Interests Groups, Campaign Contributions and Political Competition*

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Abstract

We study how interest groups affect political competition and policy outcomes. Two parties compete in an election, where each of them can receive support from an interest group in the form of monetary contributions for campaign spending in exchange for a certain position in the political spectrum. The trade-off is that more campaign spending increases the chances of winning the election but the ideology of the interest group is not aligned with that of the voter. We find, among others, that median voter's welfare decreases the closer the interest groups fundraising abilities are with each other. Thus, the interest group with the most fundraising ability increasing its fundraising ability is beneficial for the median voter.

JEL Classification: D72, D82.

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1 Introduction

Over the last 8 years, interest groups have spent over $3 billion per year in the US, with more than 10,000 interest groups registered every year.[1] The literature on interest groups so far

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[1]Office of Public Records of the US Senate, calculations made by the Center for Responsible Politics (CRP henceforth).

has focused on campaign spending as a tool for reducing voters' uncertainty about the parties (see Austen-Smith (1987) or Prat (2002b) among others), or on how interest groups affect politicians currently in office (see Martimort and Semenov (2008) and Buzard and Saiegh (2016) among others). However, there is a gap in understanding how interest groups interact with each other and with candidates during elections when interest groups directly influence parties' policy positions in exchange for campaign contributions. Grossman and Helpman (1996) address this scenario but allow interest groups to contribute to both parties.

In this paper, we develop a model of electoral competition where two interest groups each offer a contract to one of two competing political parties, specifying a campaign contribution in exchange for the adoption of a specific policy position. Our contribution is that, unlike previous work, we assume that each interest group can only influence one party, which captures scenarios where interest groups are ideologically aligned with particular parties or have strategic reasons to focus their efforts (such as gun control and abortion, discussed in depth later). The result of this is that we obtain a unique equilibrium and, thus, our model produces unique comparative statics that explain and match data. Grossman and Helpman (1996) instead allow interest groups to contribute to both parties. They find that there is equilibrium multiplicity because interest groups invest in the party they think is more likely to win, which makes the party even more likely to win, leading to a self-fulfilling prophecy where neither the amount of campaign spending nor which party this money goes to are uniquely determined.

We consider the game played between two interest groups and two political parties facing off in an election. Parties want to win the election and, in order to do so, need the support of the median voter (the voter, henceforth). The voter can be influenced in two ways. First, the closer the party's position in the political spectrum to the voter's own position, the more likely the voter is to vote for that party. Second, the higher the campaign spending of one party relative to that of the other, the higher the chances that the voter votes for this party. Each party receives a contract from one of the interest groups specifying a campaign contribution in exchange for a certain position in the political spectrum. The parties' trade-off is that accepting the contract leads to higher campaign spending but a position that is further away from that of the voter. The interest groups' trade-off is that asking for a more polarized position reduces its party's chances of winning the election and, thus, the chances of securing a policy beneficial for the interest group. However, some of this effect can be mitigated by increasing its campaign contributions to the party.

The model described above leads to a two-stage, four-player game where in the first stage each interest group simultaneously offers a contract to a party, each interest group to a different party, and then in the second stage parties simultaneously and without knowing the contract offered to the other party decide whether to accept or reject the contract they received. We solve this game by finding its unique equilibrium and then we proceed to study how the different parameters in the model affect parties' polarization, campaign spending, and voter's welfare, among others.

In our results about polarization, we find that the interest group that has a higher stake in the election (the high-valuation interest group, henceforth) forces its party to adopt a more polarized position than the other party.[2] Although a more polarized position decreases voter support, this can be partly compensated by a higher campaign spending, which the high-valuation interest group is willing to fund. The low-valuation interest group cannot afford to compete in the campaign spending dimension and, thus, asks its party for a less polarized position. Moreover, we also find that as policy salience increases, parties become less polarized. This is because as policy salience goes up, campaign spending becomes less effective at swaying the voter.

Finally, we show that uncertainty about the voter's behaviour has an asymmetric effect on polarization. On the one hand it increases polarization for the party that gets offered the contract from the higher valuation interest group, while on the other hand it decreases polarization of the other party. This is because as uncertainty about the voter goes up, adopting a more polarized position becomes less risky. The high-valuation interest group takes advantage of this by asking its party for a more polarized position, while the low-valuation interest group instead allows its party to become less polarized to be in a better position against the now more polarized opposing party.

In terms of campaign spending, we find that the high-valuation interest group contributes more to campaign spending than the other interest group. Moreover, increasing an interest group's valuation increases its campaign spending offer. However, the effect of this increase on the amount of campaign spending offered by the other interest group is ambiguous. Increasing the valuation of one interest group increases this interest group's spending offer, which initially makes the other interest group offer more spending to fight this increase off. However, as the valuation of the interest group whose valuation increases goes up, the other interest group finds it harder to compete in terms of spending and instead switches competition to the policy space. Furthermore, we also find that a higher policy salience decreases total campaign spending as more salience means campaign spending is less effective. In relative terms, however, the high-valuation interest group offers more campaign spending to its party with respect to the low-valuation interest group. This is because, as salience goes up, the

[2] We define polarization as the distance between a party's policy position and the voter's ideal point. Polarization also measures how far away parties are from each other and from the voter's ideal point in the political spectrum.

low-valuation interest group offers less campaign spending to its party, which makes every unit spent on campaign spending more effective. On top of that, we find that uncertainty about the voter also increases the relative spending of the party associated with the high-valuation interest group. This is because, as discussed in the previous paragraph, increasing uncertainty increases the polarization of the party associated with the high-valuation interest group, which implies that now this interest group has to compensate its party by offering more funds proportional to the funds the other interest group offers. Finally, as uncertainty increases total spending goes down, as more uncertainty means that campaign spending is less useful at swaying the voter.

We also find that if the two interest groups have different valuations, the party influenced by the high-valuation interest group receives greater financial support, but in return, adopts a less electorally viable position, resulting in a reduced probability of winning. The reason is that there is some overlap in the preferences of an interest group and its party as they both want the party to win the election, the party because that is what it cares about and the interest group because if its party wins then the policy implemented will be closer to its ideal policy. However, this overlap in preferences is not perfect, as the interest group cares about the policy that is implemented, but the party does not have an intrinsic preference about policy. Thus, although the party is willing to exchange a policy away from the voter in return for campaign funds, as long as its probability of winning the election remains unchanged, the interest group does not want to increase the probability that its party wins the election. Therefore, the low-valuation interest group can only offer a marginal campaign investment to its party and in exchange asks for a policy that is not very polarized, i.e. close to the voter. The party with a high-valuation interest group's outside option is thus to compete against a party that is close to the median voter with some funds for campaign spending, which leads the party with a high-valuation interest group to have a low probability of winning the election if it does not accept its interest group's contract. Since, as we shall argue, in equilibrium, parties' participation constraint binds, i.e. they expect the same probability of winning with and without accepting the contract offered by the interest group, the high-valuation interest group can take advantage of this by offering a contract where policy platform is far away from the median voter but where the compensation in campaign contributions is not large, yet higher than the campaign contributions made by the other interest group. This leads to a situation where the party with a higher campaign spending actually enjoys a lower probability of winning the election.

From the point of view of welfare measured as the utility of the voter, we find, among others that competition between interest groups, i.e. when both have similar valuations, minimizes welfare. In a nutshell, the reason for this is that when interest groups' valuations are uneven, the low-valuation interest group cannot offer as much funding as the other interest group, and, as a result, asks for a less polarized position but enjoys a higher probability of winning the election as discussed above. On the other hand, when both interest groups have a similar valuation, they face the same incentives, which leads to both of them having a similar level of polarization and similar probability of winning the election. This effect makes it so that welfare is lower than when interest groups have different valuations. In terms of the effect of salience of the election on welfare, higher salience translates into higher welfare. This is because higher salience makes campaign spending less effective and as a result parties become less polarized. Finally, we find that higher uncertainty about the voter decreases welfare.

We believe our model can help explain certain patterns observed in the US interest groups' industry. For example, looking at the issue of gun rights and gun control in the US, we find that there are two different interest groups. On the one hand, there is the gun rights interest group, which in the 2013-2014 election cycle spent in Congress over \$3.2 million, over 97% of this amount going to Republican candidates. On the other hand, there is the gun control interest group, which spent less than \$0.01 million, its entirety to Democratic candidates.[3] The attitude of Republican candidates towards gun rights is such that more than 98% of the House members reject stricter gun controls, while 90% of Democrats support stricter gun controls.[4] However, voters in America seem to side with the gun control interest group. In particular, in 2013, 55% of Americans favoured stricter gun controls, while only 6% were in favour of less strict gun controls.[5]

Our model can accommodate the situation above. There are two interest groups, each influencing a different party. The gun rights interest group has much higher valuation than the gun control interest group.[6] As a result, they spend more and also force a more polarized position on their party: Republicans favour gun rights yet this is not in line with the voter. Our model replicates this outcome, gives an explanation of how more valuation translates into more polarization, and also helps explain other phenomena, like what would be best from the voter's point of view.[7] On top of that, our model also makes testable predictions in

[3]CRP with data provided by the Federal Electoral Commission. Here we report interest groups' money that is spent in Congress as donations to candidates. The figures for total interest groups' efforts are \$17.3 million for pro-gun and \$4.2 million for gun control in 2013 and 2014.

[4]Retrieved from The Daily Beast with data from Census.gov, NRA, The New York Times, Project Vote Smart and Sunlight Foundation.

[5]From Gallup opinion piece of June 13, 2016. See also Bouton et al (2016).

[6]This may be for a variety of reasons, such as gun rights Americans being wealthier, more willing to spend money on their ideology, or the gun right interest group receiving support from gun manufacturers, like the National Rifle Association Institute for Legislative Action receiving funds from gun manufacturers Beretta, Smith & Wesson, and Ruger among others (see the report "Bloodmoney" by the Violence Policy Center).

[7]Note that although in our model we speak about the winner of the election, in this example it may be

the form of comparative statics that to our knowledge are new to the literature.

Another example where the model can be applied is the pro-life/pro-choice case. Pro-life interest groups spent over $0.7 million in the 2013-2014 election cycle, 98% of which went to Republican pro-life candidates, while the pro-choice interest group spent over $2.4 million, of which 97% was for pro-choice Democratic candidates.[8] Just as in the gun rights/gun control case, the voters' opinion seems to be in line with the opinion of the less polarized, Republican in this case, party: during the period 2013-2014, between 50-52% of Americans thought abortion should be legal only in some circumstances.[9]

The rest of the paper is organized as follows. Next, we present a review of the literature. In Section 2, we introduce the model while we calculate its unique equilibrium in Section 3. Our main results are presented as comparative statics in Section 4. In Section 5, we present a discussion on our assumptions. Finally, we conclude in Section 6. All mathematical proofs and all extensions to our main analysis are presented in the appendix.

1.1 Related Literature

There is a vast literature spanning several decades on the effects of interest groups and campaign contributions in political outcomes. Next, we summarize the subset of this literature that is related to our paper and elaborate on why our work is novel.

Grossman and Helpman's (1996) paper is perhaps the closest in the literature to our work. There are two main differences between their work and ours, in particular in the way interactions and preferences are modelled. The first main difference is that in their paper each interest group can offer contributions to both parties, while in our paper each interest group offers a contribution to one party, with each interest group contributing to a different party. This crucial difference has a big impact on the results. Namely, in Grossman and Helpman (1996), there is equilibrium multiplicity because interest groups invest in the party they think is more likely to win, which makes the party even more likely to win, leading to a situation where all interest groups invest more in one party but how much more and the identity of this party depends on the particular equilibrium, i.e. there is a self-fulfilling prophecy where neither the amount of campaign spending nor which party this money goes to are uniquely determined. By restricting each interest group to offer a contract to only one party, and this party being different for every interest group, we do not run into this

more appropriate to speak about winning support on an issue affecting some voters, not the whole election.

[8]CRP with data provided by the Federal Electoral Commission. As before, we report interest groups' money that is spent in Congress as donations to candidates. The figures for total interest groups' efforts are $1.5 million for pro-life and $4.3 million for pro-choice in 2013 and 2014.

[9]From Gallup in Depth: Abortion.

problem. Indeed, in our model, there is a unique equilibrium, which means that we can carry out equilibrium comparative statics and formulate meaningful testable predictions. Which setting is a better description of the real world depends on the particular case at hand. As we argued above, for the gun rights/gun control and pro-life/pro-choice cases, among others, it makes more sense to model interest groups the way we do in this paper. In other areas, however, it may make more sense to have all interest groups contributing to all parties.

The second main difference between Grossman and Helpman (1996) and our paper is that in their model, the campaign spending of a party affects the voter in a linear way, while in our model it affects the voter in a proportional way with respect to the campaign spending of both parties. We believe a proportional effect has better features because it allows us to study the effect of policy salience in polarization, without having to worry about the fact that salience itself may be endogenous to campaign spending. On top of that, the fact that campaign spending affects the voter in a linear way in Grossman and Helpman (1996) implies that in their model, the objective function of each party is additively separable in its own policy/campaign spending and that of the other party. In our model, proportional spending means that contributions offered by one interest group are not additively separable from the contributions made by the other interest group and vice versa. Moreover, most of the previous literature also uses a proportional formulation. As Grossman and Helpman (1996, footnote 6) themselves write, "It is perhaps more common in the literature to assume that the ratio of campaign expenditures affects the allocation of voters.".

