Regime Change and Critical Junctures*

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Abstract

In this paper we study how a society can transition between different economic and political regimes. When the current regime is elitism, the society is modeled as a collection of units of land where at each of these units there is a member of the elite and a peasant. Under the democratic regime, at each of the units of land there is a citizen whose role is to work the land and enjoy the full output he produces. At every period with some small probability a critical juncture arrives, giving a chance for a regime change. Among others, we find that a wider output gap can increase the number of different institutions that are possible after a critical juncture and that lower land profitability makes equilibria where an extractive regime continues less likely.

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1 Introduction

Consider a society where few of its members, the elite, have ownership of the different economic activities. The rest of the society, peasants, are the ones that work on each of the different economic activities producing a certain output, a share of which is given to the elite. How extractive institutions are, i.e. how the output is shared between the elite and peasants, is decided by the elite and the only chance peasants have at changing the regime is via a revolution. Examples of these societies are, for instance, feudalism where each economic activity is simply a piece of land, or dictatorships where citizens work in the different economic activities and the ministers of the regime and government officials choose how much output to extract from peasants.

Throughout the history of civilizations societies as the ones described above have been abundant, from the medieval feudalism in Europe in the 10th century, to the military dictatorships in certain Asian and African countries in the present century. A frequent feature of these societies is that whenever they have evolved to different regimes, the change was usually triggered by what is known as a critical juncture (see Collier and Collier (1991) among others). A critical juncture can be defined as a "a major event or confluence of factors disrupting the existing economic or political balance in society" (Acemoglu and Robinson (2012), see also Capoccia and Kelemen (2007)). Examples of critical junctures are events such as the discovery of the Americas, the Black Death or more recently the Arab Spring.

In this paper we present a model that tries to explain the factors that influence the possible outcomes after a critical juncture, as well as study how a society can transition between different regimes. To this end, we consider a society that is divided into different units of land. If the current economic and political regime is elitism, we assume that in each of these units of land there is a member of the elite and a peasant. The role of the member of the elite is to decide how extractive institutions in place are via setting up a tax rate. The role of the peasant is to work the land to produce a certain output which is shared between the member of the elite and the peasant according to the tax rate. If the current regime is a democracy, at each of the units of land there is a citizen whose role is to work the land and enjoy the full output he produces.[1]

At every period with some small probability a critical juncture arrives. In the event of a critical juncture, players have a chance to update their actions. Under elitism, members of the elite react to the critical juncture by adapting the institutions to the new circumstances, i.e., updating the tax rate they charge to peasants. After members of the elite have reacted to the critical juncture, peasants may stage a revolution. The revolution can be successful

[1] Thus, by democracy we mean that there is no ruling elite that extracts rent out of a ruled class.

at overruling the current regime depending on how many peasants join the revolt. If the current regime is overruled, the elite is eliminated and a democracy regime is installed: there is no distinction between members of the society and each citizen works on a unit of land and enjoys the full output he produces. When a critical juncture arrives under the democratic regime, citizens have a chance to stage a coup to revert back to elitism and become the new elite.

Our model is based on the work by Acemoglu and Robinson (2001a).[2] The main difference between our model and the various models in Acemoglu and Robinson (2000a, 2000b, 2001b) is that we explicitly model the cooperation/coordination problem faced by players (elite, peasants and citizens, depending on the current regime). In our paper, each member of the elite is free to set up any tax rate he desires for his own unit of land and, hence, members the elite are playing a *cooperation* game with each other. In Acemoglu and Robinson (2000a, 2000b, 2001b) the elite is aggregated into a single player (all members of the elite choose the same tax rate) and, thus, they do not study the elite's cooperation problem.[3]

Another difference from previous literature is that the probability that there is a regime change (successful revolution or coup) in our paper is an increasing function of the number of players that attempt the regime change. This means that peasants or citizens (depending on the current regime) play a *coordination* game with each other whenever a critical juncture arrives. In the models of Acemoglu and Robinson (2000a, 2000b, 2001b), a regime change happens if and only if a given number of players attempt a regime change. Thus, the unique equilibrium outcome in their work is that either all players attempt a regime change or none do.

Also different from previous literature (Acemoglu and Robinson (2000a, 2000b, 2001b) and relevant references herein and also Edmon (2013) and Bove et al (2017)) is that in our setting equilibria multiplicity is possible. Equilibria multiplicity should not be surprising given the inherent coordination problem that a revolution or a coup entail.[4]

The idea that regime change from transitions between elite and workers has been present in the literature for a number of years (see Rustow (1970), Guillermo and Schmitter (1986) and more recently (Lizzeri and Persico (2004) and Llavador and Oxoby (2005)) and has been

[2]See also Acemoglu and Robinson (2000a, 2000b, 2001b) and Acemoglu et al (2001).

[3]In our model, even though all members of the elite have ex-ante the same information and are ex-ante homogeneous, there will be equilibria where all members of the elite choose the same tax rate and equilibria where they do not. Therefore, if the elite behaves as one single player it will be an endogenous result of the model instead of an exogenous assumption.

[4]This is true even though several attempts have been made to reconcile the coordination problem faced by peasants and citizens with the simplifying assumption by which either all or no player attempts a regime change (i.e. the collective action problem, see also Lichbach (1995), Moore (1995) or Popkin (1979)).

confirmed empirically recently by Acemoglu et al (2020). However, this is not the only way transition between regimes can occur (see Treisman (2020) for instance).

2 The Model

2.1 Revolutions: from Elitism to Democracy

Assume that time is discrete and given by $n = 0, 1, \ldots$ over an infinite horizon. The society consists of a continuum of units of land indexed by i. In each unit of land i there is one member of the elite and a peasant. The role of the member of the elite is to set a tax rate $t_i^r \in [0, 1]$ while the role of the peasant is to work the land and pay the elite a percentage t_i^r of the output. The subscript r stands for revolution and is used to distinguish this tax rate from the tax rate t_i^c which is introduced later on when talking about coups.

The output of each unit of land i at any given time period is given by $y_i \in \mathbb{R}$. The values of $\{y_i\}_i$ are independent and identically distributed where for all i we have that y_i takes the normalized value 1 with probability $(1 - \varepsilon_n) \in (0, 1)$ and the value $y \in [0, 1]$ with probability ε_n.[5] The random variable ε_n is independent and identically distributed for every period n with mean $\mu_\varepsilon \in (0, 1)$ and support $[\underline{\varepsilon}, \overline{\varepsilon}] \subset (0, 1)$. The value of ε_n represents the likelihood of an output shock at time n while y represents the output in case of a shock. Whenever there is no ambiguity, we refer to the value of ε_n at the current period as ε. The peasant works the land every period by paying a cost $w \in [0, Ey]$ where $Ey = (1 - \mu_\varepsilon) + \mu_\varepsilon y$ is the expected output of each unit of land. Thus, in unit of land i with tax rate t_i^r the payoff of the member of the elite is given by $t_i^r y_i$ and the payoff of the peasant is given by $(1 - t_i^r) y_i - w$.

At each time period and with some probability $\delta \in (0, 1)$ a critical juncture may arrive. We focus on the case where $\delta \to 0$; critical junctures are very rare events and as such the likelihood of a future critical junctures after the current one should not affect player's decisions.[6]

During a critical juncture, and after the current state of the economy ε is known but before the actual output of each unit of land y_i for all i is observed, each member of the elite has a chance to change the tax rate for his unit of land. All members of the elite choose their tax rate simultaneously and critical junctures are the only opportunity they have at changing the tax rate they charge. After the new tax rates have been set, the realization of y_i for all i is

[5] As we shall see, two output levels instead of a continuum make interpretation of results easier as with we can talk about units of land where there is an output shock versus units of land where there is not.

[6] On top of increasing realism, this assumption simplifies the analysis considerably as it allows every critical juncture to be studied independently of each other.

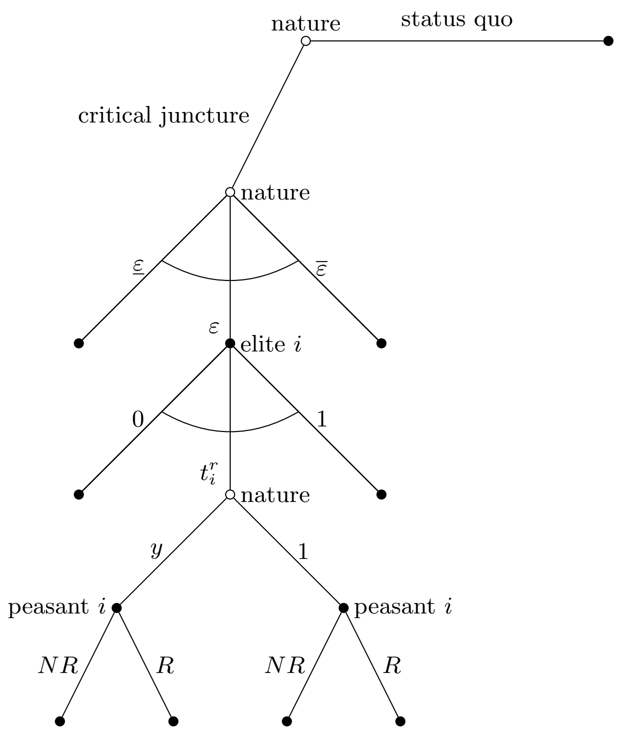
observed and peasants have a chance to revolt against the regime by simultaneously deciding whether to join the revolution or not.

When the peasant of a given unit of land revolts the output of that unit of land for the current period is 0 and the peasant does not have to pay the cost w of working the land. If a fraction x of the peasants revolt then the revolution is successful with probability $\gamma(x)$ where $\gamma : [0,1] \to [0,1]$. We assume γ is continuous and weakly increasing with $\gamma(0) = 0$ and $\gamma(1) = 1$. Throughout the paper γ is referred to as the technology of the revolution.

