

We now identify an optimal ordering rule, i.e. a rule that maximizes the achievable expected payoff of DM.[4] By Proposition 3, an ordering rule Γ is optimal if it yields the equilibrium partition of largest size among all ordering rules. By Proposition 2, an ordering rule $\hat{\Gamma}$ yields the largest achievable sender incentive compatible partition if it satisfies:

$$\hat{\Gamma} = \arg\max_{\Gamma} \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{16}$$

Denote by $N^{\max}$ the size of the largest achievable sender incentive compatible partition.

In principle, the incentive compatibility constraint of DM could complicate the search for an optimal ordering rule as some partitions that are sender incentive compatible under a given Γ might not be part of an equilibrium as DM's incentive compatibility conditions are not satisfied. To account for this potential issue, we take a two-step approach in our search for an optimal ordering rule.

We first ignore DM's incentive compatibility and find a necessary and sufficient condition for an ordering rule to solve (16). As we will show in Proposition 4 shortly, all of these ordering rules yield the same value of $\Psi_{\Gamma}^{\min}$ and the same largest sender incentive compatible partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$. Next, we show that among these ordering rules, there exist at least one

[4] Recall that a rule typically yields a set of simple partitional equilibria, and we focus, for each rule, on the equilibrium that leads to the maximum number of partitions.

such that under this largest partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive compatibility constraint is also satisfied. This second step is accomplished in two substeps, by first identifying a simple class of rules that satisfy (16) and then searching within this class.

We next show that optimal rules are random. To understand why the optimal order needs to be random, imagine an example with just two senders A and B. Consider the incentives of A when it is biased and when the state is low ($w = 0$). If A is the last in the consultation order with probability 1, then his message is only observed if the message sent by B was t_N (as otherwise DM stops consultation after the first message). Thus, A's best response is to report t_N as it induces the highest possible action of DM with probability 1.

If, however, A is the first in the consultation order, sending t_N induces DM to consult again, which runs the risk of B being unbiased and thus sending message t_1, which induces DM to play the lowest possible equilibrium action. If instead A sends message t_{N-1} it induces DM to stop consultation and play the second highest equilibrium action. A thus has a trade-off between inducing the second highest equilibrium action and risking either the highest or lowest equilibrium action.

Therefore, the optimal order should put A second in the consultation order. However, the same reasoning applies to sender B too. This means that that it would be optimal for both senders to be the second. The way to implement this is to choose an order that assigns positive probability to both the event where A is the last and the event where B is the last.

The optimal order is random in a way that balances the probability that every deterministic order occurs and each sender's probability of being biased. If in the example above A is very likely to be biased and B is very less to be biased, the optimal orders assigns a higher probability to the order where A is the last. This is achieved by equating all senders' beliefs, conditional on their message being observed, about the likelihood that DM observes only the highest message after the sender's own message, i.e., by equating $\Psi_{i,\Gamma}$ for all senders.

Proposition 4. *An order Γ satisfies (16) if and only if*

$$\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta} \sum_k \frac{p_k}{1-p_k}$$

for all $i, j \in \chi$.

Proof: See Appendix A.

The Lemma has two important features. First, in all rules satisfying (16), we have $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for any i, j, and therefore the largest sender incentive compatible partition is identical. To see this, assume there exists exists exactly one sender k such that $\Psi_{k,\Gamma} \leq \Psi_{j,\Gamma}$ for all j, with at least one strict inequality say for sender l. Then by continuity of the functions $\{\Psi_{i,\Gamma}\}_i$, which are linear equations on the probabilities of all deterministic orders with coefficients that are polynomials in $\{p_i\}_i$, it is possible to find an order Γ' where $\Psi_{k,\Gamma'} > \Psi_{k,\Gamma}$ and $\Psi_{k,\Gamma'} < \Psi_{l,\Gamma'} < \Psi_{l,\Gamma}$. This means that $\min_{i\in\chi}\Psi_{i,\Gamma'}$ is larger than $\min_{i\in\chi}\Psi_{i,\Gamma}$, which leads to a contradiction.

The second insight from Proposition 4 is that if the senders' beliefs are such that $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for all i, j, these beliefs pin down a unique admissible set of beliefs for all senders, namely $\Psi_{i,\Gamma} = \frac{\eta}{1-\eta}\sum_k \frac{p_k}{1-p_k}$ for all i. This means that given any ordering rule, we can easily check whether or not it satisfies (16).

While Proposition 4 gives necessary and sufficient conditions for a presentation order to satisfy (16), one faces an issue of dimensionality when explicitly constructing ordering rules that satisfy (16). Proposition 4 yields n equations whereas an ordering rule is pinned down by $n! - 1$ unknowns, as there are $n!$ possible deterministic orders while the probabilities of all deterministic orders must add up to 1. We take a constructive approach and identify a class of ordering rules that satisfy (16). The class builds on the concept of Latin squares, first studied in the 18th century by Korean and Swiss mathematicians Choi Seok-Jeong and Leonhard Euler.

Definition *a) A Latin square ordering rule is an ordering rule such that exactly n deterministic orders $\{d_1, \ldots, d_n\}$ have strictly positive probability and for every $i \in \chi$ and $l \in \{1, .., n\}$, there is a unique $d \in \{d_1, \ldots, d_n\}$ for which sender i occupies position l. b) A proportional Latin square ordering rule is a Latin square ordering rule such that for any $d \in \{d_1, .., d_n\}$, $\theta_d = \frac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)}$, where i is the sender who occupies position 1 in d.*

For example, for $\chi = \{A, B, C\}$ there are two possible Latin square ordering rules. One is such that only $\{\theta_{ABC}, \theta_{BCA}, \theta_{CAB}\}$ are positive and the other is such that only $\{\theta_{ACB}, \theta_{BAC}, \theta_{CBA}\}$ are positive. One can represent each of these as a square, where each row corresponds to a different deterministic order assigned positive probability. Each of the obtained squares is a Latin square. The first of these Latin square rules yields the following proportional Latin square ordering rule:

$$\begin{aligned}
\theta_{ABC} &= \frac{p_A/(1-p_A)}{\sum_{i\in\chi} p_i/(1-p_i)}, \\
\theta_{BCA} &= \frac{p_B/(1-p_B)}{\sum_{i\in\chi} p_i/(1-p_i)}, \\
\theta_{CAB} &= \frac{p_C/(1-p_C)}{\sum_{i\in\chi} p_i/(1-p_i)}.
\end{aligned}$$

Note that in any proportional Latin square ordering rule, the probability that a sender appears first in the presentation order is increasing in the sender's own reliability and decreasing in the other senders' reliability.

Lemma 3. *All proportional Latin square ordering rules satisfy (16).*

Proof: See Appendix A.

The Lemma above establishes that proportional Latin square rules achieve the maximal partition size if we ignore DM's incentive constraint. There is no known way to characterize all Latin squares of a general order n. Moreover, it is not known how many Latin squares of a particular order exist, although this number is exponentially increasing in n. Thus, since proportional Latin squares are a subset of all possible optimal ordering rules, there is no known way to characterize all optimal rules.