Other previous papers study campaign spending as a tool to inform voters about the parties' ideological positions. Austen-Smith (1987) considers a probabilistic voting model where parties compete in an election in which risk-averse voters are uncertain about the parties' position in the political space. Parties can reduce this uncertainty via campaign contributions, which are obtained from interest groups. Interest groups choose whether to contribute or not to parties after they have announced their policy positions. Baron (1994) extends this model by distinguishing between particularistic and collective policies. Grossman and Helpman (1994) consider the effects of interest groups on the party already in power (as do Schneider (2012) and Klingelhöfer (2013)), instead of on the election itself as we do. Grossman and Helpman (1999) focus on endorsements as a way of transmitting information to voters. In Besley and Coate (2001) the winner of the election can be influenced by offering direct payments only after the election. Coate (2004) studies the effect of campaign limits on welfare. Ashworth (2006) considers incumbency advantage in fundraising. Felli and Merlo (2006) consider endogenous interest groups and find that interest groups reduce polarization. As we discuss later on, this is in contrast to previous theoretical and empirical studies (see, for example, Austen Smith (1987) for theoretical evidence and Woll (2013) for empirical evidence), and also the opposite of what we find in our model.

The main difference between these models and ours is that in our paper, as in the seminal work of Grossman and Helpman (1996), interest groups directly influence policy by offering a contract that specifies a campaign contribution and a policy position, instead of parties choosing a position and then interest groups choosing how much to contribute to the parties based on the position they adopted. Furthermore, in our model there is perfect information about the parties' policies, and campaign contributions affect the voter per se because it is a tool for marketing (see Jacobson (1978), Gerber (2002) or Gerber (2004) and references therein). We present a more thorough discussion on this in the next paragraph and in section 5.

Another strand of literature considers campaign advertising as providing information about a candidates' non-policy variable (valence) (see Potter (1997) and Prat (2002a)). Prat (2002b) considers a model where a large number of interest groups (lobbies) compete in different policy dimensions. The main difference between his and our model is that in our model there is competition between interest groups; in Prat (2002b), as in perfectly competitive markets, a single interest group's action does not affect the actions of the other interest groups. In our model, as in markets with a duopoly, the opposite is true. This is one of the reasons why we assume that in our model voters have perfect knowledge about the parties. As Prat (2002b) writes: "each lobby is small enough to take (the probability with which a party wins) as given in equilibrium. Without this feature, a multi-interest group model combined with candidate signalling would be intractable".[10] In Prat (2002b), therefore, each interest group does not consider the effects of its actions on the actions of the other interest groups. We assume instead that each interest group internalizes the effect of its actions but, on the other hand, we drop candidate signalling from our model. This leads to an interesting problem in our setting: when an interest group offers a contract to its party, it has to take into account that the participation constraint of its party depends on the contract the other party has been offered by the other interest group, which in equilibrium also depends on the contract the interest group itself offers.

Other papers that study the problem of uncertainty with interest groups are Martimort and Semenov (2008) and Buzard and Saiegh (2016), who consider a model of interest groups where the ideological position of the politician is uncertain, and Felgenhauer (2010), who studies the effects of transparency in how interest groups can access information. As opposed to our model, neither of these papers model the political competition happening during elections as they all consider lobbying on already elected legislators. On top of that, we consider uncertainty in the voter's preferences, not the politicians' (see also Calvert (1985)). Other

[10]See Prat (2002b) page 168, third paragraph and Assumption 1 in page 171.

papers in this strand of literature are Esteban and Ray (2006), Bennedsen and Feldmann (2006), Martimort and Stole (2015), Lefebvre and Martimort (2017), and Schnakenberg and Turner (2018).

A paper related to our work is that of Hirsch (2023), who studies a model of electoral competition with campaign spending where the parties have extreme and opposite views but there are no interest groups. We instead consider parties that are office-motivated but trade off extreme views with the interest group's support in the form of campaign spending. Moreover, contrary to Hirsch (2023), we find that the party that exhibits the most campaign spending has a lower probability to win given the extreme policy it chooses. Hirata and Kamada (2020) consider a setting where parties choose policies first, and interest groups then decide how much to contribute. The timing of their model is the opposite to ours, which leads to an equilibrium where both parties' policies converge. We find that convergence only happens in cases where both interest groups are equally influential. Also related is Le and Yalcin (2018), who study a setting with one interest group who can influence both parties and find that the interest group will only try to influence the party whose ideology is closest to that of the interest group.

Previous work also connected to our paper includes the articles by Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), Zakharov (2009) and Serra (2010), who study models where candidates compete by choosing both policy positions and how much to invest in valence. Similarly, Groseclose (2001) studies a model where candidates compete in policy but have different valences (and Herrera et al. (2008) where there is electoral competition and campaign spending). In these papers, valence plays a similar role as campaign contributions, and as a consequence some of our results match what these authors find, such as the fact that high valence candidates (those with higher campaign contributions in our model) adopt more polarized positions. However, interest groups are absent from this literature. In our paper, campaign contributions come endogenously from the interest groups, who ask for a more favourable policy platform in return.

Finally, there is the work of Bils, Duggan and Judd (2017) who consider a dynamic model and study how polarization changes depending on how effectively interest groups can transfer money to politicians. They find that the more effective money transfers are, the higher the polarization in equilibrium. We obtain a similar observation in our static model: if money is more useful (in our model, this is due to policy salience being low) then parties are more polarized in equilibrium. A difference between Bils, Duggan and Judd (2017) and our paper is that we derive a battery of comparative statics that, to our knowledge, are new in the literature.

2 The Model

2.1 Parties and Representative Voter

There are two political parties labelled L and R. There is a representative voter (henceforth the voter) with an ideal position 0 in the $\mathbb{R}$ political spectrum. The voter evaluates two factors when choosing which party to vote for. First, the voter cares about how close the party's political position is to their own. Second, the voter can be influenced via campaign spending, so that the more a party spends during a campaign relative to the other party's spending, the higher the likelihood that the voter votes for that party. In particular, we assume that the utility the voter receives from voting for party $p \in \{L, R\}$ with political position $y_p \in \mathbb{R}$ and campaign spending $t_p \geq 0$ is given by

$$u(y_p, t_p) \quad = \quad -\lambda\, |y_p| + (1-\lambda)\frac{t_p}{t_p + t_{-p}} - \varepsilon \mathbb{1}_{p=L}. \tag{1}$$

The variable $t_{-p} \geq 0$ is the campaign spending of the other party (i.e. $-p \in \{L, R\} \setminus \{p\}$). Note that in a slight abuse of notation, we are omitting the argument t_{-p} in u. We assume that if both parties spend zero campaign spending then $\frac{t_p}{t_p+t_{-p}} = \frac{1}{2}$.

The parameter $\lambda \in [0, 1]$ represents how important political stance is relative to campaign spending. We interpret λ as policy salience; higher λ means that it is harder to sway the voter using campaign spending and easier to convince them to vote for a certain party by choosing a political stance closer to their opinion.

The parameter ε represents an uncertain partisan bias and implies that there is aggregate uncertainty about the preferences of the voter. We have that ε is distributed uniformly in $[-\gamma, \gamma]$. Ceteris paribus, the lower the value of ε the higher the chances that the voter votes for party L. The parameter $\gamma > 0$ represents how uncertain parties are about the voter. A technical assumption is the following:

Assumption. *There is sufficient uncertainty about the voter. In particular,* $\gamma > 3(1 - \lambda)$.

The voter votes for party L if and only if $u(y_L, t_L) \geq u(y_R, t_R)$. As we argue later, we assume without loss of generality that $y_L \leq 0 \leq y_R$, which implies that the voter votes for party L if and only if:

$$\varepsilon \quad \leq \quad \lambda(y_L + y_R) + (1-\lambda)\frac{t_L - t_R}{t_L + t_R}.$$

Note that in case of indifference we assume that the voter votes for party L. This has no effects on our results as the chances that the voter is indifferent between the two parties is zero. We have then that the probability that party L wins the election is given by

$$\text{Prob}_L\left(y_L, t_L, y_R, t_R\right) = \frac{\lambda(y_L + y_R) + (1-\lambda)\frac{t_L - t_R}{t_L + t_R} + \gamma}{2\gamma}, \tag{2}$$

while the probability that party R wins is

$$\text{Prob}_R\left(y_L, t_L, y_R, t_R\right) = \frac{\gamma - \lambda(y_L + y_R) - (1-\lambda)\frac{t_L - t_R}{t_L + t_R}}{2\gamma}. \tag{3}$$

Technically speaking, the probability with which party L wins the election is

$$\min\{\max\{\text{Prob}_L\left(y_L, t_L, y_R, t_R\right), 0\}, 1\},$$

and similarly for party R. However, as we shall see later on, in equilibrium, the expression $\text{Prob}_L\left(y_L, t_L, y_R, t_R\right)$ is always in $(0, 1)$ and thus we can save on notation by omitting the min and max functions.

2.2 Interest Groups

There are two interest groups, labelled l and r, with ideal positions on the political spectrum given by $-\infty$ and ∞, respectively. Each interest group $b \in \{l, r\}$ tries to influence political parties by offering a contract (y_b, t_b) that specifies a position in the political spectrum $y_b \in \mathbb{R}$ and a transfer $t_b \geq 0$. If a party accepts the contract, then this party chooses platform y_b and receives a monetary transfer to spend on campaign spending t_b.

We assume that interest group l offers its contract to party L and interest group r offers its contract to party R. In our model, parties have no budget; if a party does not accept the contract offered by its respective interest group, then it does not have funds to spend on campaigning in the election.

The profit of each interest group depends on how close the implemented policy is to its ideal position, minus the cost of the campaign contributions. In particular, the profit of interest group l is given by

$$\pi_l\left(y_L, t_L, y_R, t_R\right) = -v_l\left[y_L \text{Prob}_L\left(y_L, t_L, y_R, t_R\right) + y_R \text{Prob}_R\left(y_L, t_L, y_R, t_R\right)\right] - t_L, \tag{4}$$

where $v_l > 0$ is how much the interest group values the election. As in Prat (2002b), this parameter can be viewed as the interest group's fundraising ability.

Similarly, the profit of interest group r is given by

$$\pi_r\left(y_L, t_L, y_R, t_R\right) = v_r\left[y_L \text{Prob}_L\left(y_L, t_L, y_R, t_R\right) + y_R \text{Prob}_R\left(y_L, t_L, y_R, t_R\right)\right] - t_R, \tag{5}$$

with $v_r > 0$.

2.3 Timing and Equilibrium Concept

The timing of the game is as follows:

- **Stage 1:** Each interest group $b \in \{l, r\}$ simultaneously offers a contract (y_b, t_b) to their respective party.

- **Stage 2:** Without knowing the contract offered to the other party, each party $p \in \{L, R\}$ simultaneously decides whether to accept the contract offered by their interest group or not.

- **Stage 3:** Each party that accepts their interest group's b contract chooses position y_b and campaign spending t_b, the parties that do not accept their interest group's contract choose any position in the political spectrum.

- **Stage 4:** Given party positions and campaign spending, nature draws the value of ε, a winner of the election is declared and payoffs are realized.

Note that if a party rejects the contract offered by the interest group, then it is free to choose any position in the political spectrum. If the party is free to choose any policy position, it is a strictly dominant strategy to target the ideal policy of the voter, i.e. to choose position 0. Given this, we continue our analysis assuming without loss of generality that if a party rejects the contract offered by the interest group, then it chooses position 0.

The equilibrium concept we use is the Sub-Game Perfect Nash Equilibrium (equilibrium for short), where an equilibrium is given by the tuple $((y_l, t_l), (y_r, t_r), A_L, A_R)$ where (y_l, t_l) and (y_r, t_r) are the contracts offered by interest group l and r, respectively, and for $p \in \{L, R\}$ the function $A_p : \mathbb{R} \times \mathbb{R}^+ \to \{\text{accept}, \text{reject}\}$ determines whether party p accepts or rejects a given contract, such that:

- The position and campaign expenditure of parties L and R are given respectively by

$$(y_L, t_L) = \begin{cases} (y_l, t_l) & \text{if } A_L(y_l, t_l, y_R, t_R) = \text{accept}, \\ (0, 0) & \text{otherwise.} \end{cases}$$

$$(y_R, t_R) = \begin{cases} (y_r, t_r) & \text{if } A_R(y_L, t_L, y_r, t_r) = \text{accept}, \\ (0, 0) & \text{otherwise.} \end{cases}$$

- Given the position and campaign spending adopted by party R, A_L is such that party L maximizes profit by accepting a contract if and only if its profit is at least as high as its profit when choosing position in 0 and no campaign spending. That is, given (y_R, t_R),

$$A_L(y_l, t_l) = \begin{cases} \text{accept} & \text{if } \pi_L(y_l, t_l, y_R, t_R) \geq \pi_L(0, 0, y_R, t_R), \\ \text{reject} & \text{otherwise.} \end{cases}$$

- Given the position and campaign spending adopted by party L, A_R is such that party R maximizes profit by accepting a contract if and only if its profit is at least as high as its profit when choosing position 0 and no campaign spending. That is, given (y_L, t_L),

$$A_R(y_r, t_r) = \begin{cases} \text{accept} & \text{if } \pi_R(y_L, t_L, y_r, t_r) \geq \pi_R(y_L, t_L, 0, 0), \\ \text{reject} & \text{otherwise.} \end{cases}$$

- Given the position and campaign spending adopted by party R and the conditions under which party L accepts a contract, interest group l maximizes profit by offering contract (y_l, t_l). That is, given (y_R, t_R) and A_L,

$$(y_l, t_l) = \text{argmax}_{(y,t)} \begin{cases} \pi_l(y, t, y_R, t_R) & \text{if } A_L(y, t) = \text{accept}, \\ \pi_l(0, 0, y_R, t_R) & \text{otherwise.} \end{cases}$$

- Given the position and campaign spending adopted by party L and the conditions under which party R accepts a contract, interest group r maximizes profit by offering contract (y_r, t_r). That is, given (y_L, t_L) and A_R,

$$(y_r, t_r) = \text{argmax}_{(y,t)} \begin{cases} \pi_r(y_L, t_L, y, t) & \text{if } A_R(y, t) = \text{accept}, \\ \pi_r(y_L, t_L, 0, 0) & \text{otherwise.} \end{cases}$$

2.4 Comments on Modelling Choices

Note that our model is a probabilistic voting model where the voter has an uncertain bias towards either party. For more on probabilistic voting models and their relation with the median voter theorem, see Schofield (2007).