If a revolution is unsuccessful then those peasants that revolted are removed from the game and replaced with new ones. The society then continues to function as before with the new tax rates set by the elite. If the revolution is successful several things occur: first, peasants that did not take part in the revolution are removed from the game and replaced with new ones. Second, the elite is completely removed from the game and all peasants become citizens. Third, the political regime changes to one where citizens enjoy all the product (and costs) of the land. After a successful revolution the payoff per period of a citizen working in unit of land i is $y_i - w$.

The timing of the game at hands is summarized in Figure 1 where NR stands for not revolt while R stands for revolt. This figure is not an extensive form game representation of the model but an illustration of the order at which events take place in unit of land i.

Figure 1: Revolutions - Timing of the game for unit of land i



Assume that all agents discount future payoffs at a rate $\beta \in (0,1)$. The expected discounted stream of present and future payoffs (expected payoff hereafter) of the member of the elite in unit of land i after a critical juncture where the revolution is unsuccessful is given by

$$V^e(t_i^r) = \sum_{n=1}^{\infty}(1-\delta)^n \beta^{n-1} t_i^r Ey + \delta o^e(t_i^r),$$

where o^e is some function that is bounded below by 0 (in case there is a critical juncture at some point in the future and the elite is removed from the game), and above by $\frac{Ey}{1-\beta}$ (in case there is a critical juncture at some point in the future, the elite sets up tax rate $t_i^r = 1$ and the revolution is unsuccessful). The function o^e may include beliefs on what will happen when the next critical juncture arrives, beliefs about beliefs on what will happen in the next critical juncture, etc. However, as the purpose is to study the situation when $\delta \to 0$, we do not need to have any knowledge about o^e except for the fact that it is bounded above and below. We can rewrite the function V^e as

$$\begin{aligned} V^e(t_i^r) &= (1-\delta)\frac{t_i^r Ey}{1-(1-\delta)\beta} + \delta o^e(t_i^r), \\ \lim_{\delta \to 0} V^e(t_i^r) &= \frac{t_i^r Ey}{1-\beta}. \end{aligned}$$

Similarly, after a critical juncture where the revolution is unsuccessful the expected payoff of the peasant working in unit of land i if he did not revolt is given by

$$\begin{aligned} V^p(t_i^r) &= (1-\delta)\frac{(1-t_i^r)Ey - w}{1-(1-\delta)\beta} + \delta o^p(t_i^r), \\ \lim_{\delta \to 0} V^p(t_i^r) &= \frac{(1-t_i^r)Ey - w}{1-\beta}, \end{aligned}$$

where the function o^p belongs to the interval $\left[0, \frac{Ey}{1-\beta}\right]$ and has a similar interpretation to that of o^e.[7]

If the revolution is successful, the expected payoff of each citizen is given by

$$\begin{aligned} V^c &= (1-\delta)\frac{Ey - w}{1-(1-\delta)\beta} + \delta o^c, \\ \lim_{\delta \to 0} V^c &= \frac{Ey - w}{1-\beta}, \end{aligned}$$

where again the function o^c belongs to the interval $\left[0, \frac{Ey}{1-\beta}\right]$ and as a similar interpretation to that of o^e.

[7]The function o^p could take the value $\frac{Ey}{1-\beta}$ if the peasant becomes a member of the elite in the future. This is explained in more detail in Section 2.2 when we consider coups.

At a critical juncture, and after the elite of a given unit of land i sets up the new tax rate t_i^r, the expected payoff of the peasant working on unit of land i if a fraction x of the peasants revolt is given by $u_i^p(t_i^r, x)$ with

$$u_i^p(t_i^r, x) \quad = \quad \begin{cases} (1-t_i^r)y_i - w + \beta(1-\gamma(x))V^p(t_i^r) & \text{if he does not revolt,} \\ \beta\gamma(x)V^c & \text{if he revolts.} \end{cases}$$

Hence, the peasant working on unit of land i chooses not to revolt if and only if

$$(1-t_i^r)y_i - w - \beta[\gamma(x)V^c - (1-\gamma(x))V^p(t_i^r)] \quad \geq \quad 0.$$

Note that in case a peasant is indifferent between joining the revolution or not we assume he does not join the revolution. This assumption does not affect our results in any meaningful way.

Two different tax rates arise naturally in our setting, these are the maximum tax rate such that the peasant does not revolt when there is an output shock to its land and the maximum tax rate such that the peasant does not revolt if there is not an output shock. Let $\underline{t}^r(x)$ be the maximum tax rate such that the peasant of a given unit of land does not join the revolution if a fraction x of the peasants revolt and if there is an output shock in his unit of land.[8] We have that $\underline{t}^r(x)$ is given implicitly by

$$(1-\underline{t}^r(x))y - w - \beta[\gamma(x)V^c - (1-\gamma(x))V^p(\underline{t}^r(x))] \quad = \quad 0,$$

which as $\delta \to 0$ can be rewritten as

$$\underline{t}^r(x) \quad = \quad 1 - \frac{w + \beta\gamma(x)(Ey - 2w)}{(1-\beta)y + \beta(1-\gamma(x))Ey}. \tag{1}$$

Similarly, let $\overline{t}^r(x) \geq \underline{t}^r(x)$ be the maximum tax rate that keeps the peasant of a given unit of land from joining the revolution if a fraction x of the peasants revolt and if there is no output shock in his unit of land. Then, $\overline{t}^r(x)$ when $\delta \to 0$ is given by:

$$\overline{t}^r(x) \quad = \quad 1 - \frac{w + \beta\gamma(x)(Ey - 2w)}{(1-\beta) + \beta(1-\gamma(x))Ey}. \tag{2}$$

Note that it can happen that either $\underline{t}^r(x)$ or both $\underline{t}^r(x)$ and $\overline{t}^r(x)$ are negative for a given value of x. If $\underline{t}^r(x) < 0$ then it is not possible for the elite to keep his peasant from revolting in case of an output shock. If, on top of that, $\overline{t}^r(x) < 0$ then the peasant always revolts

[8] The fact that there are two tax rates follows from the tact that there are two different output levels. If there were more than two but countable many different outputs levels then we will have a higher number of relevant tax rates. Modifying the model along these lines will complicate the analysis but leave the qualitatively results unchanged.

regardless of the tax rate set by the member of the elite. Notice that it is never the case that $\underline{t}^r(x)$ nor $\overline{t}^r(x)$ are greater than 1 for any $x \in [0,1]$.

We restrict our attention to tax rates t such that $t \leq \overline{t}^r(0)$ for the following reason: if we allowed a tax rate t such that $t > \overline{t}^r(0)$ then the tax rate would be so extractive that the peasant would want to revolt even in the most adverse circumstances for a revolution: no other peasant revolts and there is no output shock to its unit of land.

At a critical juncture where a fraction x of the peasants revolt the expected payoff of the member of the elite of a given unit of land i is given by $u^e_{NR}(t^r_i, x)$ if his peasant does not join the revolution and by $u^e_R(t^r_i, x)$ if he does:

$$\begin{aligned} u^e_{NR}(t^r_i, x) &= t^r_i Ey + \beta(1-\gamma(x))V^e(t^r_i), \\ u^e_R(t^r_i, x) &= \beta(1-\gamma(x))V^e(t^r_i). \end{aligned}$$

Note that the subscript i is not needed in the functions u^e_{NR} and u^e_R as members of the elite do not know the realization of output y_i when they choose the tax rate.

Thus, if we denote by $u^e(t^r_i, x)$ the expected payoff of the member of the elite who owns unit of land i and by ε the value of ε_n at the critical juncture we have that

$$u^e(t^r_i, x) = \begin{cases} t^r_i Ey + \beta(1-\gamma(x))V^e(t^r_i) & \text{if } t^r_i \leq \underline{t}^r(x), \\ t^r_i(1-\varepsilon) + \beta(1-\gamma(x))V^e(t^r_i) & \text{if } t^r_i \in (\underline{t}^r(x), \overline{t}^r(x)]. \end{cases} \tag{3}$$

2.2 Coups: from Democracy to Elitism

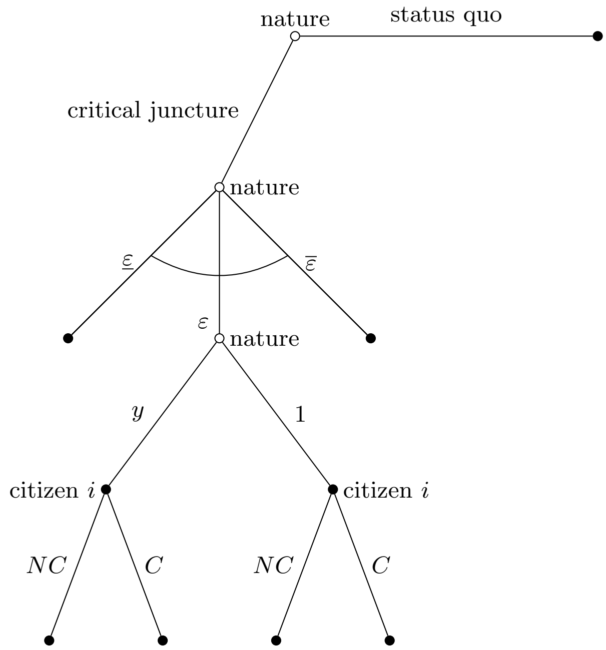
In this section we develop the model for the opposite situation as in the section above: citizens work the land and at each critical juncture they may stage a coup that could reinstate the regime where the society is split between the elite and peasants.

Assume that in each unit of land there is a citizen with an expected payoff of V^c. When a critical juncture arrives, citizens decide whether to stage a coup or not after observing the frequency of shocks ε and the production in each unit of land y_i. The timing of the game for each unit of land i is represented in Figure 2.

If a fraction $z \in [0,1]$ of the citizens stage a coup, let $\rho(z)$ be the probability that the coup is successful. As with function γ, we assume $\rho : [0,1] \to [0,1]$ to be continuous and weakly increasing with $\rho(0) = 0$ and $\rho(1) = 1$. The fact that γ and ρ are different functions is meant to represent that it may be easier to move from one regime to the other than vice versa.