The next question is whether we can find any proportional Latin square ordering rule that is, furthermore, incentive compatible for DM. The answer is positive.

Lemma 4. *Among the set of proportional Latin square rules, there is one such that given the rule and maximum partition it induces, consulting according to the order of presentation is incentive compatible for DM.*

The proof of the Lemma is as follows. There is a simple $(n-1)$-step algorithm for identifying a proportional Latin square rule Γ^* such that given Γ^* and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive conditions are satisfied. To see this, start with an arbitrary proportional Latin square ordering rule Γ and assume that the equilibrium partition is $\{t_r^*\}_{r=1}^{N^{\max}-1}$. It is easy to show using the rearrangement inequality that in the first consultation by the DM, the sender appearing in position 1 of the presentation order is the most reliable one in expectation.

Consider now the second consultation assuming that DM observed message t_N^* in the first consultation. Assess the relative reliability of the senders located in positions 2 to n of the presentation order. If the sender in position 2 is the most reliable sender, then keep the ordering rule Γ and proceed to the sender in position 3. If the most reliable sender is in position $r > 2$, then construct a new ordering rule Γ' by permutating senders in position 2 and position r in all of the n sequences that have positive probability of occuring under Γ. Note that Γ' is also a proportional Latin square rule and it is such that in the second round, the sender in position 2 is the most reliable sender. The reason is that in the second round, the expected reliability of senders in position 2 and r have now been interchanged, as is immediately clear from (11) and (12).

Repeat the procedure for the third consultation, by checking who is the most reliable sender after t_N^* has been observed in the first and second rounds. We iterate the procedure until consultation in the n-th round is reached, at which point we have a proportional Latin square rule Γ^* such that given Γ^* and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive compatibility is satisfied.

5 Extensions

5.1 Optimal Deterministic Ordering Rules

Note first that any deterministic ordering rule is trivially suboptimal. For a deterministic ordering rule Γ pinned down by $d \in D(\chi)$, recalling that $\chi_d^{i,+}$ denotes the set of senders who are consulted after sender i, we have $\Psi_{i,d} = \Pi_{j \in \chi_d^{i,+}}(1 - p_j)$ and $\Psi_d^i \neq \Psi_d^j$ for any i, j, which violates a necessary condition for optimality.

Remark 1. *Suppose only deterministic orderings are allowed. The only deterministic ordering rule that can be part of an equilibrium is such that i appears before j if $p_i > p_j$.*

The proof is as follows. A sender incentive compatible partition allows for a finer partition in equilibrium if the other senders are more likely to be biased. Thus the first sender should be the most reliable one, because that way the probability that all of the rest are biased is maximized. The same reasoning applies to all senders who follow.

This ordering rule yields the highest $\min_{i \in \chi}\{\Psi_{A,d}, \Psi_{B,d}, ..\}$ and thus the largest equilibrium size among all deterministic ordering rules. For any deterministic order pinned down by d, it is immediate that

$$\min_{i \in \chi}\{\Psi_{A,d}, \Psi_{B,d}, ..\} = \Psi_{i,\Gamma}$$

if i is the first sender consulted. It follows immediately that the most attractive deterministic ordering rule, in terms of inducing the equilibrium with the largest number of partitions, is such that the first sender consulted is the sender with the highest p_i. Indeed, for $i, i' \in \chi$, it holds true that $\Pi_{j \in \chi_{-i}}(1 - p_j) > \Pi_{j \in \chi_{-i'}}(1 - p_j)$ if and only if $p_i > p_{i'}$.

5.2 Observable Reliability Levels

Consider the situation where the online platform shares information about reviewers' reliability with consumers.[5] Our model indicates that this is not beneficial to the consumers.

Specifically, suppose that DM now knows the identity i of each sender and thus observes p_i directly. Note that the DM's equilibrium beliefs are as in the main model. The expected state when all n messages are the highest $E[\omega \,|m^1 = .. = m^n = t_N]$ is affected by senders' reliability only via η, which is independent of how exactly the entries in $\{p_1, .., p_N\}$ are allocated among individual senders.

Clearly, in any partitional equilibrium, DM consults more reliable senders first (as such DM's consultation strategy induces the optimal deterministic ordering rule discussed previously), which means that in equilibrium we must have $\Psi_\Gamma^{\min} = \Pi_{j \in \chi_{-i}}(1 - p_j)$, where i is the

[5]For instance, Yelp.com shares information about reviewers with customers and Amazon used to do so.

most reliable sender. This is strictly less than the value of $\Psi_{\Gamma'}^{\min}$ achieved by a proportional Latin square ordering rule Γ'.

5.3 Varying the Pool of Senders

We here investigate the role of the distribution of the revels of reliability assuming that an optimal ordering rule is used by the planner ($\Psi_{i,\Gamma} = \frac{\eta}{1-\eta}\sum_{k\in\mathcal{X}}\frac{p_k}{1-p_k}$ for all i). We restrict ourselves to comparing distributions that yield the same value of η, the probability that all senders are biased. We look for the optimal profile of p_i's conditional on this constraint and, furthermore, assuming a potential lower bound on the reliability of any individual $\varepsilon \in [0, 1-\eta^{\frac{1}{n}}]$. We thus solve

$$\max_{\{p_i\}_{i\in\mathcal{X}}} \quad \frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} \tag{17}$$

$$\text{s.t.} \prod_i (1-p_i) = \eta, \tag{18}$$

$$\min_{i\in\mathcal{X}} p_i \geq \varepsilon, \text{ for } \varepsilon \in [0, 1-\eta^{1/n}). \tag{19}$$

Let $\mathbf{p} = \{p_i\}_{i\in\mathcal{X}}$ and define $\eta(\mathbf{p})$ as the corresponding value of $\prod_i(1-p_i)$. Let $\Lambda(\eta, n)$ be the set of all distributions involving n senders and that yield the same value of η.

Proposition 5. *a) The solution to problem (17)-(19) is given by* $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ *for some* $i \in \{1, \ldots, n\}$ *and* $p_j = \varepsilon$ *for all* $j \neq i$.

b) Consider two profiles $\mathbf{p}, \mathbf{p}' \in \Lambda(\eta, n)$ *such that for some* i, j *we have* $p'_i > p_i$ *and* $p'_j < p_j$ *while* $p_k = p'_k$ *for all* $k \notin \{i, j\}$. *If* $p_i > p_j$, *then* $\mathbf{p}'$ *yields a weakly higher expected payoff of DM and vice versa if instead* $p_i < p_j$.

Proof: See Appendix A.