Campaign spending affects the utility of the voter in our model because they are, using the terminology of Helpman and Grossman (1996), impressionable. The fact that campaign spending enters proportionally in the utility function follows from past literature (see for instance Snyder (1984) and Baron (1989, 1994)). We discuss in more detail this and other aspects of our assumptions including our linear specification of the utility function of the voter in section 5. On top of that, we also solve the model for other functional forms of the utility function in appendix A2.

The convenience Assumption 1 is that it leads to a unique equilibrium as we show later on. Nevertheless, in appendix A3 we study situations where this assumption is not met and show that our main conclusions still hold true.

Notice that the voter only cares about how distant the implemented policy is to their ideal point but not in which direction this distance is measured. Thus, for instance, if party L chooses any position x, then the utilities of all parties and the voter are the same as if party L chose instead position $-x$. Given this, we assume without loss of generality that, in equilibrium, party L chooses a position in $(-\infty, 0]$ while party R chooses a position in $[0, \infty)$.

Parties do not have a preference over the political spectrum; they only care about the probability of winning the election. We could relax this assumption, but doing so would complicate the analysis that follows without changing our main results. On top of that, this assumption allows us to conclude that any polarization we observe in equilibrium comes from the interest groups' influence. The profit of party $p \in \{L, R\}$ is thus given by $\text{Prob}_p\,(y_L, t_L, y_R, t_R)$.

In terms of the interest groups, note that an interest group's preferred policy could be finite. As long as such finite quantity is large enough, this would not change any of our results. Furthermore, we could assume that instead of a single contract (y_b, t_b), each interest group offers a menu of contracts $(y_b, t_b(y_b))$ for all $y_b \in \mathbb{R}$. However, given that each interest group only offers a contract to its party and not to the other party, the single contract assumption is without loss of generality. This is because the interest group can just offer the contract (y_b, t_b) that maximizes its profit from the menu of optimal contracts. This contract is unique because there exists a unique contract that is a best response to the policy choice and campaign spending of the other party, as we show in the proof of Theorem 1.

Moreover, we could have assumed that parties have a fixed budget than can be topped up by the interest groups' contributions, but this will only complicate the exposition without adding any new insights. Furthermore, note that a budget for campaign spending is not required to have a positive (and potentially high) probability of winning the election.

3 Equilibrium

In order to calculate the equilibrium of the game, we solve the game backwards. First, we calculate the participation constraints in Stage 2. Second, given this information we then calculate the optimal contracts offered by the interest groups in Stage 1.

3.1 Parties' Participation Constraint

The contract offered by the interest groups must satisfy the participation constraint of the parties; otherwise, such contract is not accepted. In order to study the participation constraint, we must first find out what the outside option of the parties is. The profit of party L if it rejects the contract of the interest group when party R chooses position y_R and campaign spending $t_R > 0$ is given by

$$\begin{aligned} \text{Prob}_L\left(0, 0, y_R, t_R\right) &= \frac{\lambda y_R + (1-\lambda)\frac{0-t_R}{0+t_R} + \gamma}{2\gamma} \\ &= \frac{\lambda y_R - (1-\lambda) + \gamma}{2\gamma}. \end{aligned}$$

Therefore, the participation constraint of party L given contract (y_l, t_l) is

$$\text{Prob}_L\left(y_l, t_l, y_R, t_R\right) \geq \frac{\lambda y_R - (1-\lambda) + \gamma}{2\gamma}.$$

This implies

$$\begin{aligned} \lambda(y_l + y_R) + (1-\lambda)\frac{t_l - t_R}{t_l + t_R} &\geq \lambda y_R - (1-\lambda), \\ y_l &\geq -\frac{1-\lambda}{\lambda}\frac{2t_l}{t_l + t_R}. \end{aligned} \tag{6}$$

Similarly, the participation constraint of party R is given by

$$\text{Prob}_R\left(y_L, t_L, y_r, t_r\right) \geq \frac{\gamma - \lambda y_L - (1-\lambda)}{2\gamma}.$$

This means

$$y_r \leq \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L + t_r}. \tag{7}$$

Note that both participation constraints above, in equations (6) and (7) seem to depend only on the campaign spending of the other party, not on the other party's position. As we shall see later on, in equilibrium, these two magnitudes are related in a unique manner and, thus, the participation constraint of one party does indeed depend on the position of the other party.

If one of the parties chooses zero campaign spending, then the participation constraint of the other party is slightly different from the ones computed above. However, we do not need to consider this case because, as we show in Appendix A1, each interest group will always find it optimal to offer a contract with a positive campaign contribution, ensuring that its party always finds it optimal to accept the contract offered.

3.2 Interest Groups' Problem

Each interest group offers a contract in order to maximize its profit. In Appendix A1, we show incentive compatibility for the interest groups, i.e. both interest groups are better off by offering a contract where the party's participation constraint is satisfied than by not offering a contract (or offering one where the party's participation constraint is not satisfied). Thus, we proceed in this section by considering the case where interest groups offer a contract such that the participation constraint of their respective party holds.

By backwards induction, given position y_R and campaign spending t_R of party R and the participation constraint of party L in equation (6), interest group l offers contract (y_l, t_l) in order to maximize its profit in (4). That is, if we abuse notation by writing P_L instead of $\text{Prob}_L(y_l, t_l, y_R, t_R)$, interest group l solves:

$$\left.\begin{array}{ll} \max_{(y_l,t_l)} & -v_l\left(y_l P_L + y_R(1-P_L)\right) - t_l \\ \text{subject to:} & y_l \geq -\frac{1-\lambda}{\lambda}\frac{2t_l}{t_l+t_R}. \end{array}\right\}$$

Notice that we are not requiring $y_l \leq 0$ as it is never optimal for interest group l to offer any contract with $y_l \geq 0$, since for any such y_l, interest group l can always obtain a higher payoff by offering $y_l = 0$.

Similarly, we have that interest group r solves:

$$\left.\begin{array}{ll} \max_{(y_r,t_r)} & v_r\left(y_L P_L + y_r(1-P_L)\right) - t_r \\ \text{subject to:} & y_r \leq \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L+t_r}. \end{array}\right\}$$

3.3 Equilibrium Characterization

Solving the maximization problem of both interest groups (see Appendix A1) leads to our first result:

Theorem 1. *There exists a unique equilibrium. This equilibrium is such that interest group L offers contract (y_L, t_L) and interest group R offers contract (y_R, t_R) where both contracts are accepted and such that the participation constraint of both parties binds.*

The implicit equation for the equilibrium value of y_L is given by

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_L - (1-\lambda)}{\gamma + \lambda y_L + (1-\lambda)}\frac{-y_L}{\frac{2(1-\lambda)}{\lambda} + y_L} = 1.$$

The implicit equation for the equilibrium value of y_R is given by

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_R + (1-\lambda)}{\gamma + \lambda y_R - (1-\lambda)}\frac{\frac{2(1-\lambda)}{\lambda} - y_R}{y_R} = 1.$$

The equilibrium level of proportional campaign spending of interest group l, $x_L = \frac{t_L}{t_L+t_R}$, *is given implicitly by*

$$\frac{v_r}{v_l}\frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{\gamma - 2(1-\lambda)x_L + (1-\lambda)}\frac{x_L}{1-x_L} = 1.$$

Finally, the equilibrium total expenditure level is given by

$$t_L + t_R = v_r \frac{1-\lambda}{\lambda}\frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{\gamma} x_L.$$

It is possible to write the full specification of the equilibrium explicitly in closed form (the equilibrium values of the model's variables are given implicitly in equations (18), (19), (20) and (21) in Appendix A1). We have chosen not to do this given that the length and order of the expressions involved make the interpretation of the different equilibrium values futile. Nevertheless, such expressions are not needed for the analysis. In the next section, we carry out comparative statics on the equilibrium values as well as plot some numerical examples.

Theorem 1 states that the equilibrium is unique. Uniqueness is a desirable and convenient feature that allows us to focus the discussion that follows on the value of the different variables in equilibrium while we can safely ignore any coordination problems that could arise from equilibrium multiplicity.

According to theorem 1, in equilibrium both interest groups offer a contract that is accepted. This is because an interest group is always willing to offer a contract, as for any valuation, the increase in the profits of the interest group from possibly implementing a policy closer to the interest group's ideal position offsets the campaign costs in equilibrium. If the interest group's valuation is low, the other party will adopt a highly polarized position (as we show later on in Proposition 1). This has the effect of increasing the returns from offering funding: for the party associated with the low-valuation interest group, the loss in terms of probability of winning the election by moving away from the voter are low as the other party is itself further away from the voter and, thus, such a party will be willing to accept a low campaign contribution in exchange for such a move in the political spectrum. If the interest group's valuation is high, then such an interest group is willing to pay campaign contributions, as the potential benefit is high given the interest group's valuation.

The result in theorem 1 implies that both interest groups offer a contract that makes the parties' participation constraints bind in equilibrium. Thus, in Helpman and Grossman (1996) language, interest groups exhibit only an influence motive in equilibrium. Both the interest group and its associated party are interested in increasing the probability of winning the election. The party only cares about this magnitude, while the interest group also cares about the policy implemented and the amount of campaign spending. Thus, the higher the probability that the party wins the election, the better for the interest group, although this comes at a cost: less polarized policy and/or higher campaign spending. The reason why the interest group offers a contract delivering its party the same probability of winning as if a contract were not offered is that, because of the level of uncertainty about the voter (i.e. the parameter γ), the returns from increasing the probability of winning the election from the default no-contract level are low. In Appendix A3 we study what happens when uncertainty about the voter is low and show that our main conclusions from the comparative static analysis that follows below do not change significantly.

The fact that both interest groups offer a contract where the probability of winning the election for their party is no greater than in to the situation where they do not offer a contract has the interpretation that interest groups do not help their parties win the election but simply try to affect the policy implemented in a way that is beneficial to them. The way they do this is by offering campaign contributions, which are simply used to offset the negative effect of choosing a policy that is further away from the voter's ideal policy.

4 Comparative Statics

In this section, we perform comparative statics on the values of the different variables in equilibrium. We start each of the following sections with formal results about comparative statics (all of which are proven in Appendix A1), and then follow on with a graph illustrating these. Since most comparative statics are unambiguous and do not depend on the specific parameters used, graphs are indicative of not just the particular case they depict, but of the general behaviour of how the different equilibrium values respond to the parameters of the model.

Before we start with a full analysis of the equilibrium comparative statics, the following remark is in order:

Remark 1. *The party accepting the contract of the interest group with a higher valuation will spend more and adopt a more polarized position than the other party.*

The interest group with a higher valuation has more to gain from the election and, therefore, it offers a higher campaign contribution in equilibrium. Moreover, such an interest group also demands its party to choose a more polarized position. This is because by offering a higher campaign contribution, the interest group can afford to ask its party to move away from the voter while still having the same chances of winning the election.

4.1 Polarization

With respect to how polarized parties are, we have the following result:[11]

Proposition 1. ***Polarization***

Valuation Effect: *The higher an interest group's valuation, the more polarized its party will be and the less polarized the opposing party will be.*

Salience Effect: *The higher the policy salience, the less polarized parties will be.*

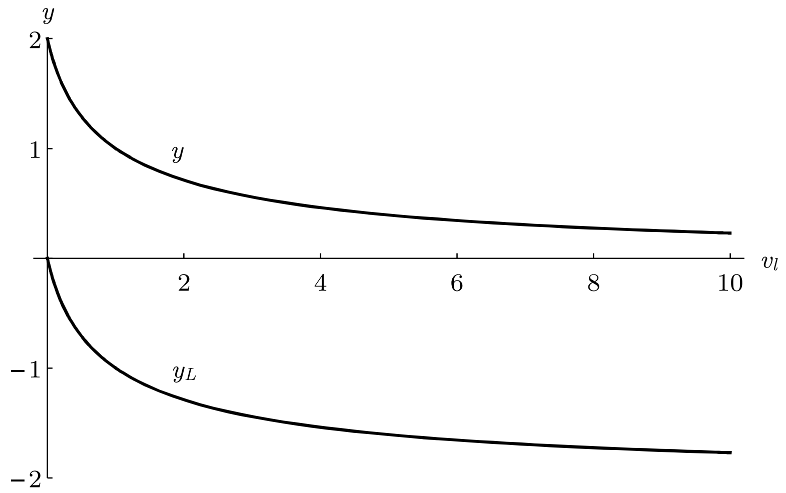
Uncertainty Effect: *The more uncertain the voter's preferences, the party whose interest group has the highest valuation becomes more polarized while the party whose interest group has the lowest valuation becomes less polarized.*

The Valuation Effect on polarization adds to Remark 1 that as one interest group increases its valuation, the opposing party becomes less polarized. This is because, as one party becomes more polarized, it also increases its relative campaign spending (as Proposition 3 shows later on). Thus, the opportunity costs of polarization increase for the other party because, on the one hand, an increase in relative campaign spending for one party means that the other party decreases its probability of winning the elections, which it can partly counter by moving closer to the voter's position. On the other hand, when a party becomes more polarized, the return from being closer to the voter increases as it becomes easier to compete in that dimension.