At a critical juncture where a fraction z of the citizens stage a coup, the expected payoff

Figure 2: Coups: Timing of the game for unit of land i



of the citizen in unit of land i is given by $u_i^c(t^c, z)$ with

$$u_i^c(t^c, z) \;=\; \begin{cases} y_i - w + \beta\left[(1-\rho(z))V^c + \rho(z)V^p(t^c)\right] & \text{if he does not join the coup,} \\ \beta\rho(z)V^e(t^c) & \text{if he joins the coup,} \end{cases}$$

where t^c is the tax rate implemented after a successful coup. We assume that before the coup is resolved, the citizens that stage a coup commit to setting up a certain tax rate if they become members of the elite, and this tax rate is the same in all units of land. This has the following interpretation: the tax rate represents the institutions in place and, thus, committing to a tax rate is the equivalent for those that stage the coup to commit to a certain manifesto which lays out the institutions to be put in place after the coup.[9].

A citizen does not have incentives to join the coup when $\delta \to 0$ if and only if

$$y_i - w + \frac{\beta}{1-\beta}\left[Ey(1-2\rho(z)t^c) - w\right] \;\geq\; 0, \tag{4}$$

which for $\rho(z) > 0$ can be rewritten as

$$t^c \;\leq\; \frac{(1-\beta)y_i + \beta Ey - w}{2\rho(z)\beta Ey}.$$

Note that in case of indifference a citizen chooses not to join the coup. This assumption is made simply for analytical convenience and does not affect our results in any meaningful way.

[9] As, for example, the "Manifiesto de Sierra Maestra" signed by Fidel Castro, Felipe Pazos y Raúl Chibás in Cuba in 1957 before the Cuban revolution concluded in 1959 (see Bonache and San Martin (1974)).

Let $\underline{t}^c(z)$ be the maximum tax rate such that the citizen of a given unit of land does not join the coup if a fraction $z > 0$ of the citizens stage a coup and if there is an output shock in his unit of land. We have that $\underline{t}^c(z)$ as $\delta \to 0$ is given by

$$\underline{t}^c(z) = \frac{(1-\beta)y + \beta Ey - w}{2\rho(z)\beta Ey}. \tag{5}$$

Similarly, let $\bar{t}^c(z) \geq \underline{t}^c(z)$ be the maximum tax rate such that a citizen does not join the coup if a fraction $z > 0$ of the citizens stage a coup and if there is no output shock in his unit of land. Then, $\bar{t}^c(z)$ when $\delta \to 0$ is given by

$$\bar{t}^c(z) = \frac{(1-\beta) + \beta Ey - w}{2\rho(z)\beta Ey}. \tag{6}$$

If $z = 0$ then as $w \in [0, Ey]$ no citizen that works on a unit of land that does not suffer an output shock wants to revolt (see equation (4)). However, if $y_i - w + \frac{\beta}{1-\beta}[Ey - w] < 0$ then, even when no other citizen joins the coup, the citizens that work in the units of land that suffer an output shock have incentives to join the coup. Moreover, it is possible that $\underline{t}^c(z) < 0$, in which case there is no tax rate that keeps the citizens that work in the units of land that suffer a production shock from joining the coup. Note that it is always the case that $\bar{t}^c(z) > 0$.

2.3 Equilibrium

In our model, we use Markov Perfect Equilibrium as the equilibrium concept. A Markov Perfect Equilibrium is a (pure strategy) sub-game perfect Nash equilibrium of the game that is played at each critical juncture. At each critical juncture, the state variables are the current frequency of shocks ε and whether the society is elitist (Section 2.1) or a democracy (Section 2.2).

Definition 1. *A Markov Perfect Equilibrium is a tuple* $\{(\{t_i^r\}_i, x), (t^c, z)\}_{\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]}$ *which for each possible value of* $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ *specifies a collection of tax rates* $\{t_i^r\}_i$ *with* $t_i^r \in [0, t_i^r(x)]$ *for each unit of land* i*, a fraction of the peasants that revolt* $x \in [0,1]$*, a tax rate* $t^c \in [0,1]$ *for all units of land, and a fraction of the citizens that stage a coup* $z \in [0,1]$ *such that:*

1. *At a critical juncture, if the society is under elitism:*
   - *Given* x*, every member of the elite maximizes* $u^e(t_i^r, x)$ *by choosing tax rate* t_i^r*.*
   - *Given* t_i^r*,* y_i *and* x*, the peasant working in unit of land* i *maximizes* $u_i^p(t_i^r, x)$ *by choosing whether to join the revolution or not, and the fraction of the peasants that choose to revolt equals* x*.*

2. *At a critical juncture, if the society is under the democratic regime:*

   - *Given t^c and z, every member of the elite maximizes $u_i^c(t^c, z)$ by choosing whether to join the coup or not, and the fraction of the citizens that choose to join the coup equals z.*

3 Analysis

Since $\delta \to 0$ and $\beta < 1$, every critical juncture can be studied independently of each other, except for the regime that results after the critical juncture, which is one of the state variables in our definition of Markov Perfect Equilibrium. Therefore, it is useful to separate the analysis of the model in two parts. The first one (Section 3.1) deals with the first component of the Markov perfect Equilibrium: the tuple $(\{t_i^r\}_i, x)$. The second part of the analysis (Section 3.2) deals with the second component of the Markov Perfect Equilibrium: the tuple (t^c, z).

3.1 Analysis: Revolutions

We refer to the tuples $(\{t_i^r\}_{iy}, x)$ that are part of a Markov Perfect Equilibrium as r-equilibria.

Proposition 1. *For any value of ε there are at most four possible r-equilibria:*

- *r-equilibrium 1:* $(\{\underline{t}^r(0)\}_i, 0)$.
- *r-equilibrium 2:* $\left(\{\overline{t}^r(\varepsilon)\}_i, \varepsilon\right)$.
- *r-equilibrium 3:* $(\{t_i^r\}_i, x)$ *with* $x \in [0, \varepsilon]$ *where a fraction* $1 - \frac{x}{\varepsilon}$ *of the elite chooses tax rate* $\underline{t}^r(x)$ *and the rest choose tax rate* $\overline{t}^r(x)$*, and* x *is such that*

$$u^e(\underline{t}^r(x), x) \quad = \quad u^e(\overline{t}^r(x), x).$$

- *r-equilibrium 4:* $(\{t_i^r\}_i, 1)$ *for any* $\{t_i^r\}_i$.

In r-equilibrium 1 (oppressive regime) we have that all members of the elite choose a tax rate $\underline{t}^r(0)$ and no peasant revolts. Thus, in this r-equilibrium the current regime whereby the peasant works the land and the elite extracts some of the revenue at no cost continues with probability one after the critical juncture. Regimes that display similar characteristics to those of this type of equilibrium can be found in North Korea, where the strength of the elite increased after the critical juncture caused by the Second World War (Acemoglu and Robinson (2012)).

Under r-equilibrium 2 (unstable regime) we have that the tax rate set up by the elite is such that only the peasants that suffer an output shock revolt. The probability that the regime changes is given by $\gamma(\varepsilon)$. The Arab Spring that caused fast transitions to democracy in some countries (such as Tunisia) while at the same time has left the current regime unstable and in a period of continued civil war (as in Syria) is an example of a situation where this equilibrium could apply to.

The third equilibrium, r-equilibrium 3 (segregated regime), is a combination of r-equilibria 1 and 2, where some members of the elite set up tax rate $\underline{t}^r(x)$ and other set up tax rate $\overline{t}^r(x)$. As $x \in [0, \varepsilon]$ implies $\underline{t}^r(x) \leq \overline{t}^r(x)$, there is segregation between the units of land where the more extractive tax rate $\overline{t}^r(x)$ is in place and where some of the peasants revolt, and the units of land where the less extractive tax rate $\underline{t}^r(x)$ is in place and where no peasant revolts. A real-world example that the model could help explain with this equilibrium was observed after the end of World War II (interpreted as a critical juncture in our model), where Korea was divided in two countries with different autocratic regimes during the Korean War.

Finally, r-equilibrium 4 (regime change) represents a situation where all peasants revolt. If all peasants revolt, then the elite can do little to stop a revolution. This equilibrium could be seen as what happened in north Vietnam after World War II, where the Democratic Republic of Vietnam was formed in 1945.

Our next result shows that there always exists an r-equilibrium and, furthermore, it states the conditions under which each of the four different r-equilibria are possible. Conditions for r-equilibria 1-3 to exist are stated implicitly, their explicit forms are presented in the appendix, where we also present the proof of the proposition.