Regarding point a), If $\varepsilon = 0$, then the solution to the problem is trivial. The objective function can always be made equal to 1 by setting $p_i = 1-\eta$ for any one $i \in \{1, .., , n\}$ and $p_j = 0$ for all $j \neq i$. In this case $\prod_i(1-p_i) = \eta$ and $\frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} = \frac{\eta}{1-\eta}\frac{1-\eta}{\eta} = 1$. This is enough to guarantee the existence of an equilibrium of any size (and recall that larger equilibria yield a higher DM expected payoff). In general, assuming a lower bound $\varepsilon > 0$, the optimal distribution is one where all probabilities take the lowest possible value but one of them, which takes the highest. Point b) compares pools of senders in which we shift reliability levels between two senders by making the more reliable sender even more reliable and the less reliable sender even less reliable, in a way that keeps η fixed. We see that such an polarizing shift is beneficial to DM, in a way that echoes point a).

5.4 Adding or Removing Senders

Form our analysis it is easy to see that removing senders who are likely to be biased is not necessarily beneficial (as advocated by some online platforms).[6] Removing a sender, regardless of its probability of being biased, has two effects. A direct effect is that the probability of learning the state $1-\eta$ decreases and this is detrimental for DM. Intuitively, removing a sender reduces the amount of information available from senders; if the sender removed is biased there is no change to this pool while if the sender removed is honest then there is a strict decrease in the information available.

An indirect effect is that, since the optimal number of partitions depends positively on $\frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i}$, removing a sender increases $\frac{\eta}{1-\eta}$ but decreases $\sum_i \frac{p_i}{1-p_i}$. Thus, removing a sender can potentially increase the number of partitions in equilibrium, which in turn increases the amount of information transmitted in equilibrium to DM. Intuitively, for every sender, having fewer (potentially) biased senders may reduce the probability that all senders presented later than him report the highest message. That is, the incentive for a biased sender to deviate to the second highest message ($m_i = t_{N-1}$) is higher when there are fewer biased senders.

Therefore, removing a sender reduces information available but may increase the information transmitted in equilibrium. The net result of these two effects is ambiguous. We can find numerical examples where the effect of removing those senders who are most likely to be biased is negative or positive. Similarly, there are also numerical examples where the effect of adding senders, even if they are likely to be unbiased, is positive or negative.

6 Conclusion

We have presented a model of information transmission between senders, some of whom can be biased, and a receiver who consults them sequentially. The model maps into the setting of online product reviews. We have shown that the optimal way to display product reviews is random in such a way that it equates the beliefs that the consumer will learn the truth about the product after reading each review, which may explain why the most reliable reviewers are not necessarily presented first. We have also seen that removing reviews that are likely to be fake is not necessarily beneficial.

The model can also be applied to the cases of user generated commenting on general issues, such as Quora or newspaper comment sections. The pool of reviewers providing opinions is a mixture of honest citizens and agenda driven partisans possibly tied to organizations. Reviewers are as such largely ex-ante identical from readers' perspective, but the platform

[6]See for instance https://www.aboutamazon.com/news/policy-news-views/.

may have access to data that allows it to estimate the reliability of individuals. The platform is typically free to decide in which order responses are shown and might condition the order on these estimates. Google's search page offers another instance of the ordering problem. For any given search query, the PageRank algorithm provides an ordered set of results. In this particular case, however, different sources typically have different levels of reliability in the eyes of readers.

The main result of this paper is to identify how to optimally order experts in a sequential consultation problem. We find that that the order should be stochastic, which implies that less reliable experts might sometimes be asked earlier. Experimental work would be called upon to qualitatively test our predictions, in order to see whether experts' behaviour is indeed driven by the pre-emption motive that drives our findings.

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8 Appendix A

8.1 Proof of Lemma 1

Step 1 From the incentives of unbiased senders, it must be true that

$$E[\omega \left| m^1 = .. = m^n = t_N \right] - t_{N-1} = t_{N-1} - \frac{t_{N-1} + t_{N-2}}{2} \tag{20}$$

and it must also be true that all thresholds between $t_0 = 0$ and t_{N-1} are equally spaced, which means that for any $K < N-1$, we have $t_r = \left(\frac{K}{N-1}\right)t_{N-1}$. Using $t_{N-2} = \left(\frac{N-2}{N-1}\right)t_{N-1}$, (20) is equivalent to:

$$\frac{E[\omega \left| m^1 = .. = m^n = t_N \right]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}.$$

Inserting the closed form expression for $E[\omega \left| m^1 = .. = m^n = t_N \right]$, we obtain for any given N and η, the unique solution value of t_{N-1} which is given by (4).

Step 2 In an equilibrium featuring the partition $\{t_r\}_{r=1}^{N-1}$, let $m(\omega^*)$ denote the message sent by an unbiased sender if the state is ω^* and $\omega^* < t_{N-1}$. Denote by $E[\omega \left| m(\omega^*) \right]K$'s expected value of the state if he encounters the equilibrium message $m(\omega^*)$. From the incentives of biased senders, we need that for every sender $i \in \chi$ and for every $\omega^* \leq t_{N-1}$, it holds true that:

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right] + (1 - \Psi_{i,\Gamma}) E[\omega \left| m(\omega^*) \right] \geq \frac{t_{N-1} + t_{N-2}}{2}. \tag{21}$$

This condition ensures that any biased sender is willing to send m_N rather than deviate to m_{N-1}, whatever the realized state. Clearly, $E[\omega \,|m(\omega^*)]$ is increasing in ω^*, so the condition is most difficult to satisfy for $\omega^* = 0$. Thus, (21) is satisfied if and only if it is satisfied at $\omega^* = 0$. We thus need that for every sender $i \in \chi$ it holds true that:

$$\Psi_{i,\Gamma} E[\omega \,|m^1 = .. = m^n = t_N] + (1 - \Psi_{i,\Gamma})\frac{t_1}{2} \geq \frac{t_{N-1} + t_{N-2}}{2}. \tag{22}$$

Recall that the size of every interval to the left of t_{N-1} is identical and given by:

$$2\left(E[\omega \,|m^1 = .. = m^n = t_N] - t_{N-1}\right).$$

We may thus rewrite the constraint (22) as

$$\begin{aligned}&\Psi_{i,\Gamma} E[\omega \,|m^1 = .. = m^n = t_N] + (1 - \Psi_{i,\Gamma})\left[E[\omega \,|m^1 = .. = m^n = t_N] - t_{N-1}\right] \\ &\geq 2t_{N-1} - E[\omega \,|m^1 = .. = m^n = t_N]\end{aligned}$$

which is equivalent to

$$\frac{E[\omega \,|m^1 = .. = m^n = t_N]}{t_{N-1}} \geq \frac{(1 - \Psi_{i,\Gamma}) + 2}{2}.$$

Now, bringing together the two conditions derived from the incentives of biased and unbiased senders, an equilibrium with N intervals exists if and only if:

$$\frac{2(N-1)+1}{2(N-1)} \geq \frac{(1 - \Psi_{i,\Gamma}) + 2}{2}.$$

8.2 Proof of Proposition 3

The proof analyses the general case of $n \geq 2$ senders. We proceed by proving point b), noting that point a) follows from step 3 in the proof of point b), and finally we prove point c).