Figure 1 plots the effect of changing v_l on the equilibrium value of y_L and y_R holding all other parameters constant.

[11] As mentioned in the introduction, we define polarization as how far away from the voter's ideal policy a party's chosen policy is. However, since the voter's ideal position is between the chosen policies of both parties in equilibrium polarization also gives a measure of how far away parties are from each other in the political spectrum.

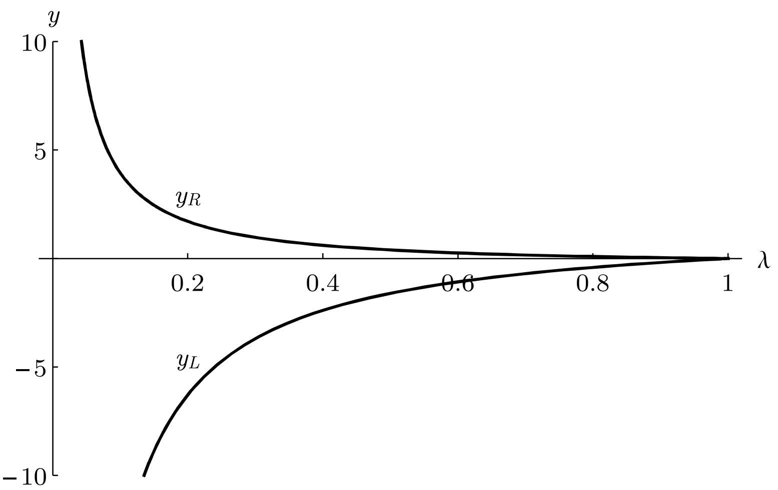
Figure 1: Valuation Effect - Polarization



Equilibrium values of y_L and y_R as v_l changes for $\lambda = \frac{1}{2}$, $\gamma = 3$ and $v_r = 1$.

The Salience Effect on polarization is such that as policy salience increases, parties becomes less polarized. This is because when policy salience increases, the voter becomes more concerned with the policy position of parties and less so with campaign spending. That is, campaign spending becomes less effective at swaying the voter and, thus, competition in the policy space becomes more fierce.

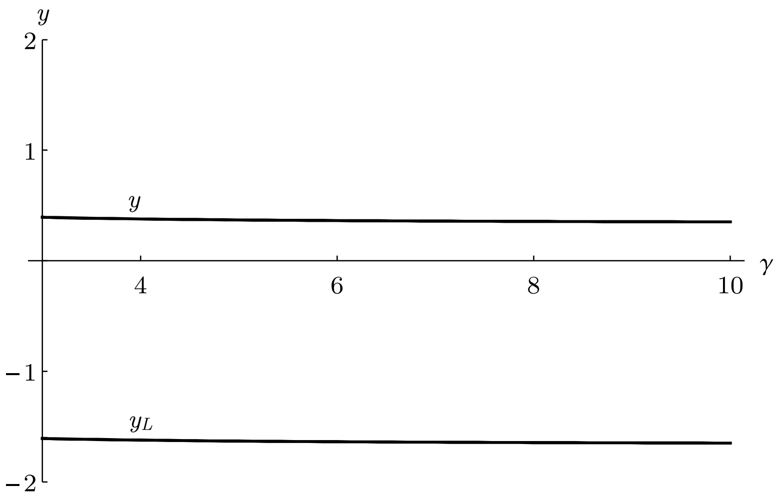
Figure 2: Salience Effect - Polarization



Equilibrium values of y_L and y_R as λ changes for $\gamma = 3$, $v_l = 5$ and $v_r = 1$.

Finally, the Uncertainty Effect on polarization implies that the interest group with a highest valuation forces its party to become more polarized, while the opposite happens for the interest group with the lowest valuation. This is because an increase in uncertainty makes adopting more polarized positions less risky. The interest group with a higher valuation offers its party a contract with a more polarized position, while the interest group with a lower valuation offers a contract asking its party for a less polarized position to, first, capitalize on the higher polarization of the other party and, second, to better compete against the higher relative campaign spending of the other party (we elaborate more on campaign spending later on).

Figure 3: Uncertainty Effect - Polarization



Equilibrium values of y_L and y_R as $\gamma > 3$ changes for $\lambda = \frac{1}{2}$, $v_l = 5$ and $v_r = 1$.

4.1.1 Expected Polarization

Apart from understanding the polarization exhibited by each party, it is also useful to understand how expected polarization, i.e. the ex-ante level of polarization of the winning party: $EP = P_L(-y_L) + P_R y_R$, changes. We have the following comparative statics:

Proposition 2. ***Expected Polarization***

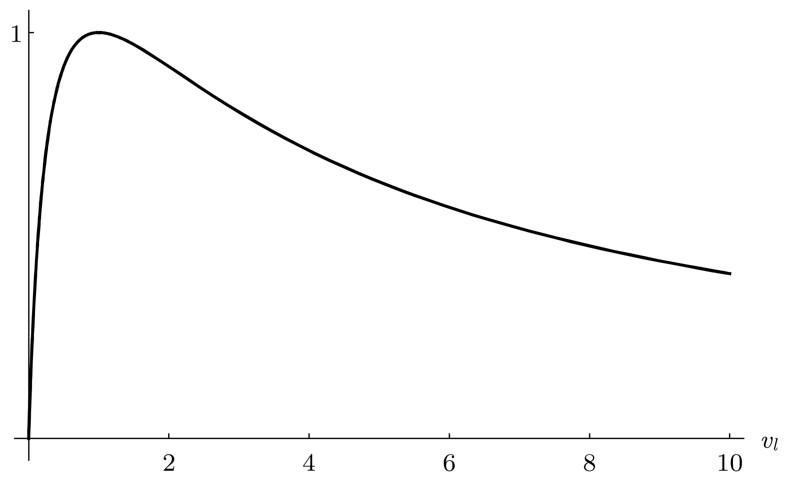
Valuation Effect: *The higher the differences in valuations, the lower the expected polarization.*

Salience Effect: *The higher the policy salience, the lower the expected polarization.*

Uncertainty Effect: *The higher the uncertainty about the voter, the higher the expected polarization.*

The Valuation Effect on expected polarization means that expected polarization is maximized when both interest groups have the same valuation. When both interest groups have the same valuation, their incentives are the same, i.e. they want a the same level of polarization given how much such polarization costs in campaign spending. If valuations are different, however, there is one interest group that asks for a more polarized position than the other interest group, but this interest group has a much lower probability of winning the election (because of being more polarized), and thus expected polarization goes down. Figure 4 illustrates the fact that expected polarization is maximized when both interest groups have the same valuation (i.e. $v_l = v_r$).

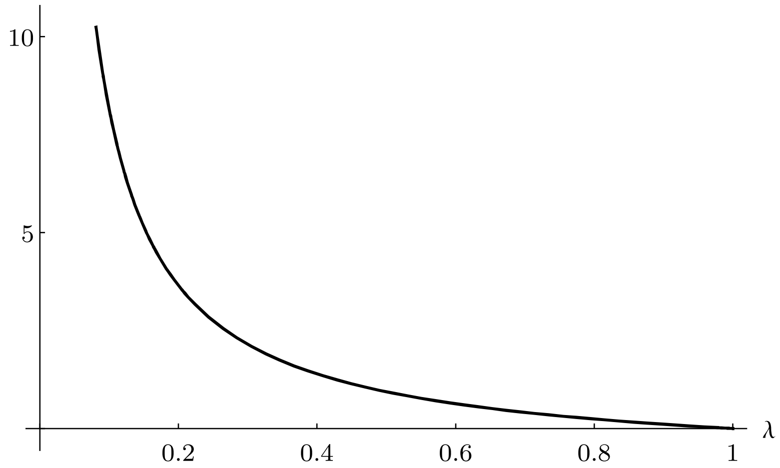
Figure 4: Valuation Effect - Expected Polarization



Equilibrium value of EP as v_l changes for $\lambda = \frac{1}{2}$, $\gamma = 3$ and $v_r = 1$.

The Salience Effect on polarization is such that in more salient elections, expected polarization goes down. More salience means that campaign spending is less useful at influencing the voter and, therefore, parties switch competition from spending to policy, i.e. they both choose a policy that is close to the voter, thus reducing expected polarization. This is in line with empirical evidence.

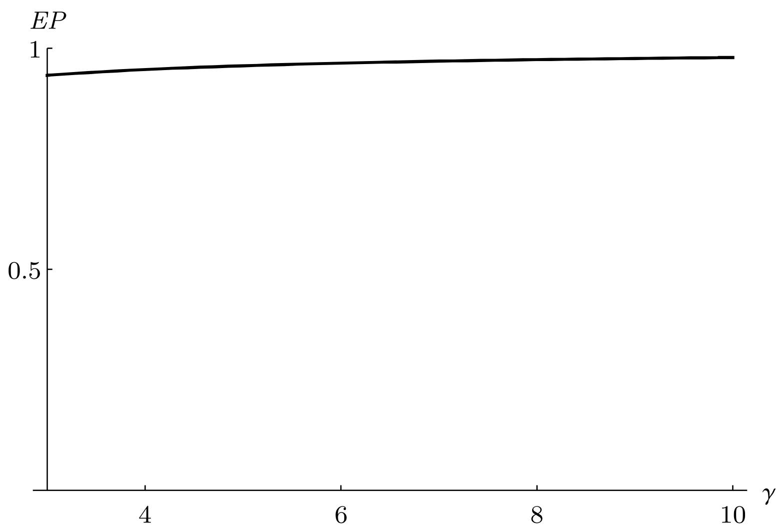
Figure 5: Salience Effect - Expected Polarization



Equilibrium value of EP as λ changes for $\gamma = 3$, $v_l = 5$ and $v_r = 1$.

The Uncertainty Effect on expected polarization implies that higher uncertainty about the voter increases expected polarization. As we saw in the previous section, when uncertainty goes up, the high-valuation interest group offers a more polarized contract, but the low-valuation interest group does the opposite. As the change in the probability with which each party wins the election becomes less sensitive to polarization the higher the uncertainty, the net effect of one party becoming more polarized and the other party becoming less polarized is that expected polarization goes up.

Figure 6: Uncertainty Effect - Expected Polarization



Equilibrium values of EP as $\gamma > 3$ changes for $\lambda = \frac{1}{2}$, $v_l = 5$ and $v_r = 1$.

4.2 Campaign Spending

In terms of campaign spending, we refer to the ratio of spending of one party by the sum of the campaign spending of both parties as relative campaign spending: $x_p = \frac{t_p}{t_p + t_{-p}}$ for each party $p \in \{L, R\}$. Absolute spending is the value of t_p, while total spending is $T = t_L + t_R$. We have the following comparative statics:

Proposition 3. ***Campaign Spending***

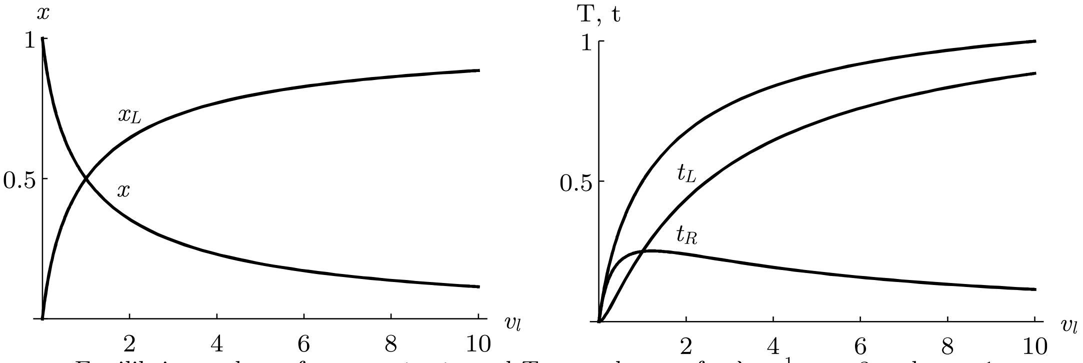
Valuation Effect: *When increasing an interest group's valuations, total spending increases, the party's relative and absolute campaign spending increases, the other party decreases its relative campaign spending, while its absolute campaign spending: (i) increases if its initial valuation is low and (ii) decreases if it is high.*

Salience Effect: *When increasing policy salience: total spending increases, the party whose interest group has a higher valuation increases its relative campaign spending and decreases its absolute campaign spending, the other party decreases its campaign spending both in relative and absolute terms.*

Uncertainty Effect: *When increasing uncertainty about the voter's preferences, total spending increases, the party whose interest group has the highest valuation increases its relative campaign spending, while the change in absolute spending is: decreasing if the difference in valuations is high and increasing if this difference is low, the other party decreases campaign spending both in relative and absolute terms.*

The Valuation Effect on campaign spending is such that as the valuation of an interest group increases, the relative and absolute campaign spending of its associated party increases. The relative campaign spending of the other party decreases, while the change on its absolute campaign spending depends on the valuation of the interest group whose valuation increases. When the valuation of an interest group increases, which causes this interest group to increase the campaign spending it offers to its party, the other interest group will ask for a less polarized position to counter this effect (Proposition 1), and will offer more campaign spending when its own valuation is close to or higher than the valuation of the interest group while it will offer less campaign spending when its own valuation is lower. That is, a strong interest group in terms of valuation will fight off an increase in the opposing interest group's campaign spending offer with an increase in its own campaign spending offer while a weaker interest group will actually offer less campaign spending and focus more on competing in the policy space (i.e. offering a contract that asks for a less polarized position). This effect can be seen in the plot of t_R on right-hand side of Figure 7.