Proposition 2. *For any value of ε there always exists an r-equilibrium. Furthermore, define*

$$\begin{aligned}\Delta u^e(x) &= u^e(\underline{t}^r(x), x) - u^e(\overline{t}^r(x), x) \\ &= \underline{t}^r(x)\varepsilon y - (\overline{t}^r(x) - \underline{t}^r(x)) \left[(1-\varepsilon) + (1-\gamma(x))\frac{\beta E y}{1-\beta}\right],\end{aligned}$$

we have the following:

1. *The tuple $(\{\underline{t}^r(0)\}_i, 0)$ is an r-equilibrium if and only if $\Delta u^e(0) \geq 0$.*
2. *The tuple $\left(\{\overline{t}^r(\varepsilon)\}_i, \varepsilon\right)$ is an r-equilibrium if and only if $\overline{t}^r(\varepsilon) \geq 0$ and $\Delta u^e(\varepsilon) \leq 0$.*
3. *The tuple $(\{t^r_i\}_i, x)$ with $x \in [0, \varepsilon]$ where a fraction $1 - \frac{x}{\varepsilon}$ of the elite chooses tax rate $\underline{t}^r(x)$ and the rest choose tax rate $\overline{t}^r(x)$ is an r-equilibrium if and only if $\Delta u^e(x) = 0$.*
4. *The tuple $(\{t^r_i\}_i, 1)$ is an r-equilibrium for any $\{t^r_i\}_i$ if and only if*

$$\frac{\beta}{1-\beta} V^c > 1 - w.$$

Proposition 2 states the conditions under which each of the different r-equilibria are possible. A crucial function is that of $\Delta u^e(x)$, which specifies what is the increase in the expected payoff of a member of the elite from choosing tax rate $\underline{t}^r(x)$ instead of tax rate $\overline{t}^r(x)$ when a fraction x of the peasants revolt. Thus, for instance, r-equilibrium $(\{\underline{t}^r(0)\}_i, 0)$ is possible if and only if the increase in expected payoff from choosing tax rate $\underline{t}^r(0)$ instead of tax rate $\overline{t}^r(0)$ when no peasant revolts is positive: i.e. all members of the elite have incentives to choose tax rate $\underline{t}^r(0)$ (when all the other members of the elite choose this tax rate) and, hence, no peasant has incentives to revolt. The second r-equilibrium, $\left(\{\overline{t}^r(\varepsilon)\}_i, \varepsilon\right)$, is possible if and only if the increase in expected payoff from choosing tax rate $\underline{t}^r(\varepsilon)$ instead of tax rate $\overline{t}^r(\varepsilon)$ when a fraction ε of the peasants revolt is negative: i.e. all members of the elite have incentives to choose tax rate $\overline{t}^r(\varepsilon)$ and, hence, only those peasants that work on a unit of land that suffers an output shock revolt. Thirdly, if for some $x \in [0, \varepsilon]$ we have that $\Delta u^e(x) = 0$ then when exactly a fraction x of the peasants revolt, members of the elite are indifferent between setting tax rate $\underline{t}^r(x)$ and tax rate $\overline{t}^r(x)$. Hence, if a fraction $1 - \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $\underline{t}^r(x)$ and the rest choose tax rate $\overline{t}^r(x)$ then exactly a fraction x of the peasants has incentives to revolt. We discuss r-equilibrium 4 in more detail later on.

As stated by Proposition 2, a necessary but not sufficient condition for r-equilibrium 2 to exist is that $\overline{t}^r(\varepsilon) \geq 0$. This is a requirement as otherwise the elite cannot set up tax rate $\overline{t}^r(\varepsilon)$ given that tax rates to belong to the interval $[0, 1]$. As sufficient condition for $\overline{t}^r(\varepsilon) \geq 0$ is that $\gamma(\varepsilon) \leq \frac{1}{2\beta}$ (see Lemma 1 in the Appendix).

For most of the rest of this section we shall focus our attention away from r-equilibrium 4 and concentrate on r-equilibria 1-3. Nevertheless, a further discussion on r-equilibrium 4 is presented at the end of this section.

3.1.1 Comparative Statics

The analysis that follows is thus focused on function Δu^e. Since this function is of high order and depends on a number of parameters, we make a few simplification so that the number of cases to be consider is not excessive.

Assumption 1. *For this section, assume that $\gamma(x) = x$, $y = 0.8$ and that $\varepsilon < \frac{1}{4}$.*

Notice that $\varepsilon < \frac{1}{4}$ implies $\mu_\varepsilon < \frac{1}{4}$ and that the only region in the domain of Δu^e that matters for the existence of the different equilibria is $x \in \left[0, \frac{1}{4}\right]$. This further simplifies the comparative statics considerably.

We have the following result:

Proposition 3. *The function Δu^{ε} is decreasing. Moreover, we have the following comparative statics:*

1. *There exists a $\bar{c}$ and a $\bar{\varepsilon}$ such that if $w < \bar{w}$ and $\varepsilon > \bar{\varepsilon}$ then $\Delta u^{e}(0) > 0$.*
2. *The function Δu^{e} is increasing in ε.*
3. *The function Δu^{e} is decreasing in w.*

The fact that Δu^{ε} is decreasing combined with point 1 in the proposition above implies that if the cost w of working the land is sufficiently small and the probability of an output shock ε is sufficiently high then the oppressive equilibrium (r-equilibrium 1) will be present. The reason is that w needs to be low so that the one period gain from joining a revolution (not paying the cost w in the present period) is not too high. However, the effect of the frequency of output shocks ε is less straightforward. On the one hand higher ε implies that the opportunity cost or joining a revolution is low as output is low on many units of land which increases the incentives to join a revolution. On the other hand, higher ε means that the unstable regime equilibrium (r-equilibrium 2) is less likely as it now requires the coordination of more peasants. That is, higher ε means there are more incentives to join the revolution, but the coordination needed to revolt is higher.

With respect to point 2 in proposition 3 we have that increasing the frequency of output shocks ε increases the incentives for the elite to choose tax rate $\underline{t}^{r}(x)$ instead of tax rate $\overline{t}^{r}(x)$ as output is lower and thus the gains from a higher tax rate are lower. As a result, if r-equilibrium 1 does not exists already, it becomes more likely as discussed above. Moreover, if r-equilibrium 1 is not present then the segregated regime equilibrium (r-equilibrium 3) also becomes more likely. This is because, as already discussed, higher ε means more incentives to revolt but also the number of peasants that have to coordinate is higher. This naturally gives rise to the segregated regime equilibrium where some peasants revolt and others do not.

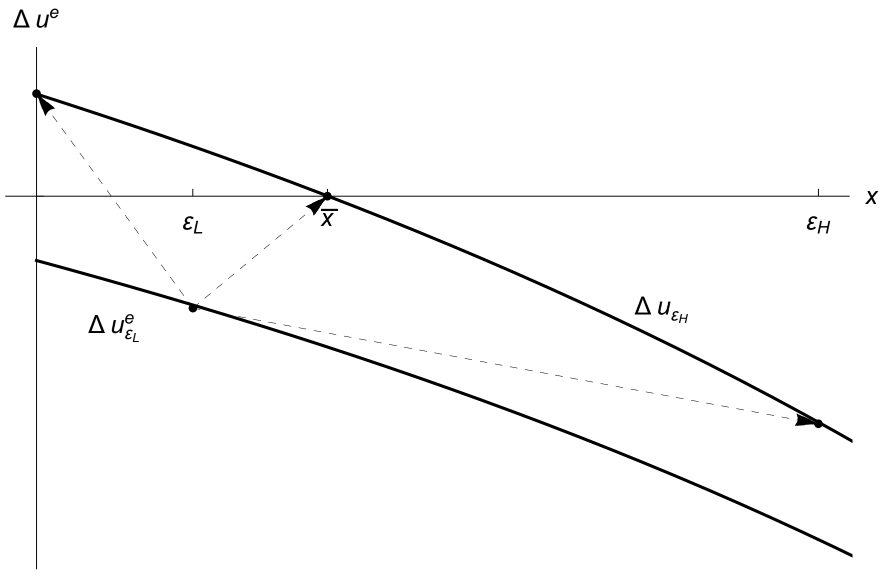
In terms of the cost of working the land w, point 3 in proposition 3, we have that increasing w makes it more attractive for the elite to choose tax rate $\overline{t}^{r}(x)$. The reason is that with a higher cost of working the land the opportunity cost of joining a failed revolution or not joining a successful revolution are lower, as the utility of the peasant (or citizen in case a successful revolution) is lower when w increases. Therefore, peasants are more likely to revolt which in turn means that the cost for the elite of setting up the higher tax rate versus the low tax rate are lower as the difference in the likelihood of a revolution between both tax rates is less pronounced.

3.1.2 Graphical Illustration of Results

To obtain a graphical representation on when r-equilibria 1-3 are possible, we present Figures 3 and 4, where the parameter values are set to $\beta = 0.95$, $y = 0.5$, $\gamma(x) = \sqrt{x}$ and $\mu_\varepsilon = 0.2$, and ε takes two possible values: $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$. In Figure 3 the cost of working the land is set to $w = 0.2$ while in Figure 4 this value is set to $w = 0.5$. As it can be checked using Lemma 1 in the Appendix, in both figures it is true that $\bar{t}^r(\varepsilon) \geq 0$. Each figure plots the function Δu^e from Proposition 2 for each of the two values of ε: $\Delta u^e_{\varepsilon_k}$ equals function Δu^e when $\varepsilon = \varepsilon_k$ with $k \in \{L, H\}$.

In Figure 3, we can see that $\Delta u^e_{\varepsilon_L}(0) < 0$, $\Delta u^e_{\varepsilon_L}(\varepsilon_L) < 0$ and there is no $x \in [0, \varepsilon_L]$ such that $\Delta u^e_{\varepsilon_L}(x) = 0$. Hence, by Proposition 2 the unique r-equilibrium (apart from r-equilibrium 4) is r-equilibrium 2. If $\varepsilon = \varepsilon_H$ then we have that $\Delta u^e_{\varepsilon_H}(0) > 0$, $\Delta u^e_{\varepsilon_H}(\varepsilon_H) < 0$ and $\Delta u^e_{\varepsilon_H}(\bar{x}) = 0$. Hence, by Proposition 2 we have that r-equilibria 1-3 are possible. In Figure 3 each possible r-equilibrium is represented by a dot and the arrows point from r-equilibrium 2 when $\varepsilon = \varepsilon_L$ to r-equilibria 1-3 when $\varepsilon = \varepsilon_H$.

Figure 3: $\beta = 0.95$, $y = 0.5$, $w = 0.2$, $\gamma(x) = \sqrt{x}$, $\mu_\varepsilon = 0.2$, $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$.