8.2.1 Point b): Effect of N

Step 1 Recall that in equilibrium, we have:

$$t_{N-1}\left(\frac{2(N-1)+1}{2(N-1)}\right) = E[\omega \,|m^1 = .. = m^n = t_N].$$

In what follows, define $f(N, \eta) := t^*_{N-1}$, where t^*_{N-1} is given as in (4).

Step 2 $\Pi_{DM}(N, \eta)$ is given by minus the following sum:

$$\begin{aligned}&(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \\ &+ (1-\eta)\int_{f(N,\eta)}^{1}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\ &+ \eta\int_0^1\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.\end{aligned} \tag{23}$$

This can be further decomposed into the following elements:

$$\begin{aligned}
&(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \\
&+\eta\int_0^{f(N,\eta)}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\
&+\int_{f(N,\eta)}^{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\
&+\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.
\end{aligned} \tag{24}$$

Consider the four lines that constitute expression (24) above. The expression in the last line is decreasing in N, as we shall show in next step. In step 3, we prove that the sum of the three expressions appearing in the first, second and third line is also decreasing in N, which proves point a).

Step 3 Consider:

$$\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.$$

Note first that $\int_t^1(\omega-t)^2\,d\omega=-\frac{1}{3}(t-1)^3$ is trivially decreasing in t. Now, we need to show that $f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)$ is increasing in N. Note that:

$$\begin{aligned}
&\frac{\partial\left(\left(\frac{2(N-1)+1}{2(N-1)}\right)f(N,\eta)\right)}{\partial N} \\
&=\frac{1}{4N^2(1-\eta)(N-1)^2\sqrt{4\eta N^2-4\eta N+1}}G_0(\eta,N),
\end{aligned}$$

where

$$\begin{aligned}
G_0(N,\eta)=&\sqrt{4\eta N^2-4\eta N+1}-2N\eta-2N \\
&-2N\sqrt{4\eta N^2-4\eta N+1}+2N^2\eta+2N^2+1.
\end{aligned}$$

We simply need to show that $G_0(N,\eta)>0$. Simple algebraic manipulation shows that this is equivalent to proving that $-4N^2(\eta-1)^2(N-1)^2<0$, which is always true.

Step 4 Consider the three expressions appearing in the first, second and third line of (24). The sum of these equals:

$$\begin{aligned}
T(N,\eta)=\frac{1}{192N^3(\eta-1)^3}\frac{2N-1}{(N-1)^3}&\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^3 \\
&\left(4\eta N^2-4\eta N+1\right).
\end{aligned}$$

We want to prove that $T(N,\eta)$ is always decreasing in N. Note that:

$$\frac{\partial T(N,\eta)}{\partial N} = \frac{1}{(\eta-1)^3}\frac{1}{192N^4(N-1)^4}$$
$$\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^2 G_1(\eta,N),$$

where

$$G_1(N,\eta) =$$
$$10N-10N^2\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}+8N\eta-3\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}$$
$$+10N\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}-24N^2\eta+16N^3\eta-12N^2+8N^3$$
$$+50N^2\eta\sqrt{4\eta N^2-4\eta N+1}-80N^3\eta\sqrt{4\eta N^2-4\eta N+1}$$
$$+40N^4\eta\sqrt{4\eta N^2-4\eta N+1}-10N\eta\sqrt{4\eta N^2-4\eta N+1}-3.$$

To show that $\frac{\partial T(N,\eta)}{\partial N}<0$, we simply need to show that $G_1(N,\eta)>0$. Simple algebraic manipulation shows that this in turn equivalent to proving that:

$$-4N^2(\eta-1)^2(N-1)^2\left(4N\eta-16N-4N^2\eta+16N^2+3\right)<0,$$

which is always true.

8.2.2 Point c): Effect of η

Step 1 Consider expression (24). The expression appearing in the last line is trivially increasing in η, as proved now. We have:

$$\frac{\partial\left(f(N,\eta)\right)}{\partial\eta}$$
$$= -\frac{1}{2N(\eta-1)^2\sqrt{4\eta N^2-4\eta N+1}}G_0(\eta,N),$$

where $G_0(N,\eta)$ was defined earlier in our analysis of comparative statics with respect to N. We wish to prove that the above is negative. This is equivalent to showing that $G_0(N,\eta)>0$, which we already proved is true.

Step 2 Consider the three expressions appearing in the first, second and third line of (24). We now show that the sum of these three expressions (denoted $T(N,\eta)$) is increasing in η. Note that:

$$\frac{\partial T(N,\eta)}{\partial\eta} = \frac{1}{192N^3}\frac{2N-1}{(N-1)^3}\frac{1}{(\eta-1)^4}$$
$$\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^2 G_2(\eta,N),$$

where

$$G_2(N,\eta) = \begin{array}{c} 10N - 10N^2\sqrt{4\eta N^2 - 4\eta N + 1} + 8N\eta - 3\left(4\eta N^2 - 4\eta N + 1\right)^{\frac{3}{2}} \\ +10N\sqrt{4\eta N^2 - 4\eta N + 1} - 24N^2\eta + 16N^3\eta - 12N^2 + 8N^3 \\ +10N^2\eta\sqrt{4\eta N^2 - 4\eta N + 1} - 10N\eta\sqrt{4\eta N^2 - 4\eta N + 1} - 3. \end{array}.$$

To show that $\frac{\partial T(\eta,N)}{\partial \eta} > 0$, we simply need to show that $G_2(\eta, N) > 0$. Simple algebraic manipulation shows that this in turn equivalent to proving that

$$\left(4N\eta - 16N - 4N^2\eta + 16N^2 + 3\right) > 0,$$

which is always true.

8.3 Proof of Proposition 4

The proof is decomposed into two lemmas which together yield the result. Recall that we focus on ordering rules that satisfy:

$$\Gamma = \arg\max_{\Gamma} \min_{i\in\chi} \Psi_{i,\Gamma}. \tag{25}$$

The first Lemma below shows that any ordering rule Γ that solves (25) is such that $\Psi_{i,\Gamma}$ is equalized across senders. We then show that equalizing $\Psi_{i,\Gamma}$ for all senders pins down a unique value for $\Psi_{i,\Gamma}$ for all i, given by $\frac{\eta}{1-\eta}\sum_j \frac{p_j}{1-p_j}$. This in turn means that any order which achieves this value solves (25).

The lemmas below use the fact that for any order Γ it is true that in equilibrium $Q_{i,\Gamma}(1-p_i)\Psi_{i,\Gamma} = \eta$ for all i. That is, the probability that sender i is consulted (which in equilibrium equals the probability that all senders before i are biased), times the probability that i is biased, times the probability that all senders after i are biased, equals η, i.e. the probability that all senders are biased. Also note that $Q_{i,\Gamma}(1-p_i)\Psi_{i,\Gamma} = \eta$ implies that $\max_\Gamma \min_{i\in\chi} \Psi_{i,\Gamma}$ is equivalent to $\min_\Gamma \max_{i\in\chi}(1-p_i)Q_{i,\Gamma}$.