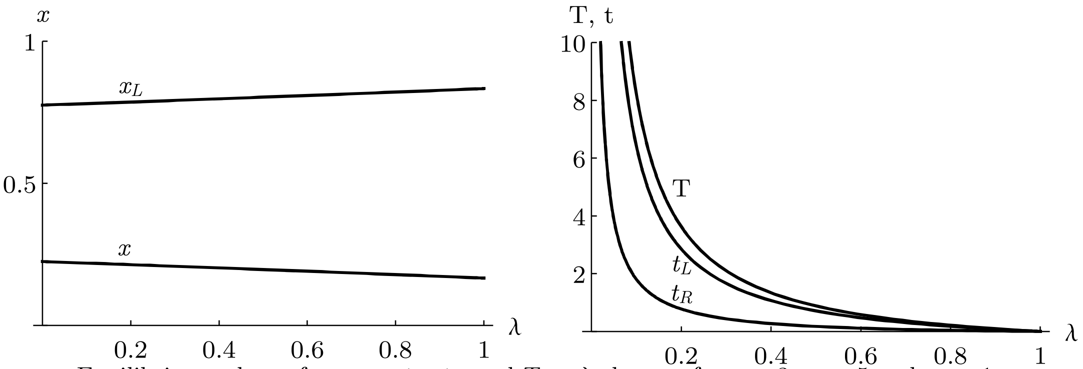
Figure 7: Valuation Effect - Campaign Spending



Equilibrium values of x_L, x_R, t_L, t_R and T as v_l changes for $\lambda = \frac{1}{2}$, $\gamma = 3$ and $v_r = 1$.

The Salience Effect on campaign spending means that as the election becomes more salient, total campaign spending decreases. For the interest group with the lowest valuation, this translates into both offering lower absolute and relative campaign spending. For the interest group with the highest valuation, its offer of relative spending increases and the absolute campaign spending decreases. Increasing salience makes campaign spending less useful in terms of swaying the voter which leads to a situation where both interest groups offer less campaign spending. However, the high-valuation interest group decreases spending less than the other interest group.

Figure 8: Salience Effect - Campaign Spending

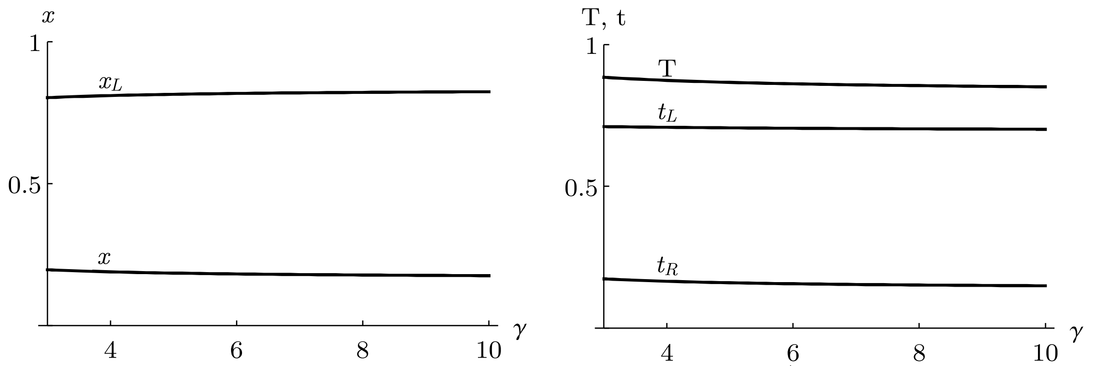


Equilibrium values of x_L, x_R, t_L, t_R and T as λ changes for $\gamma = 3$, $v_l = 5$ and $v_r = 1$.

The Uncertainty Effect on campaign spending implies that an increase in uncertainty about the behavior of the voter leads to the interest group with the highest valuation to offer more relative campaign spending, and the interest group with the lowest valuation to offer less of it. This is because uncertainty makes the interest group with the highest valuation to ask its party to adopt a more polarized position (see proposition 1) which then means that it has to compensate the party by increasing the relative campaign spending it offers. The relationship between uncertainty and total campaign spending is negative, which is in line with previous work, both theoretical (Martimort and Semenov (2008)) and empirical (Buzard and Saiegh (2016)), that finds that uncertainty about the voter decreases total spending.

In terms of the parties' absolute campaign spending, the campaign spending of the party associated with the low-valuation interest group decreases. The absolute campaign spending of the party associated with the high-valuation interest group also decreases but only when the valuation of its interest group is sufficiently high. In Figure 9 below we have that for $v_L = 5$ absolute campaign spending decreases but we have numerical examples for which $v_l > v_r$ yet t_L increases with γ.[12] When uncertainty increases, campaign spending becomes less useful as the voter's decision is influenced more by chance. However, when the absolute campaign spending of the other party decreases, the returns from increasing absolute campaign spending go up, more so when the interest group's valuation is low as this means total spending is also low. This is why an increase in uncertainty may lead to an increase in campaign spending for the party associated with the high-valuation interest group but only when such valuation is not too high.

Figure 9: Uncertainty Effect - Campaign Spending



Equilibrium values of x_L, x_R, t_L, t_R and T as $\gamma > 3$ changes for $\lambda = \frac{1}{2}$ and $v_l = 5$ and $v_r = 1$.

4.3 Welfare

Next, we study how interest groups affect the welfare of the voter. We define welfare W as the expected utility of the voter. Thus, from (1) we have

$$W = P_L\left(\lambda y_L + (1-\lambda)\frac{t_L}{t_L+t_R}\right) + P_R\left(-\lambda y_R + (1-\lambda)\frac{t_R}{t_L+t_R}\right).$$

Using the fact that $y_L = -\frac{1-\lambda}{\lambda}\frac{2t_L}{t_L+t_R}$ and $y_R = \frac{1-\lambda}{\lambda}\frac{2t_R}{t_L+t_R}$, we can rewrite the welfare in equilibrium as

$$W = -\frac{\lambda}{2}\left(P_L(-y_L) + P_R y_R\right). \qquad (8)$$

[12]For the exact expression for $\frac{\mathrm{d}t_L}{\mathrm{d}\gamma}$ see the proof of Proposition 3 in Appendix A1.

Notice that the expression $P_L(-y_L) + P_R y_R$ is simply expected polarization (EP). Hence, the formulation in (8) highlights the negative effects of interest groups on the voter, the higher the polarization caused by the interest groups the lower the welfare of the voter. We have the following comparative statics for welfare:

Proposition 4. ***Welfare***

Valuation Effect: *The higher the differences in valuations, the higher the welfare*

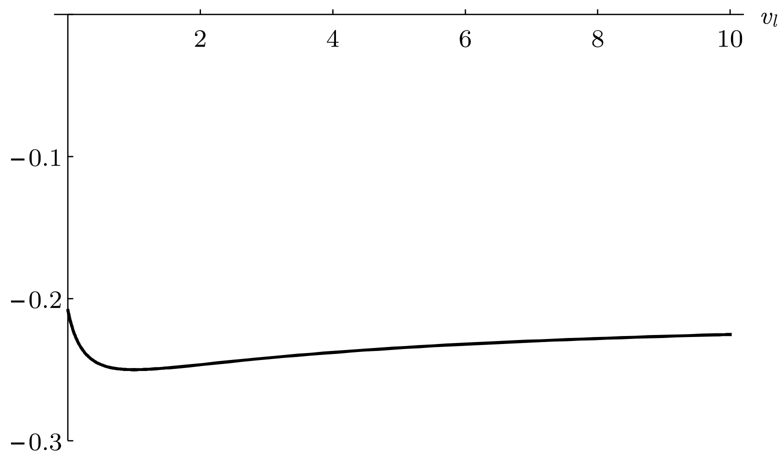
Salience Effect: *The higher the policy salience, the higher the welfare.*

Uncertainty Effect: *The higher the uncertainty about the voter, the lower the welfare.*

The Valuation Effect on welfare implies that, from the point of view of the voter, it is better to have an interest group with a higher valuation than the other interest group, rather than to have two interest groups with similar valuations. When one interest group dominates the other, in the sense that it has a higher valuation, the party associated with the low-valuation interest group has a higher chance of winning the election than the party associated with the high-valuation interest group. The high-valuation interest group can counter some, but not all, of this effect via campaign spending. There is the possibility that the election is won by the high-valuation, high-polarization interest group. However, since in equilibrium higher polarization means lower probability of winning the election, the chances of the highly polarized party winning are small. Thus, in this case, the voter faces a not very polarized party with a high chance of winning the election and a highly polarized party with a low chance of winning the election. Compare this with a situation where both interest groups have similar valuations: the voter faces two parties that are both relatively polarized and each with similar chance of winning the election.

Holding everything else constant but the valuation of one interest group, the voter's welfare is minimized when this valuation equals the other interest group's valuation. This can be seen both in equation (27) in Appendix A1, which is maximized when $\frac{t_L}{t_L+t_R} = \frac{1}{2}$ (this only happens if $v_l = v_r$ by Remark 1), and in Figure 10, where a minimum is reached at $v_l = v_r$.

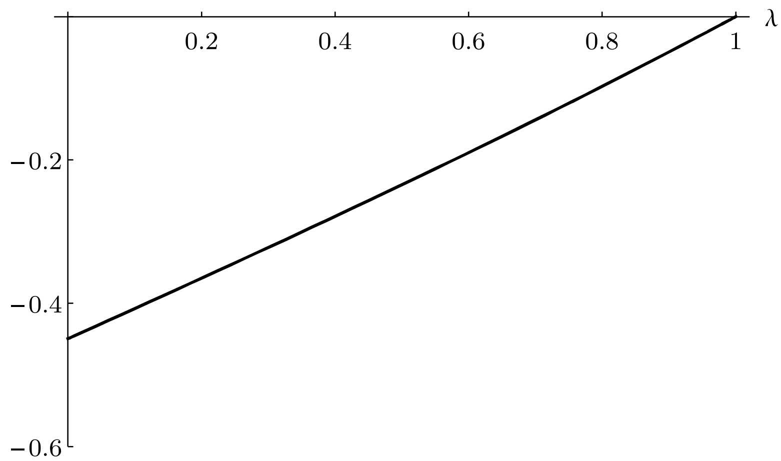
Figure 10: Valuation Effect - Welfare



Equilibrium value of W as v_l changes for $\lambda = \frac{1}{2}$, $\gamma = 3$ and $v_r = 1$.

The Salience Effect on welfare means that as the election becomes more salient, the welfare increases. The reason for this is that interest groups "buy" polarization via campaign spending. That is, they can offer contracts with polarized positions because they can counter the negative effect of this via campaign spending. If salience goes up, then campaign spending becomes less effective, and as a result, parties become less polarized. This increases the welfare of the voter.

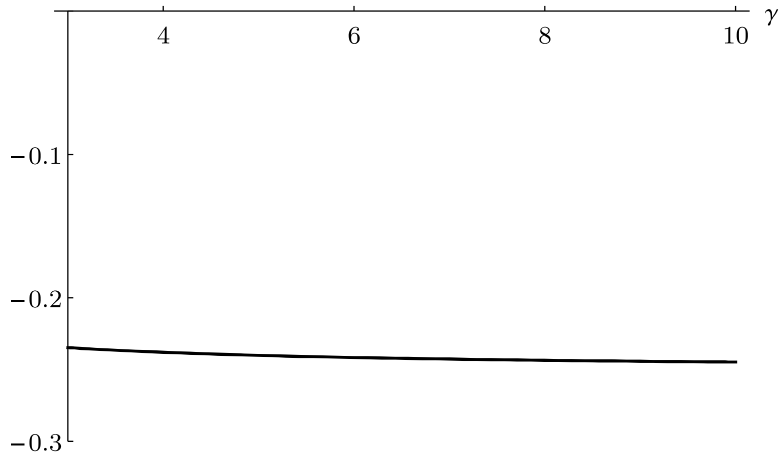
Figure 11: Salience Effect - Welfare



Equilibrium value of W as λ changes for $\gamma = 3$, $v_l = 5$ and $v_r = 1$.

The Uncertainty Effect on welfare means that as there is more uncertainty about the voter, their welfare goes down. As we argued above, more uncertainty means that one party becomes more polarized while the other becomes less polarized. However, more uncertainty also means that the probability with which each party wins the election is less responsive to changes in polarization. Thus, increasing uncertainty increases the gap in how polarized parties are, but the gap in the probabilities with which each party wins increases by a lower factor. Therefore, welfare decreases.