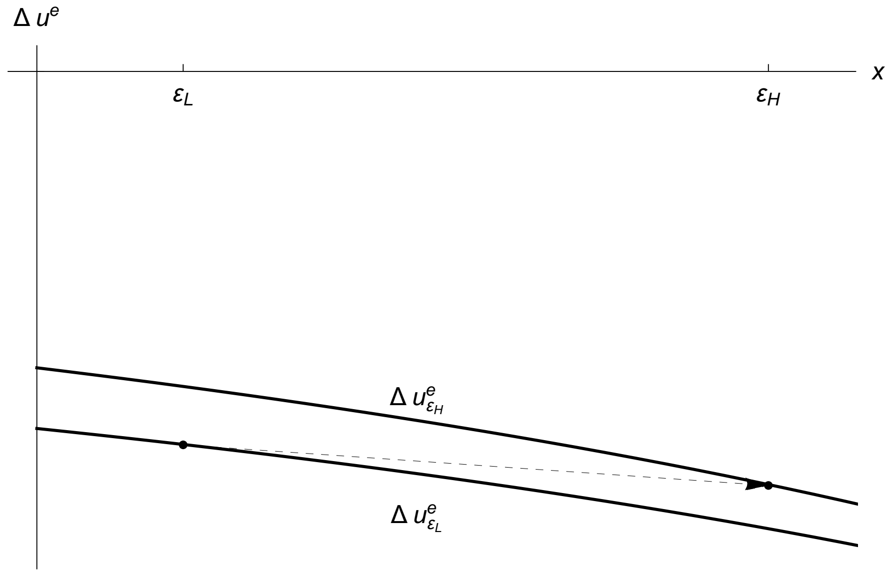
Suppose that at a given critical juncture where the current regime is elitism it is true that $\varepsilon = \varepsilon_L$. Ignoring r-equilibrium 4, this would lead members of the elite to set a tax rate $\bar{t}^r(\varepsilon_L)$. Consider now that the elitist regime survives the revolution initiated by a fraction ε_L of the peasants. Suppose a new critical juncture arrives and assume that this time the society is going through a period of economic crisis that makes output shocks more likely: $\varepsilon_H > \varepsilon_L$. This new critical juncture can lead to three very distinct situations. Firstly, the society could move to an r-equilibrium where the tax rate is $\underline{t}^r(0)$ and such that no peasant revolts. Secondly, the society could move to an r-equilibrium where tax rate is $\overline{t}^r(\varepsilon_H)$ and where the chances of the revolution to be successful are $\gamma(\varepsilon_H) = 25\%$ and, therefore, a regime change is likely. Thirdly, the society could move to a situation where each unit of land belongs to either one of two different groups: one where tax rate $\underline{t}^r(\bar{x})$ is in place and as such no peasant revolts, and another one where tax rate $\overline{t}^r(\bar{x})$ is in place and where a fraction ε_H of the peasants revolts.

Hence, the arrival of a critical juncture when the frequency of shocks increases creates different alternatives in the possible history that the society could follow. In this respect, our model illustrates Acemoglu and Robinsons' (2013) statement that "A critical juncture is a double-edged sword that can cause a sharp turn in the trajectory of a nation. On the one hand it can open the way for breaking the cycle of extractive institutions and enable more inclusive ones to emerge... Or it can intensify the emergence of extractive institutions...".

The reason why the frequency of shocks affects the r-equilibria that are possible in this case is the following. During times of expansion (ε_L), if a member of the elite charges the high tax rate $\overline{t}^r$ then the probability that his peasant revolts is small given that output shocks are infrequent. Hence, members of the elite are better off by charging a high tax rate and having the risk that their peasant revolts than by charging a smaller tax rate that ensures that their peasant does not revolt. On the other hand, during times of economic crisis (ε_H), members of the elite would like to set up a tax rate so that no peasant revolts (r-equilibrium 1) as the number of the peasants that could revolt in this case is more significant and, hence, a regime change would be likely. However, if peasants coordinate and many of them revolt then the elite can only stop from revolting those that do not suffer an output shock (r-equilibrium 2). An intermediate case also exists where in some units of land no peasant revolts and in some others a fraction of the peasants revolt (r-equilibrium 3).

Figure 4 plots the same situation as Figure 3 when instead the cost of working the land is given by $w = 0.5$. As we can observe, there is a unique r-equilibrium (except for r-equilibrium 4) given by $(\{\overline{t}^r(\varepsilon)\}, \varepsilon)$. Hence, with respect to Figure 3, increasing the cost of working the land implies that the r-equilibrium where no peasant revolts is less likely to be present. This suggests that societies that enjoy lower land profitability $(Ey - w)$ are less prone to equilibria multiplicity as r-equilibria 1 and 3 are less likely to be present.

Figure 4: $\beta = 0.95$, $y = 0.5$, $w = 0.5$, $\gamma(x) = \sqrt{x}$, $\mu_\varepsilon = 0.2$, $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$.



In both figures above we have ignored r-equilibrium 4, which is the equilibrium where all peasants revolt. The reason for which we analyse r-equilibrium 4 separately is that this equilibrium is present in all the graphs plotted above. This is not surprising, however, if all peasants revolt, then the revolution is successful and, hence, every single peasant wants to revolt.[10] A question that may arise is that of whether or not it is possible for r-equilibrium 4 to be the unique equilibrium:

Proposition 4. *If* $(1-\beta)y + \beta Ey < w$ *then r-equilibrium 4 is the unique equilibrium.*

3.2 Analysis: Coups

In this section we analyze the second component of the Markov Perfect Equilibrium. We refer to the tuples $(\{t_i^c\}_i, z)$ that are part of a Markov Perfect Equilibrium as c-equilibria. The next result characterizes the set of c-equilibria and states the conditions under which each of the possible c-equilibria can be present.

Proposition 5. *For any value of* ε *there always exists a c-equilibrium and there are at most three possible c-equilibria:*

- *c-equilibrium 1:* $(\{t^c\}_i, 0)$ *for any* $t^c \in [0,1]$*. This c-equilibrium exists if and only if*

$$(1-\beta)y + \beta Ey - w \quad \geq \quad 0.$$

[10]Unless his discount factor is such that he prefers the one-off payoff from working the land today instead of the future stream of payoffs from eliminating the elite and becoming a citizen.

- *c-equilibrium 2:* $(\{t^c\}_i, \varepsilon)$ *for any* $t^c \in (\underline{t}^c(\varepsilon), \overline{t}^c(\varepsilon)]$. *This c-equilibrium exists if and only if*

$$(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - w \quad < \quad 0.$$

- *c-equilibrium 3:* $(\{t^c\}_i, 1)$ *for any* $t^c > \overline{t}^c(1)$. *This c-equilibrium exists if and only if*

$$(1-\beta) - \beta Ey - w \quad < \quad 0.$$

In c-equilibrium 1 (stable democracy) we have that no citizen attempts a coup and the society continues to function under the democratic regime after the critical juncture with probability one. In this situation, citizens do not have incentives to stage a coup for one main reason: no other citizen joins the coup. c-equilibrium 1 is not possible if $y - w + \frac{\beta}{1-\beta}[Ey - w] < 0$ or, in other words, if the expected payoff of a citizen who works on a unit of land that suffers a production shock is negative. In this case, each citizen working on a unit of land that suffers an output shock would be better off staging a coup even if he is the only citizen doing so. This would imply that all citizens that suffer an output shock stage a coup, which contradicts the fact that in a c-equilibrium 1 no citizen stages a coup.

The behavior of most democracies in the western world could fall into this type of equilibrium. After a critical juncture, like the financial crisis in 2008, the country continues to function as a democracy. The role of a financial crisis in the institutions in a democratic regime is discussed further after we describe c-equilibrium 2 and c-equilibrium 3.

The second c-equilibrium, c-equilibrium 2 (unstable democracy), is such that those citizens that work on a unit of land that suffers an output shock join the coup. In this case, the tax rate to be set by the future elite is such that only a fraction of the citizens wants to stage a coup. The coup is successful at changing the current regime back to elitism with a probability of $\rho(\varepsilon)$. This type of equilibrium has similar characteristics as those observed in democratic countries where a critical juncture has caused a civil war, like the civil war that started in South Sudan in 2013 after a minister was removed from his duties and staged a coup.[11]

Finally, in c-equilibrium 3 (regime change) citizens from all units of land join the coup and a regime change happens with probability 1. The regime change implies that from next period on the society is back to elitism. This equilibrium was observed in Guatemala when after World War II a successful coup d'état changed the regime from democracy to a military dictatorship.[12]. This equilibrium is present for most sensible parameter values because if all

[11] See, for instance, `http://www.aljazeera.com/video/africa/2013/07/20137287019670555.html` and `http://www.trust.org/item/20131223195244-2j16n/?source=hptop#`.

[12] See `https://www.cia.gov/library/center-for-the-study-of-intelligence/kent-csi/vol44no5/html/v44i5a03p.htm`

citizens join the coup then the coup will succeed with probability one.[13] Hence, the only factor that may stop a citizen from joining the coup is a very low discount factor β, so that the cost of not having any output today is greater than the future benefits of joining the elite. The fact that c-equilibrium 3 is a possibility for most sensible parameter values should not be surprising and has a very natural interpretation: even in the most robust democracies in the developed world, if all politically active people coordinate to change the regime there will be a regime change.

Proposition 5 also implies the following. Suppose that a severe economic crisis causes a critical juncture. In the language of the model, the fact that the economic crisis is severe implies that ε is high, which in turn implies that Ey is low. This makes c-equilibrium 1 less likely and c-equilibrium 3 more likely. However, whether c-equilibrium 2 is more or less likely depends on how likely the coup is to succeed given the number of citizens that join the coup (function ρ).

3.2.1 Comparative Statics

As opposed to the section on revolutions, comparative statics for coups are straightforward from proposition 5 and require no additional assumptions and little to no algebra. This is why the proposition below is presented with no proof.

Proposition 6. *We have the following comparative statics:*

1. *Increasing μ_ε makes c-equilibria 1 and 3 less likely, and c-equilibrium 2 more likely.*
2. *Increasing w makes c-equilibrium 1 less likely and c-equilibria 2 and 3 more likely.*
3. *Increasing β makes c-equilibria 1 less likely and c-equilibrium 3 more likely while the effect on c-equilibria 2 is ambiguous.*

According to point 1 in the proposition above, increasing the frequency of output shocks makes the unstable democracy equilibrium more likely while the other equilibria become less likely. Increasing the average frequency of output shocks makes the opportunity cost of a coup decrease (hence c-equilibrium 1 is becomes more likely), and it also decreases the gain if the coup is successful (hence why c-equilibrium 3 is also less likely). In terms of c-equilibrium 2, this type of equilibrium becomes more likely as one the one hand the opportunity cost of a coup decreases but at the same time the benefits from a coup decrease.