Lemma 5. *If an ordering rule* Γ *solves (25) then* $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ *for all* i, j *and consequently* $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ *for all* i, j*.*

Proof. Assume the contrary, this means that there is a rule Γ that solves (25) but for which there exists a non-empty set of senders $\hat{\chi}$ such that $(1-p_j)Q_{i,\Gamma} = (1-p_k)Q_{k,\Gamma}$ for all $i, k \in \hat{\chi}$, and $(1-p_i)Q_{i,\Gamma} > (1-p_j)Q_{j,\Gamma}$ for all $i \in \hat{\chi}$ and $j \notin \hat{\chi}$. Note that $\hat{\chi} \subset \chi$ as if $\hat{\chi} = \chi$ the lemma holds true.

There exists an $i \in \hat{\chi}$ and a $j \notin \hat{\chi}$ such that i acts immediately before j in some deterministic order, call it d, that occurs with positive probability, $\theta_d > 0$. Otherwise, all senders in $\hat{\chi}$

act consecutively and after all senders not in $\hat{\chi}$ in all deterministic orders that have positive probability, in which case $\min_{j\notin\chi}(1-p_j)Q_{j,\Gamma} > \max_{j\in\chi}(1-p_i)Q_{i,\Gamma}$, i.e. the sender not in $\hat{\chi}$ that is least likely to be asked is more likely to be asked than the sender in $\hat{\chi}$ that is most likely to be asked. This contradicts the definition of the set $\hat{\chi}$.

Create a new ordering rule Γ' identical to Γ but such that order d has probability $\hat{\theta}_d = \theta_d - \varepsilon$ for some small $\varepsilon > 0$, and order d', which is the same as d but where the positions of i and j are swapped, has probability $\hat{\theta}_{d'} = \theta_{d'} + \varepsilon$ (note that $\theta_{d'}$ cannot be 1 as otherwise j always acts before i and $(1-p_j)Q_{j,\Gamma} > (1-p_i)Q_{i,\Gamma}$, a contradiction to $i \in \hat{\chi}$).

Notice that Γ and Γ' are such that $Q_{k,\Gamma} = Q_{k,\Gamma'}$ for all $k \neq i,j$ and $Q_{i,\Gamma'} < Q_{i,\Gamma}$ and $Q_{j,\Gamma'} > Q_{j,\Gamma}$. Since $Q_{k,\Gamma}$ for all sender k is linear in the probabilities of each deterministic order, by continuity ε can be chosen such that $(1-p_i)Q_{i,\Gamma} > (1-p_j)Q_{j,\Gamma'}$.

We have just proven that there exists an ordering rule Γ' with associated set $\hat{\chi}'$, defined similarly to $\hat{\chi}$, such that $\hat{\chi}'$ includes all senders in $\hat{\chi}$ except i, and no other senders.

If $\hat{\chi}$ only has one sender skip this step of the proof and go to the next paragraph. Otherwise, note that given Γ' and $\hat{\chi}'$, we have that again there exists an $i' \in \hat{\chi}'$ and a $j' \notin \hat{\chi}'$ such that i' acts immediately before j' in some deterministic order that occurs with positive probability. Repeat the reasoning in this proof to generate an order Γ'' with associated set $\hat{\chi}''$, defined similarly to $\hat{\chi}$, such that $\hat{\chi}''$ includes all senders in $\hat{\chi}'$ except i', and no other senders.

Iterating on this process, after a finite number of steps equal to the number of senders in $\hat{\chi}$, we are left with an order $\hat{\Gamma}$ such that $\max_{i\in\chi}(1-p_i)Q_{i,\hat{\Gamma}} < \max_{i\in\chi}(1-p_i)Q_{i,\Gamma}$. Therefore, Γ does not solve $\min_\Gamma \max_{i\in\chi}(1-p_i)Q_{i,\Gamma}$, which means it does not solve $\max_\Gamma \min_{i\in\chi} \Psi_{i,\Gamma}$, a contradiction. □

Lemma 6. *Let Γ be an ordering rule that solves (25). Then $\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta}\sum_j \frac{p_j}{1-p_j}$ for all i,j.*

Proof. Notice first that for any ordering Γ with its respective $\{Q_{j,\Gamma}\}_j$ we have

$$\sum_j p_j Q_{j,\Gamma} = 1 - \eta. \tag{26}$$

The left hand side is the probability that DM learns the truth; the sum for every sender of the probability that this sender is asked and tells the truth (notice that when $w = 0$ it is not possible for two senders to be asked and both tell the truth, as when one does so consultation stops). The right hand side is the same but expressed differently; it is the probability that at least one sender is honest (i.e. not true that all senders are biased).

Since (25) implies $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for any i, j by lemma 5, we have $Q_{j,\Gamma} = Q_{i,\Gamma}\frac{1-p_i}{1-p_j}$. This leads to

$$\sum_j Q_{j,\Gamma} = (1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j}. \tag{27}$$

On top of that, (25) implies, via lemma 5, $\sum_j (1-p_j)Q_{j,\Gamma} = n(1-p_i)Q_{i,\Gamma}$ for any i. If we combine (26) and (27) with this observation we obtain

$$\begin{aligned}(1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j} - (1-\eta) &= n(1-p_i)Q_{i,\Gamma} \\ (1-p_i)Q_{i,\Gamma} &= \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}},\end{aligned}$$

for all i. Combined with the fact that $Q_{i,\Gamma}(1-p_i)\Psi_{i,\Gamma} = \eta$ gives the desired result. □

8.4 Proof of Lemma 3

For a given Latin square ordering rule, given sender i and position l, let d_l be the unique deterministic order such that $\theta_{d_l} > 0$ and sender i occupies position l. Furthermore, let $\{1_{d_l}, 2_{d_l}, \ldots, (l-1)_{d_l}\}$ be the senders who occupy positions $\{1, 2, \ldots, l-1\}$ respectively in deterministic order d_l. Note that $l_{d_l} = i$, as sender i occupies position l in order d_l by definition.

We have

$$(1-p_i)Q_{i,\Gamma} = (1-p_i)\left(\theta_{d_1} + \theta_{d_2}(1-p_{1_{d_2}}) + \ldots + \theta_{d_n}\prod_{j\neq i}(1-p_j)\right).$$

The term in parenthesis in the right-hand side is the probability that sender i is asked first, θ_{d1}, plus the probability that sender i is asked second and whichever sender is asked first is biased, $\theta_{d_2}(1-p_{1d_2})$, etc.