Figure 12: Uncertainty Effect - Welfare



Equilibrium value of W as $\gamma > 3$ changes for $\lambda = \frac{1}{2}$, $v_l = 5$ and $v_r = 1$.

5 Discussion on the Voter

The voter's behavior is characterized by their utility function in (1). This utility function has three important aspects worth discussing. First, is the fact the the voter cares about campaign spending. In this respect, we follow Helpman and Grossman (1996) in that voters are impressionable, i.e. they can be influenced by campaign spending per se. In line with previous empirical literature (see Jacobson (1978) or Gerber (1998) and references therein), this assumption is motivated by the fact that campaign spending by itself increases the likelihood that the voter votes for the party.[13] In this literature, campaign spending is thought of as a marketing tool that makes the party known and liked by the voter. Another strand of literature takes campaign spending as a tool to give information about the party to the voter (see, for example, Austen Smith (1987), Prat (2002b) and Ashworth (2006)

[13]Traditionally, the literature agreed that the effect of campaign spending on votes was greater for the challenger than for the incumbent (see, for example, Abramowitz (1988) and Jacobson (1990)). This has been shown to depend on how close the race is (Erikson and Palfrey (2000)) and, depending on the election, it has been shown not to be true at all (see Gerber (2002)). Levitt (1994) shows that campaign spending has little effect on electoral outcomes, a situation which can be modelled in our setting via assuming a high value for λ. However, recent evidence has questioned Levitt's (1994) finding (Gerber (2004)).

among others). Contrary to this second strand of literature, we do not specify how campaign spending affects the voter, and focus instead on the game played between the two interest groups and the two parties. As discussed in the introduction, this is motivated by the fact that we want to study the interaction between the interest groups when they both, together with the parties, determine the probability with which each party wins the election and they endogenize this when choosing their actions. As argued by Prat (2002b), a model with more than one interest group when each of them internalizes the fact that its action affects the probability with which a party wins and, thus, the actions of the other interest groups, together with voter's uncertainty about the parties, will be intractable. We chose to drop uncertainty from the picture.

Second, our specification of the utility function of the voter is linear in the policy space and proportional in campaign spending. There are two reasons why we chose such a specification. The first reason is so that the magnitudes of campaign spending do not matter. That is, if two parties spend the same, then regardless of what level of spending this is, the outcome is the same. This is motivated by the fact that we want to have a single measure of how important policy is relative to campaign spending, i.e. λ. If the effect of campaign spending were absolute then more campaign spending by both parties will make the election less salient, making it impossible for us to talk about how salience affects polarization and spending. Moreover, most of the previous literature also models the effect of campaign spending as proportional (see for instance Snyder (1984) and Baron (1989, 1994)). The second reason is that if both components of the utility function are proportional, then as we show in the Appendix A2 there is no equilibrium in pure strategies where at least one interest group offers a contract. If both components are linear, then again there is no equilibrium where at least one interest group offers a contract to its respective party. We also consider this case in Appendix A2. We are not interested in situations where no interest group offers a contract to its respective party, as an equilibrium where no interest group offers a contract delivers no insights on the relation between interest groups and parties, and on how the different parameter values affect the outcome of the election in the presence of interest groups.

Third, we model uncertainty about the voter as a parameter that affects his entire utility function. This is meant to replicate the fact that the voter could have a bias towards either party, as well as represent unobservables that might influence the voter's behaviour (see Calvert (1985) or Wittman (1983) for earlier political economy models that use the stochastic preference framework). An alternative would be to have uncertainty only about his ideal political position (see the seminal work of Lindbeck and Weibull (1987) and Coughlin (1992)), or only about how campaign spending affects his utility, or both. The fact that there is no uncertainty about the voter's ideal policy position means that any polarization observed in equilibrium cannot be attributed to the uncertainty about the voter's true policy preference. This allows us to get a better picture of how interest groups can influence polarization. Furthermore, notice that if instead one assumed that parties were uncertain about the voter's ideal policy position, then parties' outside option to reject the contract from the interest group would involve choosing a policy position that could depend on the position chosen by the other party. This would greatly complicate the analysis.

6 Conclusions

In this paper, we studied a model of interest group and electoral competition in which two parties compete in an election, where support from the voter can be sought through a political stance close to the voter's ideal point and through campaign spending. Campaign spending comes from the contributions of two interest groups, each of which offers a contract specifying a donation in exchange for a position in the political spectrum. If the contract is accepted, the party then receives funds to use for campaign spending, but it commits its political position. In our results, we found that the model delivers a unique equilibrium with comparative statics that match and explain empirical observations.

In our results we found that the interest group that has a higher valuation makes its party adopt a more polarized position than the other party and also offers more campaign contributions. Furthermore, we found that the higher the policy salience, the less polarized parties will be, and the lower campaign spending will be, although the high-valuation interest group will increase its contributions relative to the low-valuation interest group. On top of that, we showed that uncertainty about the voter increases polarization and relative campaign spending but only for the party that gets offered the contract from the higher-valuation interest group while it will decrease the polarization and relative campaign spending of the other party. Finally, in terms of welfare, we found, among others, that competition between interest groups minimizes voters' welfare.

In terms of empirical observations, our model can help explain certain patterns observed in the US interest groups' industry, like the gun rights/gun control interest group and the pro-life/pro-choice interest group. For example, in the gun rights/gun control interest group, the interest group that has a higher valuation is the gun rights interest group. Accordingly, they spend significantly more on contributions than the gun control interest group. Moreover, our comparative statics also help explain why issues that attract more monetary efforts from the interest groups are not the same as those that attract the most attention from the public. As discussed in the main text, in 2016 the health and pharmaceutical industry interest group is the one that spends the most in the US, yet health care is only the fourth item on the priority list for US voters. Finally, in line with previous work both theoretical and empirical, we found that uncertainty decreases campaign spending but increases polarization.

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Appendix

A1 - Proofs

Proof of Theorem 1. Using Kuhn-Tucker, the maximization problem of interest group l becomes

$$\max_{(y_l,t_l)} \quad -v_l\left(y_l P_L + y_R(1-P_L)\right) - t_l + \mu_l \left[y_l + \frac{1-\lambda}{\lambda}\frac{2t_l}{t_l+t_R}\right],$$

with complementary conditions

$$\begin{aligned} \mu_l \left[y_l + \frac{1-\lambda}{\lambda}\frac{2t_l}{t_l+t_R}\right] &= 0, \\ y_l + \frac{1-\lambda}{\lambda}\frac{2t_l}{t_l+t_R} &\geq 0, \\ \mu_l &\geq 0. \end{aligned}$$

If we denote $P_L^{(x)}$ as the partial derivative of $\text{Prob}_L\left(y_l, t_l, y_R, t_R\right)$ with respect to variable x, we have that the first order conditions of the problem are

$$\begin{aligned} -v_l P_L - v_l y_l P_L^{(y_l)} + v_l y_R P_L^{(y_l)} + \mu_l &= 0, \\ -v_l y_l P_L^{(t_l)} + v_l y_R P_L^{(t_l)} - 1 + \mu_l \left[\frac{1-\lambda}{\lambda}\frac{2t_R}{(t_l+t_R)^2}\right] &= 0. \end{aligned}$$

Eliminating the value of μ_l leads to

$$1 = v_l \frac{1-\lambda}{\lambda} P_L \frac{2t_R}{(t_l+t_R)^2}. \tag{9}$$

We now have two cases to consider depending on whether the multiplier μ_l is strictly positive or zero:

CASE 1: $\mu_l > 0$.

Since $\mu_l > 0$, the participation constraint of party L is binding. Hence, $P_L = \frac{\lambda y_R - (1-\lambda)+\gamma}{2\gamma}$. Thus, from (9) and knowing that equation (6) binds leads to

$$1 = v_l \frac{1-\lambda}{\lambda}\frac{\lambda y_R - (1-\lambda)+\gamma}{2\gamma}\frac{2t_R}{(t_l+t_R)^2}, \tag{10}$$

$$y_l = -\frac{1-\lambda}{\lambda}\frac{2t_l}{t_l+t_R}. \tag{11}$$

These are the implicit functions for the optimal contract offered by interest group l as a best response to party R position y_R and campaign spending t_R.

CASE 2: $\mu_l = 0$.

In this case, the participation constraint of party L may not hold with equality, as $\mu_l = 0$. Thus, equation (9) becomes

$$-v_l P_L - v_l y_l \frac{\lambda}{2\gamma} + v_l y_R \frac{\lambda}{2\gamma} \quad = \quad 0.$$

Substituting the value of P_L, we have

$$-v_l \frac{\lambda(y_L + y_R) + (1-\lambda)\frac{t_L - t_R}{t_L + t_R} + \gamma}{2\gamma} - v_l y_l \frac{\lambda}{2\gamma} + v_l y_R \frac{\lambda}{2\gamma} \quad = \quad 0.$$

Which leads to

$$y_l \quad = \quad -\frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{t_l - t_R}{t_l + t_R}.$$

Together with equation (9), we have then that the implicit optimal contract in this case is

$$1 \quad = \quad v_l \frac{1-\lambda}{\lambda}\frac{2t_R}{(t_l + t_R)^2} P_L, \tag{12}$$

$$y_l \quad = \quad -\frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{t_l - t_R}{t_l + t_R}. \tag{13}$$

Proceeding in a similar fashion as above, we can compute the optimal contract offered by interest group r as a best response to party L position y_L and campaign spending t_L. The Kuhn-Tucker, the maximization problem of interest group r is

$$\max_{(y_r, t_r)} \quad v_r\left(y_L P_L + y_r(1-P_L)\right) - t_r + \mu_r\left[y_r - \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L + t_r}\right],$$

with complementary conditions

$$\mu_r\left[y_r - \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L + t_r}\right] \quad = \quad 0,$$

$$y_r - \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L + t_r} \quad \geq \quad 0,$$

$$\mu_r \quad \geq \quad 0.$$

The maximization problem above leads to another two cases to consider depending on whether or not the constraints of the maximization problem bind. These are:

CASE 3: $\mu_r > 0$.

In this case the implicit functions for the optimal contract offered by interest group r as a best response to party L position y_L and campaign spending t_L are given implicitly by

$$1 \quad = \quad v_r \frac{1-\lambda}{\lambda}\frac{\gamma - \lambda y_L - (1-\lambda)}{2\gamma}\frac{2t_L}{(t_L + t_r)^2}, \tag{14}$$

$$y_r \quad = \quad \frac{1-\lambda}{\lambda}\frac{2t_r}{t_L + t_r}. \tag{15}$$

CASE 4**:** $\mu_r = 0$.

The implicit functions for the optimal contract in this case are

$$1 = v_r \frac{1-\lambda}{\lambda}(1-P_L)\frac{2t_L}{(t_L+t_r)^2}, \tag{16}$$

$$y_r = \frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{t_L - t_r}{t_L + t_r}. \tag{17}$$

When both interest groups offer optimal contracts, we have a possible of four potential candidates for equilibrium (two cases per interest group). Given that the problem of both interest groups is symmetric, except for the valuations v_l and v_r, we can reduce the number of potential candidates for equilibrium to three. Only one of these three candidates turns out to be an equilibrium. We deal with this candidate below and prove that neither of the other two candidates is valid for equilibrium in lemma 1 later in the appendix.

CANDIDATE 1 **(cases** 1 **and** 3**):**

First of all, note that in an equilibrium where both contracts are accepted, $y_l = y_L$, $y_r = y_R$, $t_l = t_L$ and $t_r = t_R$. Next, dividing equation (10) by equation (14), we get

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_L - (1-\lambda)}{\lambda y_R - (1-\lambda) + \gamma}\frac{t_L}{t_R} = 1.$$

Using the fact that equations (11) and (15) imply $y_R - y_L = \frac{2(1-\lambda)}{\lambda}$ and $\frac{y_L}{y_R} = -\frac{t_L}{t_R}$, we obtain

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_L - (1-\lambda)}{\gamma + \lambda y_L + (1-\lambda)}\frac{-y_L}{\frac{2(1-\lambda)}{\lambda} + y_L} = 1. \tag{18}$$

This is, the implicit equation for the equilibrium value of y_L in this candidate. Proceeding in a similar fashion, we obtain

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_R + (1-\lambda)}{\gamma + \lambda y_R - (1-\lambda)}\frac{\frac{2(1-\lambda)}{\lambda} - y_R}{y_R} = 1. \tag{19}$$

If we denote by $x_L = \frac{t_L}{t_L + t_R}$ the campaign spending of party L relative to that of party R, we have by equation (11) that $y_L = -2\frac{1-\lambda}{\lambda}x_L$ and, by equation (18)

$$\frac{v_r}{v_l}\frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{\gamma - 2(1-\lambda)x_L + (1-\lambda)}\frac{x_L}{1-x_L} = 1. \tag{20}$$

Finally, we have that by equation (14), the equilibrium value for $t_L + t_R$ in terms of x_L is

$$t_L + t_R = v_r\frac{1-\lambda}{\lambda}\frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{\gamma}x_L. \tag{21}$$

As already mentioned, the fact that neither Candidate 2 (cases 2 and 4) nor Candidate 3 (cases 1 and 4 and cases 2 and 3) are valid for an equilibrium is proven in lemma 1 below.

Note that the equations in (19)-(e. candidate 1 T) are quadratic. Moreover, they all have two solutions, only one of which leads to a positive real number, and, thus, each gives a unique admissible equilibrium value for its respective variable.