[13] With regard to the interpretation of the model, note that all citizens joining the coup means that in every unit of land all those who are politically active attempt a regime change to turn the rest (politically inactive) people into peasants.

Point 2 in proposition 6 states that higher cost of working the land makes a coup more likely (whether it is a coup where only some citizens participate or all). The reason is that as w goes up, becoming a member of the elite becomes more attractive as members of the elite do not pay the cost of working the land yet they enjoy a fraction of the benefits.

Finally, point 3 states that a higher discount factor makes a coup where all citizens participate more likely, and it may also make a coup where only some citizens participate more likely. The reason is that as β goes up, the benefits of becoming a member of the elite go up; the per period payoff of being a member of the elite is higher and thus a higher discount factor amplifies this effect. The reason why c-equilibrium 2 may not necessarily become more likely is that on the one hand higher β increases the incentives to stage a coup as just discussed but on the other hand it also makes it more costly to stage a coup as the current period loss from the coup increases with β.

3.2.2 Graphical Illustration of Results

To get a better understanding of the result in Proposition 5, define the functions

$$\Delta t_1(\varepsilon) = (1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - w$$

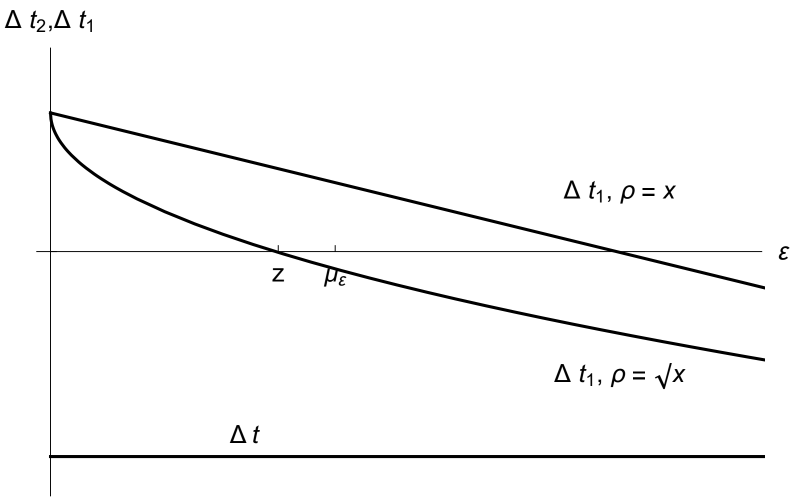
and

$$\Delta t_2 = (1-\beta) - \beta Ey - w.$$

Thus, according to Proposition 5, c-equilibrium 1 is possible if and only if $\Delta t_1(0) \geq 0$, c-equilibrium 2 is possible if and only if $\Delta t_1(\varepsilon) < 0$ and, finally, c-equilibrium 3 is possible if and only if $\Delta t_2 < 0$. Figure 5 below depicts the functions Δt_1 and Δt_2 for $\rho(z) = \sqrt{z}$ and $\rho(z) = z$.[14] This figure uses the same parameter values as Figure 3.

[14] While Δt_2 does not depend on ε the figure also plots it to assess whether c-equilibrium 3 exits or not.

Figure 5: $\beta = 0.95$, $y = 0.8$, $w = 0.2$, $\mu_\varepsilon = 0.25$, $\rho(x) = \sqrt{x}$ and $\rho(x) = x$.



Both Δt_1 lines in Figure 5 depict a similar situation: c-equilibrium 1 and c-equilibrium 3 are both possible while c-equilibrium 2 is possible if and only if ε is high enough at the critical juncture. The difference between the two is that the infimum value of ε for which c-equilibrium 2 is possible (referred to as z in the graph) is lower under the function $\rho(z) = \sqrt{z}$ than under the function $\rho(z) = z$. That is, c-equilibrium 2 is more likely to exist under the function $\rho(z) = \sqrt{z}$ than under the function $\rho(z) = z$. The interpretation of this is that if a coup is easier to succeed given how many citizens join it, then ceteris paribus citizens have more incentives to join the coup. In c-equilibrium 2 not all citizens want to join the coup because although the are some chances that the coup is successful, there is still the possibility that the coup fails. Thus, when the citizens that work on a unit of land that does not suffer an output shock decide whether to join the coup or not, the fact that the coup may fail makes them not to want to join the coup. The citizens that work on a unit of land that suffers and output shock have less to lose (lower opportunity cost from joining the coup) and, hence, because the coup has some chances of succeeding, they have enough incentives to join it.

4 Discussion on the Modelling

Each unit of land can be considered as a territory but also as, for example, a certain economic activity or market. The member of the elite of a given unit of land represents the few agents (or a single agent) that own the unit of land or that are given exclusive rights to exploit it. The tax rate represents the institutions in place. The fact that different members of the elite can set up a different tax rate in their land means that the elite does not act as one single player as in previous papers (see Acemoglu and Robinson (2001a) among others). A split elite happened, for example, in the case of the colonization of America, where the elite in the south (exclusive) was very different to the elite in the north (inclusive) (see Acemoglu and Robinson (2012)).[15]

The peasant in a unit of land represents all the agents that are responsible for the actual exploitation of the unit of land. The output produced is then split between the two sides, elite and peasant, according to the tax rate. Moreover, at every period and at every unit of land, an output shock that lowers the output of the unit of land may be present. The parameter μ_ε captures the long term output gap while the value of ε represents the current state of the economy. A high value of ε corresponds to a recession as output shocks are frequent whereas a low value of ε represents a period of economic expansion because output shocks are rare.

We assume that after a failed revolution all peasants that revolted are removed from the game and, similarly, if a revolution succeeds then the elite is removed from the game. Whenever a revolution fails revolutionaries receive very severe punishments (such as torture and death during the Tibetan unrest in 2008)).[16] Similar treatment is given to members of the elite after a successful revolution (from the beheading of aristocrats during the French revolution to the hanging of dictator Saddam Hussein). We assume that in case of a successful revolution, peasants that did not join the revolt are removed from the game. This aims to highlight the loss in private benefits of not joining a successful revolution. As Tullock (1971) writes: "...(revolutionaries) generally expect to have a good position in the new state which is to be established by the revolution. Further, ...(leaders) continuously encourage their followers in such views. In other words, they hold out private gains to them.".

During a critical juncture, the elite has a chance to "update" or "modernize" to retain its power. This takes the form of an opportunity for each member of the elite to change how extractive institutions are with the peasant, i.e., change the tax rate. After the elite has responded to the new situation posed by the critical juncture, peasants decide whether to revolt or not.

When a critical juncture takes place, it is assumed that members of the elite know the frequency of shocks ε but not which units of land suffer an output shock. We assume that at a critical juncture, members of the elite know the current state of the economy (expansion, recession, depression, etc.). Such state is represented in the model by the current value of ε. However, the elite ignores which specific units of lands (or sectors, markets, etc.) of the economy will suffer an output shock.[17]

[15] For more on split elites see also Acemoglu and Robinson (2016) and Albertus (2015) among others).

[16] See the US Department of State 2010 Human Rights Report:
http://www.state.gov/j/drl/rls/hrrpt/2010/eap/154382.htm.

[17] For example, in the case of the Black Death it was known that the economy was going through a shock

In terms of coups, if a citizen joins a successful coup, then he becomes a member of the elite and a peasant arrives to his unit of land (we discuss this in more depth below). Moreover, if a citizen does not join the coup and the coup is successful then he becomes a peasant and a member of the elite arrives to his unit of land. Finally, citizens that join an unsuccessful coup are removed from the game and replaced with new ones.

5 Conclusions

In this paper we studied regime change in the presence of critical junctures. In our results, we used the different equilibria that may arise in the model to explain different institutional settings that have appeared at various points in time in the history of civilizations. On top of that, we have used several current and historical examples to illustrate how each of the different equilibria of the model matches the different institutions observed in different countries. As opposed to previous literature, we have explicitly considered the cooperation/coordination problem faced by peasants in the presence of an elite. This led to a richer set of possible equilibria and allowed us to characterize a wide variety of institutions.

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Appendix

Proof of Proposition 1. We proceed by characterizing the equilibria first in situations where one or both tax rates $\bar{t}^r$ and $\underline{t}^r$ are negative and then proceed to characterize equilibria when these two values are positive.

Any r-equilibrium $(\{t_i^r\}_i, x)$ where x is such that $\bar{t}^r(x) < 0$ must have $x = 1$, as if x peasants revolt and $\bar{t}^r(x) < 0$ then all peasants have incentives to revolt regardless of the tax rate set by members of the elite (see equation (2)). Thus, in r-equilibrium either $(\{t_i^r\}_i, 1)$ with $\bar{t}^r(1) < 0$ or $(\{t_i^r\}_i, x)$ with $\bar{t}^r(x) \geq 0$ for $x \leq 1$.

If $\bar{t}^r(1) < 0$ and all peasants revolt then the tax rates set by the elite $\{t_i^r\}_i$ are irrelevant as there is no tax rate that will keep a peasant from revolting. Furthermore, if $x = 1$ and $\bar{t}^r(1) < 1$ then all peasants have incentives to revolt regardless of the tax rate. This is r-equilibrium 4, $(\{t_i^r\}_i, 1)$, in the proposition.