In a Latin square ordering rules the probabilities of the orders d_l are determined only by whichever sender acts first:

$$\begin{aligned}(1-p_i)Q_{i,\Gamma} &= (1-p_i)\left(\frac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)} + \frac{p_{1_{d_2}}/(1-p_{1_{d_2}})}{\sum_j p_j/(1-p_j)}(1-p_{1_{d_2}}) \right. \\ &\quad \left. + \ldots + \frac{p_{1_{d_n}}/(1-p_{1_{d_n}})}{\sum_j p_j/(1-p_j)}\prod_{j\neq i}(1-p_j)\right) \\ &= \frac{1}{\sum_j \frac{p_j}{1-p_j}}\left(p_i + p_{1_{d_2}}(1-p_i) + \ldots + p_{1_{d_n}}\prod_{j\neq 1_{d_n}}(1-p_j)\right).\end{aligned}$$

The term in parenthesis in the right-hand side is the probability that sender i is unbiased, p_i, plus the probability that i is biased times the probability that sender 1_{d_2} (the sender that is is asked first when i is asked second) is unbiased, and so on until the probability that all senders except 1_{d_n} are biased times the probability that 1_{d_n} is unbiased. This is the probability that there is at least one unbiased sender, $1-\eta$. Therefore,

$$(1-p_i)Q_{i,\Gamma} \quad = \quad \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}$$

for all sender i. That is, we have just shown that

$$(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma} = \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}$$

for all i, j, which is a necessary and sufficient condition for an optimal random order.

8.5 Proof of Proposition 5

8.5.1 Point a)

Consider the constrained optimization problem defined in (17), (18) and (19). We use Kuhn-Tucker:

$$L(\{p_i\}, \lambda, \{\mu_i\}) = \frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} + \lambda(\prod_i (1-p_i) - \eta) - \sum_i \mu_i(p_i - \varepsilon)$$

with $\lambda \geq 0$ and $\mu_i \geq 0$ for all i. Since the problem is symmetric for $\{p_i\}$ we can assume without loss of generality that the first $k \in \{1, 2, \ldots, n\}$ probabilities are strictly greater than ε and the last $n-k$ probabilities are equal to ε. In other words, $\{\mu_i\}_{i=1}^k = 0$ and $\{\mu_i\}_{i=k+1}^n > 0$ for some k. The problem is then to solve the K-T conditions for any k, and then choose the k that maximizes the objective function. Note we cannot have $k = 0$ as this would mean $\prod_i (1-p) = (1-\varepsilon)^n$, which is not in general equal to η. We have for all i

$$\begin{aligned}
\frac{\partial L}{\partial p_i} &= \frac{\eta}{1-\eta}\frac{1}{(1-p)^2} - \lambda\frac{\eta}{1-p_i} - \mu_i \\
&= 0 \\
\frac{\partial^2 L}{\partial^2 p_i} &= \frac{\eta}{1-\eta}\frac{-2}{(1-p)^3} - \lambda\frac{\eta}{(1-p_i)^2} \\
&< 0.
\end{aligned}$$

Note that for those i for which $\mu_i = 0$ we have $\frac{\eta}{1-\eta} - \lambda\eta(1-p_i) = 0$, which implies $p_i = 1 - \frac{1}{\lambda(1-\eta)}$. That is, at the optimum those p_i which are not ε all are equal to some value p given by $p = 1 - \frac{1}{\lambda(1-\eta)}$.

Therefore, we have $\eta = (1-p)^k(1-\varepsilon)^{n-k}$. This means

$$p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}.$$

Thus, at the optimum we have that the first k probabilities are equal to p and the rest are equal to ε. We are left to calculate what is the optimal k. For given k we have that the objective function is equal to

$$\psi = \frac{\eta}{1-\eta}\left[k\left(\left(\frac{(1-\varepsilon)^{n-k}}{\eta}\right)^{\frac{1}{k}} - 1\right) + (n-k)\frac{\varepsilon}{1-\varepsilon}\right]$$

Let us study the behaviour of this expression as a function of k. Taking the derivative with respect to k and using the fact that $p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}$ we obtain

$$\frac{\partial\psi}{\partial k} \propto \frac{p}{1-p} - \frac{\varepsilon}{1-\varepsilon} - \frac{1}{1-p}\log\frac{1-\varepsilon}{1-p},$$

where p depends on k.

Notice that $\frac{\partial\psi}{\partial k}_{p=\varepsilon} = 0$ and that $\frac{\partial^2\psi}{\partial k\partial p} = -\frac{1}{(1-p)^2}\log\frac{1-\varepsilon}{1-p} < 0$. Hence, we have that $\frac{\partial\psi}{\partial k}$ is decreasing in p and equal to 0 at the lowest possible value for p. Therefore, it is negative. This means that the k that maximizes Ψ is the minimum possible. That is, $k = 1$. Therefore, the optimal solution is $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ for any one $i \in \{1, \ldots, n\}$ and $p_j = \varepsilon$ for all $j \neq i$.

8.5.2 Point b)

Assume that for some $\varepsilon > 0$ we have that p_i increases to $p_i' = p_i + \varepsilon$ and that p_j decreases to $p_j' = p_j - \rho(\varepsilon)$, where $\rho(\varepsilon)$ solves

$$(1-p_i-\varepsilon)(1-p_j+\rho(\varepsilon)) = (1-p_i)(1-p_j).$$

which is equivalent to

$$\rho(\varepsilon) = \left(\frac{(1-p_i)}{(1-p_i-\varepsilon)} - 1\right)(1-p_j).$$

It is easy to show that

$$\frac{p_i'}{1-p_i'} + \frac{p_j'}{1-p_j'} > \frac{p_i}{1-p_i} + \frac{p_j}{1-p_j}.$$

Therefore, $\Psi^{*'} > \Psi^*$.

9 Appendix B - Proof of Proposition 1

9.1 Preliminary definitions

Recall that m^l denotes the message appearing in position l of the presentation order. Consider an observed history h in which DM consulted k senders, first consulting the sender located in position l, then the sender in position l', then the sender in position l'', etc. We denote such a history by the k-entries vector $h = \{m^l, m^{l'}, m^{l''}, ..\}$. We denote the rth entry of h by h_r. We say that a history has length k if DM consulted k times. We say that two histories h and h' are *comparable* if, across these two histories, DM faced the same presentation order, followed the same order of consultation, and has consulted the same number of times (so the histories have the same length). The action rule α pins down the action $\alpha(h)$ taken by DM if he stops consulting and chooses an action after h.

Definition 1. *senders' strategies induce monotonic beliefs when for any two comparable histories h and $h'$of length $k \in \{1, .., n\}$, if it holds true that there is some $i \in \{1, \dots k\}$ such that $h_j = h'_j$ for all $j \neq i$ and $h_i > h'_i$ then it holds true that $E[\omega|h] \geq E[\omega|h']$, assuming that DM's beliefs are formed via Bayes rule and the senders' strategy profile.*

The following examples illustrate the above definition.