Second order conditions:

Next, we show that the second order conditions of the interest groups' maximization problem are satisfied for candidate for equilibrium 1. In order to check for the second order conditions, we first compute the determinant of the Hessian matrix of interest group l:

$$\begin{aligned} |H| &= \begin{vmatrix} \frac{\partial^2 \pi_l}{\partial^2 y_l} & \frac{\partial^2 \pi_l}{\partial y_l \partial t_l} \\ \frac{\partial^2 \pi_l}{\partial t_l \partial y_l} & \frac{\partial^2 \pi_l}{\partial^2 t_l} \end{vmatrix} \\ &= \frac{(1-\lambda) v_l^2 t_R}{\gamma^2 (t_l + t_R)^4} \left(2\lambda (y_R - y_l)(t_l + t_R) - (1-\lambda) t_R\right) \end{aligned}$$

In a critical point, the second derivatives with respect to y_l and t_l need to be negative, while the determinant of the Hessian needs to be positive. The former is always satisfied in this model, so now we focus on the latter: the sign of the determinant of the Hessian.

For candidate 1, we have that $y_R - y_L = \frac{2(1-\lambda)}{\lambda}$ and, hence, the determinant of the Hessian is positive if and only if $4(t_L + t_R) - t_R > 0$, which is trivially satisfied. A similar calculation shows that the second order conditions are also satisfied for interest group r and, hence, the equilibrium candidate 1 fulfils the second order conditions requirement for maximum.

Incentive compatibility:

Finally, we are left to show that both interest groups are indeed better off offering a contract as opposed to not offering one. We focus on interest group l, as the calculations for interest group r follow the same logic.

First, note that since in candidate 1 interest group l is offering a contract such that the participation constraint of party L binds, it is true that the probability with which party L wins the election is the same whether interest group l offers such contract or not. We have that the profit of interest group l in candidate 1 is $-v_l y_L P_L - v_l y_R (1 - P_L) - t_L$, while if interest group l offers no contract then party L chooses position $y_L = 0$ and no campaign spending $t_L = 0$. Therefore, l's profit is $-v_r y_R (1 - P_L)$. Thus, interest group l is better off offering the contract if and only if $-v_l y_L P_L - t_L \geq 0$. By equation (9), we can rewrite this as

$$-y_L \frac{\lambda}{1-\lambda} \frac{(t_L + t_R)^2}{2t_R} - t_L \quad \geq \quad 0.$$

Substituting the value of y_L from equation (11), we have

$$t_L \frac{t_L + t_R}{t_R} - t_L \geq 0,$$

which is true since $t_L, t_R \geq 0$.

Note that we have also just shown that interest group l can always get a better payoff by offering a contract than by not offering one, regardless of what party R or interest group r do. Thus, there cannot be an equilibrium where interest group l offers no contract, or a contract that will be rejected by party L.

Finally, note that in equilibrium candidate 1, we have $P_L = \frac{\gamma+(1-\lambda)(2x_L-1)}{2\gamma}$. Therefore, given our assumption $\gamma > 3(1-\lambda)$, it is always true that $P_L \in (0,1)$. □

Lemma 1. *Neither Candidate* 2 *(cases* 2 *and* 4*) nor Candidate* 3 *(cases* 1 *and* 4*) are an equilibrium.*

Proof. **CANDIDATE** 2 **(cases** 2 **and** 4**):**

Equations (13) and (17) imply $y_R + y_L = -\frac{1-\lambda}{\lambda}\frac{t_L - t_R}{t_L + t_R}$, which implies $P_L = \frac{1}{2}$. Together with equation (12), this means $v_l \frac{1-\lambda}{\lambda}\frac{t_R}{(t_L+t_R)^2} = 1$. Similarly, by equation (16), we have $v_r \frac{1-\lambda}{\lambda}\frac{t_L}{(t_L+t_R)^2} = 1$. These two equations combined imply that $\frac{v_l}{v_r} = \frac{t_L}{t_R}$. Thus, we have that in equilibrium

$$y_L = -\frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{v_l - v_r}{v_l + v_r}, \quad (22)$$

$$y_R = \frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{v_l - v_r}{v_l + v_r}. \quad (23)$$

Moreover, the fact that $\frac{v_l}{v_r} = \frac{t_L}{t_R}$ and equation (12) imply

$$x_L = \frac{v_L}{v_L + v_R}, \quad (24)$$

$$t_L + t_R = \frac{1-\lambda}{\lambda}\frac{v_L v_R}{v_L + v_R}. \quad (25)$$

For this candidate to be valid, the participation constraint of both interest groups needs to be satisfied. By equation (6), it must be true in equilibrium that $y_L \geq -\frac{1-\lambda}{\lambda}\frac{2t_L}{t_L+t_R}$. From equation (22) this implies

$$\begin{aligned} -\frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{v_l - v_r}{v_l + v_r} &\geq -\frac{1-\lambda}{\lambda}\frac{2t_L}{t_L + t_R} \\ &\geq -\frac{1-\lambda}{\lambda}\frac{2v_l}{v_l + v_r}. \end{aligned}$$

Which is true if and only if

$$-\gamma + 1 - \lambda + 2(1-\lambda)\frac{v_l}{v_l + v_r} \geq 0.$$

Similarly, from equations (7) and (23) we have that this candidate for equilibrium is valid if and only if

$$\gamma - (1-\lambda) - 2(1-\lambda)\frac{v_r}{v_l + v_r} \quad \leq \quad 0.$$

Thus, combining these two inequalities we have that a necessary condition for this candidate to be valid is that $\gamma \leq 2(1-\lambda)$, which is not true by assumption.

CANDIDATE 3 (cases 1 and 4):

First, we compute the equilibrium values for the variables of the model. Although not necessary for the proof of the lemma, this calculations will prove useful when we discuss the comparative statics of other candidates for equilibrium later on.

From equations (11) and (17)

$$\begin{aligned} y_L &= -\frac{1-\lambda}{\lambda}\frac{2t_L}{t_L + t_R}, \\ y_R &= \frac{\gamma}{2\lambda} - \frac{1-\lambda}{2\lambda}\frac{t_L - t_R}{t_L + t_R}. \end{aligned}$$

Dividing equations (10) and (16), and using the fact that the participation constraint on party L binds, we have

$$1 \quad = \quad \frac{v_l}{v_r}\frac{\lambda y_R - (1-\lambda) + \gamma}{\gamma - \lambda y_R + (1-\lambda)}\frac{t_R}{t_L}.$$

From equation (17), we have $x_L = \frac{\lambda}{1-\lambda}\left(\frac{\gamma}{2\lambda} + \frac{1-\lambda}{2\lambda} - y_R\right)$. Thus, from the equation above

$$1 \quad = \quad \frac{v_l}{v_r}\frac{\lambda y_R - \frac{1-\lambda}{\lambda} + \gamma}{\gamma - \lambda y_R + \frac{1-\lambda}{\lambda}}\frac{\frac{\lambda}{1-\lambda}y_R - \frac{\gamma}{2(1-\lambda)} + \frac{1}{2}}{\frac{\gamma}{2(1-\lambda)} + \frac{1}{2} - \frac{\lambda}{1-\lambda}y_R}, \qquad (26)$$

which gives implicitly the equilibrium value of y_R.

Equations (11) and (17) together imply $y_L = 2y_R - \frac{\gamma}{\lambda} - \frac{1-\lambda}{\lambda}$, which is the equilibrium value of y_L in terms of y_R.

Equation (11) implies $x_L = -\frac{\lambda}{2(1-\lambda)}y_L$, which gives the equilibrium value of x_L in terms of y_L. Finally, from equation (10) we have

$$t_L + t_R \quad = \quad v_l\frac{1-\lambda}{\lambda}\frac{\lambda y_R - (1-\lambda) + \gamma}{\gamma}(1 - x_L).$$

For this candidate to be valid, the participation constraint of interest group R needs to be satisfied. From equations (7) and (17), we have that this candidate for equilibrium is valid if and only if

$$\gamma - (1-\lambda) - 2(1-\lambda)\frac{t_R}{t_L + t_R} \quad \leq \quad 0.$$

Given that $\frac{t_R}{t_L+t_R} \in [0,1]$, a necessary condition for the inequality above is that $\gamma - (1-\lambda) - 2(1-\lambda) \leq 0$. However, this needs $\gamma \leq 3(1-\lambda)$, which is ruled out by assumption. □

Proof of Remark 1. Assume $v_l > v_r$, the proof for the case where $v_l < v_r$ follows a similar logic as below and is hence omitted.

From equilibrium equations (18) and (19), we have that

$$\frac{v_r}{v_l}\frac{\gamma - \lambda y_L - (1-\lambda)}{\gamma + \lambda y_R - (1-\lambda)}\frac{-y_L}{y_R} = 1.$$

Thus, if $v_l > v_r$ then it must be that $-y_L > y_R$, i.e. party L is more polarized. By equations (11) and (15) this also means $t_L > t_R$ as we wanted to show. □

Proof of Proposition 1. We prove all statements from the point of view of party L. The proofs for party R follow the same logic and are thus omitted.

Valuation Effect

This is already proved in the proof of Remark 1 above.

Salience Effect

The fact that $y_L < 0$ implies that the left hand side of equation (18) depends positively on λ. Since that term depends negatively on y_L, we have that λ and y_L move in the same direction: higher λ implies higher y_L as we wanted to show.

Uncertainty Effect

The left hand side of equation (18) depends negatively on y_L, while it depends positively on γ if and only if $2\lambda y_L - 2(1-\lambda) \geq 0$. Since, by equation (11), it is true that $y_L = -\frac{1-\lambda}{\lambda}\frac{2t_L}{t_L+t_R}$, we have that the left hand side of (18) depends positively on γ if and only if $\frac{t_L}{t_L+t_R} \leq \frac{1}{2}$, which happens only if $v_l \leq v_r$ by remark 1.

Thus, if $v_l > v_r$ then y_L depends negatively on γ (y_L becomes more polarized as γ increases), while if $v_l < v_r$ it depends positively on γ, as we wanted to show. □

Proof of Proposition 2. We proceed by proving the valuation and uncertainty effects together, and then prove the salience effect.

Valuation and Uncertainty Effects

As we show later on in the proof of Proposition 4, we can rewrite the welfare of the voter as $W = -\frac{\lambda}{2}\left(P_L(-y_L) + P_R y_R\right)$. Therefore, $W = -\frac{\lambda}{2}EP$, which implies that the comparative statics for the expected polarization with respect to the valuations of the interest groups and the uncertainty parameter have opposite signs to those for the welfare. Thus, we omit this part of the proof and refer to the proof of Proposition 4 later on.

Salience Effect

From the proof of Proposition 4, we know that $W = -\frac{\lambda}{2}EP$ and that increasing λ increases welfare. Thus, $\frac{\mathrm{d}W}{\mathrm{d}\lambda} = -\frac{EP}{2} - \frac{\lambda}{2}\frac{\mathrm{d}EP}{\mathrm{d}\lambda} > 0$. This implies $\frac{\mathrm{d}EP}{\mathrm{d}\lambda} < -\frac{EP}{\lambda} < 0$. That is, increasing salience decreases expected polarization. □

Proof of Proposition 3. We prove all statements from the point of view of party L assuming $v_l > v_r$. The proofs for $v_l \leq v_r$ and/or party R follow the same logic and are thus omitted.

Valuation Effect

Given $y_L = -\frac{1-\lambda}{\lambda}\frac{t_L}{t_L+t_R}$ and what is stated in proposition 1 (higher v_l implies lower y_L and lower y_R), we have that that higher v_l implies higher $\frac{t_L}{t_L+t_R}$ and lower $\frac{t_R}{t_L+t_R}$.

From (21), and since higher v_l implies higher x_L, we have that higher v_l implies higher $t_L + t_R$. Moreover, higher x_L and higher $t_L + t_R$ implies higher t_L.

In terms of t_R, we have that $x_R = \frac{t_R}{t_L+t_R}$ which means $\frac{\mathrm{d}x_R}{\mathrm{d}v_l} = \frac{\frac{\mathrm{dt_R}}{\mathrm{d}v_l}T - t_R\frac{\mathrm{d}T}{\mathrm{d}v_l}}{T^2}$. Moreover, from equation (21), we have

$$\frac{\mathrm{d}T}{\mathrm{d}v_l} = v_r\frac{1-\lambda}{\lambda}\frac{2(1-\lambda)}{\gamma}\frac{\mathrm{d}x_L}{\mathrm{d}v_l}x_L + v_r\frac{1-\lambda}{\lambda}2P_L\frac{\mathrm{d}x_L}{\mathrm{d}v_l}.$$

Thus, the expression above together with the expression for $\frac{\mathrm{d}x_R}{\mathrm{d}v_l}$, and using the fact that $\frac{\mathrm{d}x_R}{\mathrm{d}v_l} = -\frac{\mathrm{d}x_L}{\mathrm{d}v_l}$, leads after some algebra to

$$\begin{aligned}
\frac{\mathrm{d}t_R}{\mathrm{d}v_l} &= \frac{\mathrm{d}x_L}{\mathrm{d}v_l}\left(x_R v_r\frac{1-\lambda}{\lambda}\left(\frac{2(1-\lambda)}{\gamma}x_L + 2P_R\right) - T\right) \\
&= \frac{\mathrm{d}x_L}{\mathrm{d}v_l}v_r\frac{1-\lambda}{\lambda}\left(2P_R(1-2x_L) + \frac{2(1-\lambda)}{\gamma}x_L(1-x_L)\right) \\
&= \frac{\mathrm{d}x_L}{\mathrm{d}v_l}v_r\frac{1-\lambda}{\lambda}\left(1-2x_L + \frac{1-\lambda}{\gamma}\left(-6x_L^2 + 6x_L - 1\right)\right).
\end{aligned}$$

Notice now that equation (20) implies that, if v_l goes to zero, so does x_L, and that as v_l grows large, x_L goes to 1. Therefore, as v_l goes to zero, the right hand side of the expression above converges to $1 - \frac{1-\lambda}{\gamma}$ which, since $\gamma > 3(1-\lambda)$, is a positive expression. Moreover, as v_l goes to one, the right hand side of the expression above converges to $-1 - \frac{1-\lambda}{\gamma}$, i.e. negative. The fact that the term in brackets in the right hand side of the expression above is decreasing in x_L because $\gamma > 3(1-\lambda)$ completes the proof for t_R.