Consider r-equilibria $(\{t_i^r\}_i, x)$ with $\bar{t}^r(x) \geq 0$. If $\underline{t}^r(x) < 0$ then it must be that all members of the elite set a tax rate equal to $\bar{t}^r(x)$. If all members of the elite choose tax rate $\bar{t}^r(x)$ and $x > \varepsilon$ then there are peasants that revolt even though their land did not suffer an output shock. If this is true then it must be that all peasants revolt; this leads to the only possibility of $\big(\{\bar{t}^r(1)\}_i, 1\big)$, which is a particular case of r-equilibrium 4. If, on the other hand, $x < \varepsilon$, then there are peasants that do not revolt even though their land suffered an output shock, a contradiction to the definition of $\bar{t}^r(x)$. Thus, if all members of the elite set up tax rate $\bar{t}^r(x)$ then the only possibility is $x = \varepsilon$ so that the chosen tax rate is given by $\bar{t}^r(\varepsilon)$. This is r-equilibrium 2 in the proposition.

Consider now r-equilibria $(\{t_i^r\}_i, x)$ with $\underline{t}^r(x) \geq 0$.Members of the elite either set a tax rate equal to $\underline{t}^r(x)$ or to $\bar{t}^r(x)$. We have now three possibilities: all members of the elite choose tax rate $\underline{t}^r(x)$, all choose $\bar{t}^r(x)$, or some choose $\underline{t}^r(x)$ and others choose $\bar{t}^r(x)$. We have already dealt with the case where all members of the elite set tax rate $\bar{t}^r(x)$ in the paragraph above. Thus, we focus now on the situations where either all members of the elite choose tax rate $\underline{t}^r(x)$, or some choose $\underline{t}^r(x)$ and others choose $\bar{t}^r(x)$.

In any r-equilibrium $(\{\underline{t}^r(x)\}_i, x)$ it must be that $x = 0$ as if $x > 0$ and all members of the elite set up tax rate $\underline{t}^r(x)$ then when a fraction x of the peasants revolt, by the definition of $\underline{t}^r(x)$ in equation (1) no peasant has incentives to revolt, which represents a contradiction to the fact that $x > 0$. Thus, the only possibility is that $(\{\underline{t}^r(0)\}_i, 0)$, which is r-equilibrium 1 in the proposition.

Finally, consider any r-equilibrium $(\{t_i^r\}_i, x)$ where some members of the elite choose $\underline{t}^r(x)$ and others choose $\bar{t}^r(x)$. Then it must be that exactly a fraction $1 - \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $\underline{t}^r(x)$ and a fraction $\frac{x}{\varepsilon}$ choose tax rate $\bar{t}^r(x)$. Otherwise, if fraction $\omega \neq \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $\bar{t}^r(x)$ then a proportion $\omega\varepsilon \neq x$ of the peasants choose to revolt (those peasants working on a unit of land subject to tax rate $\bar{t}^r(x)$ and where there is an output shock). This represents a contradiction to the fact that $(\{t_i^r\}_i, x)$ is an r-equilibrium. Thus, the only possibility is that $(\{t_i^r\}_i, x)$ where a fraction $1-\frac{x}{\varepsilon}$ of the members of the elite choose tax rate $\underline{t}^r(x)$ and the rest choose tax rate $\bar{t}^r(x)$. This is r-equilibrium 3 in the proposition, which is only possible if $x \leq \varepsilon$ and no member of the elite that chooses tax rate $\underline{t}^r(x)$ wants to choose tax rate $\bar{t}^r(x)$ and vice-versa, i.e. $u^e(\underline{t}^r(x), x) = u^e(\bar{t}^r(x), x)$. $\square$

Proof of Proposition 2. We prove the result separately for each of the different r-equilibria and then prove that at least one of them is always present.

1. $(\{\underline{t}^r(0)\}_i, 0)$:

   From equation (2), we have that

   $$(1 - \underline{t}^r(0))y - w \quad = \quad \frac{(1 - \underline{t}^r(0))Ey - w}{1 - \beta}$$

   Hence, if $\underline{t}^r(0) = 0$ then the right hand side of the equation above is greater than the left hand side, while if $\underline{t}^r(0) = 1$ then the opposite occurs. Thus, since the expression above is continuous in $\underline{t}^r(0)$, by Bolzano's Theorem we have that $\underline{t}^r(0) \in [0, 1]$.

   The tuple $(\{\underline{t}^r(0)\}_i, 0)$ is an r-equilibrium if both no peasant wants to revolt and no member of the elite wants to set up tax rate $\bar{t}^r(0)$. By definition, if a fraction 0 of the peasants revolt then $\underline{t}^r(0)$ is such that no peasant wants to revolt.

Moreover, no member of the elite sets up tax rate $\bar{t}^r(0)$ if and only if $u^e(\underline{t}^r(0), 0) \geq u^e(\bar{t}^r(0), 0)$. This can be rewritten as $\Delta u^e(0) \geq 0$. The explicit functional form for this condition $\Delta u^e(0) \geq 0$ is

$$1 - \frac{w + \beta\gamma(0)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(0))Ey}\varepsilon y - \left(\frac{w + \beta\gamma(0)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(0))Ey} - \right.$$
$$\left. \frac{w + \beta\gamma(0)(Ey - 2c)}{(1-\beta) + \beta(1-\gamma(0))Ey}\right)\left[(1-\varepsilon) + (1-\gamma(0))\frac{\beta Ey}{1-\beta}\right] \geq 0.$$

2. $\left(\{\bar{t}^r(\varepsilon)\}_i, \varepsilon\right)$:

   Firstly, it must be that $\bar{t}^r(\varepsilon) \geq 0$ as otherwise the elite is not able to set up tax rate $\bar{t}^r(\varepsilon)$. The tuple $\left(\{\bar{t}^r(\varepsilon)\}_i, \varepsilon\right)$ is an r-equilibrium if both no peasant wants to revolt and no member of the elite wants to set up tax rate $\underline{t}^r(\varepsilon)$. By definition, if a fraction ε of the peasants revolt then $\bar{t}^r(\varepsilon)$ is such that only those peasant who suffer an output shock revolt, i.e. a fraction ε of the peasants want to revolt.

   Moreover, no member of the elite sets up tax rate $\underline{t}^r(\varepsilon)$ if and only if $u^e(\bar{t}^r(\varepsilon), \varepsilon) \geq u^e(\underline{t}^r(\varepsilon), \varepsilon)$. This can be rewritten as $\Delta u^e(\varepsilon) \leq 0$. The explicit functional form for this condition is

$$1 - \frac{w + \beta\gamma(\varepsilon)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(\varepsilon))Ey}\varepsilon y - \left(\frac{w + \beta\gamma(\varepsilon)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(\varepsilon))Ey} - \right.$$
$$\left. \frac{w + \beta\gamma(\varepsilon)(Ey - 2c)}{(1-\beta) + \beta(1-\gamma(\varepsilon))Ey}\right)\left[(1-\varepsilon) + (1-\gamma(\varepsilon))\frac{\beta Ey}{1-\beta}\right] \leq 0.$$

3. $(\{t_i^r\}_i, x)$ where a fraction $1 - \frac{x}{\varepsilon}$ of the elite chooses tax rate $\underline{t}^r(x)$ and the rest choose tax rate $\bar{t}^r(x)$:

   By definition, if a fraction x of the peasants revolt then $\underline{t}^r(x)$ and $\bar{t}^r(x)$ are such that no peasant on a unit of land where the tax rate is $\underline{t}^r(x)$ wants to revolt and only those peasants that work on a unit of land where the tax rate is $\bar{t}^r(x)$ and that suffer an output shock revolt. Thus, since a fraction $\frac{x}{\varepsilon}$ of the members of the elite set up tax rate $\bar{t}^r(x)$ and a fraction ε of those have their peasants revolt, we have that a fraction x of the peasants revolt, as required.

   Moreover, no member of the elite wants to set up a tax rate different from the one he is currently setting if and only if $u^e(\underline{t}^r(x), x) = u^e(\bar{t}^r(x), x)$, which can be rewritten as $\Delta u^e(x) = 0$. The explicit functional form for this condition is

$$1 - \frac{w + \beta\gamma(x)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(x))Ey}\varepsilon y - \left(\frac{w + \beta\gamma(x)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(x))Ey} - \right.$$
$$\left. \frac{w + \beta\gamma(x)(Ey - 2c)}{(1-\beta) + \beta(1-\gamma(x))Ey}\right)\left[(1-\varepsilon) + (1-\gamma(x))\frac{\beta Ey}{1-\beta}\right] = 0.$$

Finally, we must show that both $\underline{t}^r(x)$ and $\overline{t}^r(x)$ are positive as otherwise this r-equilibrium is not possible. If x is such that $u^e(\underline{t}^r(x), x) = u^e(\overline{t}^r(x), x)$ then it must be that

$$\varepsilon \underline{t}^r(x) y \quad = \quad (1-\varepsilon)(\overline{t}^r(x) - \underline{t}^r(x)) + \beta(1-\gamma(x))[V^e(\overline{t}^r(x) - V^e(\underline{t}^r(x))].$$

Since $\underline{t}^r(x) \leq \overline{t}^r(x)$ for all $x \in [0,1]$ implies $V^e(\overline{t}^r(x)) - V^e(\underline{t}^r(x)) \geq 0$ then it is true that $(1-\varepsilon)(\overline{t}^r(x) - \underline{t}^r(x)) + \beta(1-\gamma(x))[V^e(\overline{t}^r(x) - V^e(\underline{t}^r(x))] \geq 0$ for all x and, hence, $\varepsilon \underline{t}^r(x) y \geq 0$. This implies that $\underline{t}^r(x) \geq 0$ and, as $\underline{t}^r(x) \leq \overline{t}^r(x)$, it is also true that $\overline{t}^r(x) \geq 0$ as required.

4. $(\{t_i^r\}_i, 1)$ is an r-equilibrium for any $\{t_i^r\}_i$:

   From the proof of Proposition 1, the tuple $(\{t_i^r\}_i, 1)$ is an r-equilibrium for any $\{t_i^r\}_i$ if and only if $\overline{t}^r(1) < 0$. Hence, the definition of V^c and equation (2) when $x = 1$ together with the fact that $\gamma(1) = 1$ leads to the desired result.