Example 1. *Given $n = 5$, if $h = \{m, m', m''\}$ and $h' = \{m, \widetilde{m}', m''\}$ with $m' > \widetilde{m}'$ then if senders' strategies induce monotonic beliefs it must be that $E[\omega|h] \geq E[\omega|h']$.*

Example 2. *Given $n = 5$, if $h = \{m, m', m''\}$ and $h' = \{m, m', m'', m'''\}$ then even if senders' strategies induce monotonic beliefs we cannot establish an ordinal relation between $E[\omega|h]$ and $E[\omega|h']$.*

Example 3. *Given $n = 5$, if $h = \{m, m', m''\}$ and $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ with $m' > \widetilde{m}'$ and $m'' > \widetilde{m}''$ then if senders' strategies induce monotonic beliefs it must be that $E[\omega|h] \geq E[\omega|h']$.*

Example 4. *Given $n = 5$, if $h = \{m, m', m''\}$ and $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ with $m' < \widetilde{m}'$ and $m'' > \widetilde{m}''$ then even if senders' strategies induce monotonic beliefs we cannot establish an ordinal relation between $E[\omega|h]$ and $E[\omega|h']$.*

Definition 2. *An equilibrium is monotone if sender strategies are monotonic and induce monotonic beliefs.*

Definition 3. *An equilibrium is partitional if it satisfies the following description. There is a sequence of strictly increasing thresholds $\{t_0, t_1, \dots, t_N\}$ with $N > 1$, $t_0 = 0$ and $t_N = 1$ such that the following holds true. For any two comparable histories h and $h'$of length k, if it holds true that there is some $i \in \{1, \dots k\}$ such that $h_i = h'_i$ for $i \neq j$ and $h_j > h'_j$ where either $h_j, h'_j \in [t_k, t_{k+1})$ for some $k \in \{0, \dots, N-1\}$ or $h_j, h'_j \in [t_{N-1}, 1]$, then we have $\alpha(h) = \alpha(h')$.*

9.2 Proof

In what follows, as stated in the main text, we restrict ourselves to symmetric and monotone equilibria. The proof is decomposed into three Lemmas. The Lemma 7 establishes that there can never be a subset of the state space for which DM learns the state perfectly if he meets an unbiased sender. Lemma 8 uses this property to show that any equilibrium must be partitional. Lemma 9 , building on this, shows that for any informative equilibrium that satisfies our restrictions there exist an an outcome equivalent simple partitional equilibrium.

Lemma 7. *(No Perfect Communication on an Interval) There exists no symmetric and monotone equilibrium where there is a non-degenerate interval $\widetilde{A}$ such that if $\omega \in \widetilde{A}$ then if DM consults an unbiased sender he stops consultation and plays $\alpha = \omega$.*

Proof. The proof proceed with the following steps, where each steps is detailed on each of the following paragraphs. Step 1, assume the contrary and define $\sup \widetilde{A}$ as the supremum of the state for which perfect communication is possible. Step 2, if state is $\omega = \sup \widetilde{A}$ then unbiased sender believes action must be $\sup \widetilde{A}$. Step 3, define $m_{\widetilde{A}}$ as the message that induces action arbitrarily close to $\sup \widetilde{A}$. Step 4, the action played is strictly higher than $\sup \widetilde{A}$ when all messages are greater or equal to $\sup \widetilde{A}$. Step 5, if $\omega = \sup \widetilde{A}$ all messages are greater or equal to $\sup \widetilde{A}$.

Begin by assuming the contrary, then there is a possibly uncountable collection of disjoint non-degenerate sets $\{\widetilde{A}_i\}_i$ such that if $\omega \in \widetilde{A}_i$ for some i then DM stops consultation and plays m. For all i let $\sup \widetilde{A}_i$ be the supremum of set $\widetilde{A}_i$. Create an increasing sequence in $[0, 1]$ by ordering increasingly the set of all suprema $\{\sup \widetilde{A}_i\}_i$. Since such sequence is bounded by 1, by the monotone convergence theorem it converges to its supremum. Let $\sup \widetilde{A}$ be such supremum.

If the state is $\omega = \sup \widetilde{A}$ then an unbiased sender believes with probability 1 in equilibrium that the action of DM must be $\sup \widetilde{A}$ once he stops consultation. To see this notice first that by monotonicity this action is greater or equal than $\sup \widetilde{A} - \varepsilon$ for all small enough $\varepsilon > 0$. If $\sup \widetilde{A} = 1$ then the action is $\sup \widetilde{A}$ with certainty, and if $\sup \widetilde{A} < 1$ but the action played is not $\sup \widetilde{A}$ with some probability then the expected action must by strictly higher than $\sup \widetilde{A}$. That is, there exists an $\varepsilon > 0$ such that the action played is $\sup \widetilde{A} + \varepsilon$ with some probability p, in which case the unbiased sender's best response is in $m \in ((1 - p) \sup \widetilde{A} + p(\sup \widetilde{A} - \varepsilon), \sup \widetilde{A}) \subset \widetilde{A}$. Thus, there is a deviation incentive of unbiased senders, a contradiction.

For small $\varepsilon > 0$ there exist a message $m_{\widetilde{A}}(\varepsilon)$ such that if sent by a sender DM stops consultation and plays $\sup \widetilde{A}(\varepsilon) - \varepsilon$. By monotonicity $m_{\widetilde{A}}(\varepsilon)$ is increasing in ε and, furthermore, it is bounded above by 1. Thus, it converges to its supremum. Let $m_{\widetilde{A}}$ be such supremum.

By monotonicity, if the state is $\omega = \sup \widetilde{A}$ then unbiased senders send message $m \geq m_{\widetilde{A}}$. On top of that, for any state by monotonicity biased senders always send message $m \geq m_{\widetilde{A}}$. A biased sender believes that if consulted then for any state the expected action played by DM in equilibrium, say α, is at least $\sup \widetilde{A}$, as otherwise he can guarantee any action arbitrarily close to $\sup \widetilde{A}$ by sending message $m_{\widetilde{A}}$. In equilibrium, by monotonicity, if the state is $\omega \in [0, \sup \widetilde{A})$ DM only plays an action of at least $\sup \widetilde{A}$ if he consults biased senders only, and less than $\sup \widetilde{A}$ otherwise, say $\hat{\alpha} < \sup \widetilde{A}$ at most. Using incentive compatibility for biased senders we must have $\delta\alpha + (1-\delta)\hat{\alpha} \geq \sup \widetilde{A}$, which since $\delta \in (0,1)$ and $\hat{\alpha} < \sup \widetilde{A}$ means $\alpha > \sup \widetilde{A}$. That is, the equilibrium action of DM must be strictly higher than $\sup \widetilde{A}$ when all messages he receives are $m_{\widetilde{A}}$ or above regardless of the state of nature.