For the proof that at least one the four possible r-equilibria is always present, consider first the case where $\overline{t}^r(\varepsilon) < 0$. In this situation, we have that if ε peasants revolt then all peasants revolt as no tax rate can keep a peasant from revolting. Hence, since for any given tax rate the incentives to revolt are increasing in the number of peasants that revolt, $\overline{t}^r(\varepsilon) < 0$ implies that if all peasants revolt then all peasants have incentives to revolt and the elite can do nothing to stop peasants from revolting. This means that if $\overline{t}^r(\varepsilon) < 0$ then r-equilibrium 4 exists.

Consider now the case with $\overline{t}^r(\varepsilon) \geq 0$. If $\underline{t}^r(\varepsilon) < 0$ then $(\{\overline{t}^r(\varepsilon), \varepsilon\})$ is an r-equilibrium as if ε peasants revolt members of the elite maximize their expected payoff by choosing $\overline{t}^r(\varepsilon)$. If $\overline{t}^r(\varepsilon) \geq 0$ and $\underline{t}^r(\varepsilon) \geq 0$ and neither $(\{\underline{t}^r(0), 0\})$ nor $(\{\overline{t}^r(\varepsilon), \varepsilon\})$ are r-equilibria it must be because $\Delta u^e(0) < 0$ and $\Delta u^e(\varepsilon) > 0$ (see the first part of the proof). However, since the function $\Delta u^e(x)$ is continuous, by Bolzano's Theorem there exists a $\bar{x} \in (0, \varepsilon)$ such that $\Delta u^e(\bar{x}) = 0$. Moreover, since $\underline{t}^r(\varepsilon) \geq 0$ and $\overline{t}^r(\varepsilon) \geq 0$ and both $\underline{t}^r(x)$ and $\overline{t}^r(x)$ are decreasing functions in their argument (see equations (1) and (2) and recall that γ is an increasing function), then for all $x \leq \varepsilon$ it must be that $\underline{t}^r(x) \geq 0$ and $\overline{t}^r(x) \geq 0$. In particular, $\underline{t}^r(\bar{x}) \geq 0$ and $\overline{t}^r(\bar{x}) \geq 0$. Thus, the tuple $(\{t_i^r\}_i, \bar{x})$ where a fraction $1 - \frac{\bar{x}}{\varepsilon}$ of the elite chooses tax rate $\underline{t}^r(\bar{x})$ and the rest choose tax rate $\overline{t}^r(\bar{x})$ is an r-equilibrium. □

Lemma 1. *A sufficient condition for* $\overline{t}^r(\varepsilon) \geq 0$ *is that* $\gamma(\varepsilon) \leq \frac{1}{2\beta}$.

Proof of Lemma 1. Using equation (2) we have that $\overline{t}^r(\varepsilon) \geq 0$ if and only if

$$(1 - 2\beta\gamma(\varepsilon))(Ey - w) \quad \geq \quad -(1-\beta)(1 - Ey).$$

Thus, as $Ey - w \geq 0$ and $1 - Ey \geq 0$ a sufficient condition for the inequality above to be satisfied is that $1 - 2\beta\gamma(\varepsilon) \geq 0$, which leads to the condition in the lemma. $\square$

Proof of Proposition 3. First, we prove that Δu^e is a decreasing function, we have that $\frac{\mathrm{d}\Delta u^e(x)}{\mathrm{d}x}$ is proportional ($\propto$) to:

$$\begin{aligned}
&-2 + \mu(1+2c) + \beta(1-\mu)(-2+4c+\mu) + \frac{\varepsilon(-2+\mu)}{(2+\gamma(x)\beta(-2+\mu)-\beta\mu)^2} \\
&\big[\beta(2-\mu)(2+2\beta-2\gamma(x)(2+(2-\gamma(x)(3-\beta))\beta) - (1-\gamma(x))^2\beta(3+\beta)\mu + \\
&(1-\gamma(x))^2\beta^2\mu^2) + 2c(-2+\beta(-2\beta+4\gamma(x)(2+\gamma(x)(-2+\beta)\beta) \\
&+2(-1+\gamma(x))^2\beta(1+\beta)\mu - (-1+\gamma(x))^2\beta(-1+2\beta)\mu^2))\big] .
\end{aligned}$$

With the conditions given in Assumption 1 it can be shown that $\frac{\mathrm{d}\Delta u^e(x)}{\mathrm{d}x} < 0$ as desired.

Next, to proof the first point of the proposition simplify the value of $\Delta u^e(0)$ to obtain

$$\frac{\varepsilon}{2} - \frac{w(1-\varepsilon)}{1+\beta(1-\mu_\varepsilon)} - \frac{2w\varepsilon}{2-\beta\mu_\varepsilon},$$

which simple algebra shows is positive for

$$w < \frac{-2(1-\beta) + 3\beta\mu_\varepsilon + \beta^2\mu_\varepsilon - \beta^2\mu_\varepsilon^2}{-16 - 4\beta + 10\beta\mu_\varepsilon}$$

and

$$\varepsilon > \frac{2w(2-\beta\mu_\varepsilon)}{2(1-\beta) - 2w\beta(2-\mu_\varepsilon) - 3\beta\mu_\varepsilon - \beta^2\mu_\varepsilon(1-\mu_\varepsilon^2)}.$$

With respect to point 2, we have that

$$\frac{\mathrm{d}\Delta u^e(x)}{\mathrm{d}\varepsilon} = \bar{t}^r(x) - (1-y)\underline{t}^r(x),$$

which is positive as $y < 1$ and $\bar{t}^r(x) > \underline{t}^r(x)$.

Finally, in terms of point 3 we have

$$\begin{aligned}
\frac{\mathrm{d}\Delta u^e(x)}{\mathrm{d}w} \quad &\propto (1-2\beta\gamma(x))\left[((1-\beta) - y\left(2-\beta\left(2-(1-\gamma(x))Ey\right)\right))\varepsilon\right. \\
&\left.-(1-y)\left(2+\beta+(1-\gamma(x))\beta Ey\right)\right],
\end{aligned}$$

which given Assumption 1 is negative. $\square$

Proof of Proposition 4. r-equilibria 1-3 do not exists if $\bar{t}^r(0) < 0$ as in this case $\underline{t}^r(x) \leq \bar{t}^r(x) < 0$ for all $x \in [0,1]$ and the elite cannot set up tax rates $\underline{t}^r(x)$ or $\bar{t}^r(x)$ for any $x \in [0,1]$. Using equation (2) we have that $\bar{t}^r(0) < 0$ if and only if

$$y + \beta(Ey - y) < w.$$

Combining the fact that an r-equilibrium always exists (Proposition 2) with the inequality above gives the desired result. $\square$

Proof of Proposition 5. Any c-equilibrium $(t^c, 0)$ must be such that $t^c \leq \underline{t}^c(0)$ as otherwise when no citizen joins the coup then all citizens that suffer an output shock have incentives to join the coup and, hence, at least a fraction ε of the citizens join the coup, a contradiction. Since $t^c \leq \underline{t}^c(0)$ if and only if $\underline{t}^c(0) \geq 0$, this c-equilibrium is possible if and only if $(1-\beta)y + \beta Ey - w \geq 0$ (see equation (4)).

In any c-equilibrium (t^c, z) where $z \in (0,1)$ it must be that $z = \varepsilon$ as if $z > \varepsilon$ then there are citizens working on a unit of land that does not suffer an output shock that decide to join the coup. This means that all citizens have incentives to join the coup as argued above and, therefore, we must have that $z = 1$, a contradiction. Similarly, if $z < \varepsilon$ then there are citizens working on a unit of land that suffers an output shock that decide not to join the coup. As discussed above, this implies that no citizen has incentives to join the coup and, therefore, we must have that $z = 0$, a contradiction.

Moreover, any c-equilibrium (t^c, ε) must have $t^c \in (\underline{t}^c(\varepsilon), \overline{t}^c(\varepsilon)]$. Otherwise, if $t^c > \overline{t}^c(\varepsilon)$ then when a fraction ε of the citizens join the coup we have that all citizens have incentives to join the coup and, hence, $z = 1$, a contradiction. By the same token, if $t^c \leq \underline{t}^c(\varepsilon)$ then when a fraction ε of the citizens join the coup we have that no citizen has incentives to join the coup and, hence, $z = 0$, a contradiction. Thus, any c-equilibrium where the fraction of the citizens that join the coup belongs to the interval $(0,1)$ must be such that (t^c, ε) with $t^c \in (\underline{t}^c(\varepsilon), \overline{t}^c(\varepsilon)]$. This c-equilibrium is possible if $\underline{t}^c(\varepsilon) < 1$ and $\overline{t}^c(\varepsilon) \geq 0$ (otherwise t^c would be outside the interval $[0,1]$). From equation (5) we have that $\underline{t}^c(\varepsilon) < 1$ if and only if $(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - w < 0$. Moreover, from equation (6) as $w \in [0, Ey]$ it is always the case that $\overline{t}^c(\varepsilon) \geq 0$.

Finally, any c-equilibrium with $(t^c, 1)$ must be such that $t^c > \overline{t}^c(1)$ as otherwise when all citizens join the coup then all citizens that do not suffer an output shock do not have incentives to join the coup and, hence, at most a fraction ε of the citizens join the coup, a contradiction. This c-equilibrium is possible if and only if $\overline{t}^c(1) \in [0,1)$. From equation (6) we have that $\overline{t}^c(1) < 1$ if and only if $(1-\beta) - \beta Ey - w < 0$. Moreover, as $w \in [0, Ey]$ it is always the case that $\overline{t}^c(1) \geq 0$.

In order to prove that an equilibrium always exists it suffices to observe that if c-equilibrium 1 does not exists then it follows that $(1-\beta)y + \beta Ey - w < 0$, which implies $(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - w < 0$ and, hence, c-equilibrium 2 exists. $\square$