Assume $\omega = \sup \widetilde{A}$, then DM receives all messages equal to or above $\sup \widetilde{A}$, which by the paragraph above means he plays an action strictly greater than $\sup T$, but this is incompatible with unbiased senders' equilibrium beliefs. As we showed previously if the state is $\omega = \sup \widetilde{A}$ then an unbiased sender believes with probability 1 that the action played by DM is $\sup \widetilde{A}$. □

Lemma 8. *All informative, symmetric and monotone equilibria are partitional.*

Proof. The proof proceed with the following steps, where each steps is detailed on each of the following paragraphs. Step 1, by contradiction, there exists two histories with increasing actions in some interval $\widetilde{A}$ for some sender j. Step 2, biased senders always send at least $\sup \widetilde{A}$. Step 3, define function $\hat{\alpha}(h_j)$ as the increasing action in history $h_{-j} \times h_j$ as a function of $h_j \in \widetilde{A}$. Step 4, if the image set of $\hat{\alpha}$, i.e. I, contains an interval, we contradict Lemma 7. Step 5, if the image set of $\hat{\alpha}$, i.e.I, does not contain any interval, we still contradict Lemma 7 I must be dense in some subset of $I \cap R$. Step 5.1, I contains no intervals but for at least one point in I there is another one in I infinitesimally close by. Step 5.2, for all error $\varepsilon > 0$ and for some states not in I sender can induce an action ε-close to the state. Step 5.3, there is then an interval with full communication, a contradiction.

Assume there is an equilibrium that is not partitional. This means that there exists a non-degenerate interval $\widetilde{A}$ and a pair of comparable equilibrium histories h_{-j}, h'_{-j} for all senders but j with $h_i = h'_i$ for senders $i \neq j$, such that for all $h_j, h'_j \in \widetilde{A}$ we have $h_j \neq h'_j$ implies $\alpha(h) \neq \alpha(h')$.

By monotonicity for all $h_j, h'_j \in \widetilde{A}$ with $h_j > h'_j$ we have $\alpha(h) > \alpha(h')$. Assume henceforth without loss of generality that $h_j > h'_j$. Also by monotonicity in equilibrium for any state biased senders always send message of at least $\sup \widetilde{A}$. This is because in at least one equilibrium history (the one given in step one of the proof) it leads to a strictly higher action than anything below $\sup \widetilde{A}$, and for any other equilibrium history it leads, by monotonicity, to an action at least as high as any other message below $\sup \widetilde{A}$.

For any history h_{-j} where some sender j is not consulted, define the function $\hat{\alpha} : \widetilde{A} \to I$ as a strictly increasing map between h_j and the action played in history $h = h_{-j} \times h_j$. We have that $\hat{\alpha}$ is increasing and that $I \subseteq [\hat{\alpha}(\inf \widetilde{A}), \hat{\alpha}(\sup \widetilde{A})]$.

If there exists an $x \in \mathring{I}$ and $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$ then I contains intervals. That is, there exists an $x \in \mathring{I}$ and an $\varepsilon > 0$ such that for all state $\omega \in (x, x+\varepsilon)$ there exists a message $m \in \widetilde{A}$ such that $\hat{\alpha}(m) = \omega$. Since biased senders always send message $\sup \widetilde{A}$, after observing m DM learns the that sender is unbiased and his strict best response is to stop consultation and play $\hat{\alpha}(m)$. This contradicts Theorem 1.

Assume instead that for all $x \in \mathring{I}$ there exists no $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$. That is, I contains no intervals.

Note that for all $\delta > 0$ there exists an $x, x' \in I$ with $x < x' < x + \delta$. This is because otherwise there exists a $\delta > 0$ such that for all $x \in I$ we have $(x, x + \delta) \not\subset I$. This means that I has at most $\frac{\sup I - \inf I}{\delta}$ elements. This is a contradiction as $\hat{\alpha}$ is a strictly increasing mapping from a set with infinitely many elements so its domain I must also have infinitely many elements.

We have that for all $\varepsilon > 0$ if we take $\delta \in (0, 2\varepsilon)$ and $x, x' \in I$ such that $x < x' < x + \delta$ then for all $\hat{x} \in (x, x')$ with $\hat{x} \notin I$ either $|x - \hat{x}| < \frac{\delta}{2} < \varepsilon$ or $|x' - \hat{x}| < \frac{\delta}{2} < \varepsilon$.

Notice that since $x, x' \in I$ there exists $m, m' \in \widetilde{A}$ respectively such that $\hat{\alpha}(m) = x$ and $\hat{\alpha}(m') = x'$. That is, we have found that for any $\varepsilon > 0$ and any state ω in the interval $[x, x']$ we can find a message that induces an action at least ε-close to ω. This means that again we have found an interval where there is full communication of the state, a contradiction to theorem 1.

□

Lemma 9. *In all informative, symmetric and monotone equilibria, there exists a sequence of strictly increasing thresholds* $\{t_0, t_1, \ldots, t_{m-1}, t_m\}$ *with* $t_0 = 0$ *and* $t_m = 1$ *such that:*

1. *Biased senders always send a message in* $[t_{N-1}, t_N]$*,*

2. *unbiased senders all send the same message,*

3. *DM keeps consulting as long as he has received messages in* $[t_{N-1}, t_N]$*, and stops consulting either once he has received a message not in* $[t_{N-1}, t_N]$*, or when he has consulted all senders,*

4. *if DM observes a message not in* $[t_{N-1}, t_N]$*, say it belongs to* $[t_{k-1}, t_k]$ *with* $k \in \{1, \ldots, N-1\}$*, he then plays an action* $\alpha(k)$ *that is strictly increasing in* k*. If DM only*

observes messages in $[t_{N-1}, t_N]$*, he then plays an action* $\alpha(N) > \alpha(k)$ *for all* $k \in \{1, \ldots, N-1\}$*.*

Proof. Take any informative partitional equilibria. If for any two comparable equilibrium histories h and h' such that there is some sender i where for all $i \neq j$ and $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ (or $h_i = h'_i = t_N$) and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$ and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) = \alpha(h')$, then we can redefine the partitions as $\{t_0, \ldots, t_r, t_s, t_{s+1}, t_N\}$ without-loss of generality. If for any two comparable equilibrium histories $\hat{h}$ and $\hat{h}'$ where again all messages but one are in the same interval and the action is not increasing, we can again redefine the partitions eliminating the cut-offs where the action of DM is non-increasing.

Continuing in this fashion we get to a partition $t_0, t_1, \ldots, t_{m-1}, t_m$ with $t_0 = 0$ and $t_m = 1$ such that there exists two comparable equilibrium histories h and h' where there is some sender i if for all $i \neq j$ we have $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$ and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) > \alpha(h')$.

By monotonicity, we have then that biased senders always send message $m \in [t_{N-1}, t_N]$ for a given state. Therefore, in equilibrium, any message that is not in $[t_{N-1}, t_N]$ was sent by an unbiased sender with probability 1. Moreover, since strategies are symmetric and senders do not observe other senders' messages, i.e. the history of observed messages, all unbiased senders send the same message for given state of the world. This means that in a partitional equilibrium there are only two messages ever observed by DM, the one sent by biased senders, and the one sent by unbiased senders.

Therefore, since messages not in $[t_{N-1}, t_N]$ are only ever sent by unbiased senders, once DM observes a message not in $[t_{N-1}, t_N]$ he does not have incentives to keep consulting senders, as he has learned as much as he could in equilibrium. Thus, he stops consultation.

Note finally that it is immediate that for any given equilibrium of the form described in the above Lemma, there exists a unique outcome equivalent simple partitional equilibrium. □