Salience Effect

The left hand side of equation (20) depends positively on x_L, while it depends positively on λ if and only if $x_L \leq \frac{1}{2}$, which happens if and only if $v_l \leq v_r$ by remark 1. Thus, higher λ implies higher x_L (i.e. higher $\frac{t_L}{t_L+t_R}$, and lower $\frac{t_R}{t_L+t_R}$) if and only if $v_l > v_r$ (this is true since $\gamma > 3(1-\lambda)$ implies $P_L \in (0,1)$, and so both the numerator and the denominator are positive in the left hand side of (20)).

As we are assuming $v_l > v_r$, x_L depends positively on λ by the paragraph above. Notice that in equation (20), we have $\gamma+2(1-\lambda)x_L-(1-\lambda) = 2\gamma-(\gamma-2(1-\lambda)x_L+(1-\lambda))$. Thus, if λ increases then x_L increases and according to (20) it must be that $\gamma+2(1-\lambda)x_L-(1-\lambda)$ decreases. Since equation (14) can be rewritten in equilibrium as

$$t_L + t_R \quad = \quad v_r \frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{2\gamma}(-y_L),$$

and given that by proposition 1 it is true that y_L increases with λ (and, hence, $-y_L$ decreases with λ), we have then that $t_L + t_R$ decreases with λ.

Given that, as we have just shown, increasing λ decreases $\frac{t_R}{t_L+t_R}$ and $t_L + t_R$, it must be that t_R also decreases with λ.

Finally, it remains to prove that t_L decreases with λ. We have just shown that $\gamma + 2(1-\lambda)x_L - (1-\lambda)$ must be decreasing in λ. In other words:

$$-2x_L + 2(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} + 1 \quad < \quad 0.$$

This implies $\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} < \frac{2x_L-1}{2(1-\lambda)}$. On top of that, we can rewrite (21) as

$$t_L \quad = \quad v_r \frac{1-\lambda}{\lambda} \frac{\gamma + 2(1-\lambda)x_L - (1-\lambda)}{\gamma} x_L^2.$$

Therefore, using the fact that $\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} < \frac{2x_L-1}{2(1-\lambda)}$ leads to

$$\begin{aligned}
\frac{\mathrm{d}t_L}{\mathrm{d}\lambda} &= -v_r\frac{1}{\lambda^2}\frac{\gamma+2(1-\lambda)x_L-(1-\lambda)}{\gamma}x_L^2 + v_r\frac{1-\lambda}{\lambda}\frac{-2x_L+2(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\lambda}+1}{\gamma}x_L^2 \\
&\quad +v_r\frac{1-\lambda}{\lambda}\frac{\gamma+2(1-\lambda)x_L-(1-\lambda)}{\gamma}2x_L\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} \\
&< -v_r\frac{1}{\lambda^2}\frac{\gamma+2(1-\lambda)x_L-(1-\lambda)}{\gamma}x_L^2 + v_r\frac{1-\lambda}{\lambda}\frac{-2x_L+2(1-\lambda)\frac{2x_L-1}{2(1-\lambda)}+1}{\gamma}x_L^2 \\
&\quad +v_r\frac{1-\lambda}{\lambda}\frac{\gamma+2(1-\lambda)x_L-(1-\lambda)}{\gamma}2x_L\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} \\
&\propto -\frac{1}{\lambda^2}x_L^2+\frac{1-\lambda}{\lambda}2x_L\frac{\mathrm{d}x_L}{\mathrm{d}\lambda} \\
&< -\frac{1}{\lambda^2}x_L^2+\frac{1-\lambda}{\lambda}2x_L\frac{2x_L-1}{2(1-\lambda)} \\
&\propto -\frac{x_L}{\lambda}+2x_L-1 \\
&< -x_L+2x_L-1 \\
&< 0.
\end{aligned}$$

Hence, $\frac{\mathrm{d}t_L}{\mathrm{d}\lambda} < 0$ as required.

Uncertainty Effect

By proposition 1, if γ increases then y_L decreases if and only if $v_l > v_r$. Thus, by equation 11, an increase in γ leads to an increase in x_L and a decrease in x_R.

Using implicit differentiation in equation (20), and after some algebra, we have that

$$\begin{aligned}
2\gamma(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\gamma} &= \frac{\gamma-(1-\lambda)(2x_L-1)}{x_L} \\
&\quad +\frac{1}{2}\frac{\gamma-(1-\lambda)(2x_L-1)}{x_L}\frac{\gamma+(1-\lambda)(2x_L-1)}{1-x_L}\frac{\mathrm{d}x_L}{\mathrm{d}\gamma}.
\end{aligned}$$

From equation (21), we obtain that

$$\frac{\mathrm{d}T}{\mathrm{d}\gamma} \propto \frac{(1+2(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\gamma})\gamma-(\gamma+(1-\lambda)(2x_L-1))}{\gamma^2}x_L+\frac{\gamma+(1-\lambda)(2x_L-1)}{\gamma}\frac{\mathrm{d}x_L}{\mathrm{d}\gamma}.$$

Thus, after substituting the expression for $2\gamma(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\gamma}$ derived, above we obtain $\frac{\mathrm{d}T}{\mathrm{d}\gamma} \propto 1-\frac{\gamma-(1-\lambda)(2x_L-1)}{2\gamma}\frac{1}{1-x_L}$. It can be shown that the right hand side is decreasing in x_L, zero at $x_L=\frac{1}{2}$ and negative at x_L close to 1. Hence, $\frac{\mathrm{d}T}{\mathrm{d}\gamma}<0$ for $v_l > v_r$.

Given that x_R decreases and that T decreases, it must be that t_R decreases. Thus, it only remains to show what happens with t_L when γ changes. Multiplying both sides of equation (21) by t_L and dividing by T, we have

$$\frac{\mathrm{d}t_L}{\mathrm{d}\gamma} \propto \frac{(1+2(1-\lambda)\frac{\mathrm{d}x_L}{\mathrm{d}\gamma})\gamma-(\gamma+(1-\lambda)(2x_L-1))}{\gamma^2}x_L^2+\frac{\gamma+(1-\lambda)(2x_L-1)}{\gamma}2x_L\frac{\mathrm{d}x_L}{\mathrm{d}\gamma}.$$

Using again the expression for $2\gamma(1-\lambda)\frac{x_L}{\gamma}$, we obtain

$$\frac{\mathrm{d}t_L}{\mathrm{d}\gamma} \propto 2 - \frac{\gamma-(1-\lambda)(2x_L-1)}{2\gamma}\frac{1}{1-x_L}.$$

The right hand side of the expression above is decreasing in x_L, positive at $x_L = \frac{1}{2}$ and negative at x_L close to 1. Thus, $\frac{\mathrm{d}t_L}{\mathrm{d}\gamma}$ is positive if x_L is low enough and negative otherwise. Since x_L increases the higher v_l is relative to v_r and is $\frac{1}{2}$ when $v_l = v_r$, the result follows. □

Proof of Proposition 4. Assume that $v_l \geq v_r$ (which by remark 1 implies $x_L \geq \frac{1}{2}$). The proof for the case where $v_l \leq v_r$ follows a similar logic and is therefore omitted.

From equations (11) and (15), and after some rearrangement we can rewrite W in equilibrium as

$$W = \frac{1-\lambda}{2\gamma}\left(1-\gamma-\lambda-4(1-\lambda)(1-x_L)x_L\right). \tag{27}$$

Valuation Effect:

By proposition 3, we have that higher v_l implies higher x_l, which by equation (27) and the fact that $x_L \geq \frac{1}{2}$ leads to higher welfare. On the other hand, higher v_r implies lower x_l, which by equation (27), and the fact that $x_L \geq \frac{1}{2}$, leads to lower welfare.

The second statement follows from the observation that (27) decreases as x_L approaches $\frac{1}{2}$, and the fact that x_L approaches $\frac{1}{2}$ as the two valuations v_l and v_r get closer to each other.

Salience Effect:

Assume again without loss of generality that $v_l \geq v_r$. Using equation (27), we have that the partial derivative of W with respect to λ is

$$\begin{aligned}\frac{\partial W}{\partial \lambda} &= \frac{1}{2\gamma}\left(\gamma-2(1-\lambda)-8(1-\lambda)x_L^2-4(1-\lambda)^2\frac{\partial x_L}{\partial \lambda}+8(1-\lambda)x_L(1+(1-\lambda)\frac{\partial x_L}{\partial \lambda}\right)\\ &= \frac{1}{2\gamma}\left(\gamma-2(1-\lambda)+8(1-\lambda)x_L(1-x_L)+4(1-\lambda)^2(2x_L-1)\frac{\partial x_L}{\partial \lambda}\right)\end{aligned}$$

By proposition 3, we have that $\frac{\partial x_L}{\partial \lambda} > 0$ and by remark 1 that $x_L \geq \frac{1}{2}$. This, together with the fact that $\gamma > 3(1-\lambda)$ by assumption, leads to $\frac{\partial W}{\partial \lambda} > 0$ as desired.

Uncertainty Effect:

Recall that in equilibrium $y_R - y_L = \frac{2(1-\lambda)}{\lambda}$, which implies $\frac{\mathrm{d}y_R}{\mathrm{d}\gamma} = -\frac{\mathrm{d}y_L}{\mathrm{d}\gamma}$, and that $P_L + P_R = 1$ implies $\frac{\mathrm{d}P_R}{\mathrm{d}\gamma} = -\frac{\mathrm{d}P_L}{\mathrm{d}\gamma}$. Thus, from equation (8) we have

$$\frac{\mathrm{d}W}{\mathrm{d}\gamma} \propto -\left(-\frac{\mathrm{d}P_L}{\mathrm{d}\gamma}(y_L+y_R)+\frac{\mathrm{d}y_L}{\mathrm{d}\gamma}(P_R-P_L)\right).$$

The fact that $v_l > v_r$ implies $-y_L > y_R$ as shown in Remark 1, together with the fact that $P_L - P_R = \frac{\lambda}{2\gamma}(y_L + y_R)$ we have that

$$\frac{\mathrm{d}W}{\mathrm{d}\gamma} \propto -\frac{\mathrm{d}P_L}{\mathrm{d}\gamma} + \frac{\lambda}{2\gamma}\frac{\mathrm{d}y_L}{\mathrm{d}\gamma}.$$

Since $P_L = 1 - P_R = 1 - \frac{\gamma - \lambda y_L - (1-\lambda)}{2\gamma}$, we can calculate $\frac{\mathrm{d}P_L}{\mathrm{d}\gamma}$ and substitute above to obtain:

$$\frac{\mathrm{d}W}{\mathrm{d}\gamma} \propto \lambda y_L + (1-\lambda).$$

Since $v_l > v_r$ implies $-y_L > y_R$ which in turn implies $P_R > P_L$, we must have that $P_R > \frac{1}{2}$. In other words, $\frac{\gamma - \lambda y_L - (1-\lambda)}{2\gamma} > \frac{1}{2}$. This means that $\lambda y_L < -(1-\lambda)$, which proves that $\frac{\mathrm{d}W}{\mathrm{d}\gamma}$ is negative.

□

A2 - Alternative Utility Functions

Next we study how alternatives to the utility function that we use in the main text may influence our results. In particular, we focus on two variants of the utility function in equation (1), one where both terms on y and t appear in a proportional fashion, and another one where both these terms appear linearly.

Both Terms are Proportional

Given that $y_L \leq 0$ and $y_R \geq 0$, the utility function in this case is given by

$$u(p) \quad = \quad -\lambda \frac{|y_p|}{y_R - y_L} + (1-\lambda)\frac{t_p}{t_L + t_R} - \varepsilon \mathbb{1}_{p=L}$$

where we assume that, if $y_L = y_R$, the term $\frac{|y_p|}{y_R - y_L}$ equals $\frac{1}{2}$.

The probability that the voter votes for party L is given by

$$P_L \quad = \quad \frac{\lambda \frac{y_l + y_R}{y_R - y_l} + (1-\lambda)\frac{t_l - t_R}{t_l + t_R} + \gamma}{2\gamma}.$$

The participation constraints can be derived in a similar fashion as in the main text. They are:

$$\begin{aligned} \frac{y_l}{y_R - y_l} &\geq -\frac{1-\lambda}{\lambda}\frac{t_l}{t_l + t_R}, \\ \frac{y_r}{y_r - y_L} &\leq \frac{1-\lambda}{\lambda}\frac{t_r}{t_L + t_r}. \end{aligned}$$

Already from both equations above, it can be seen that there is no equilibrium where both of these are satisfied with equality, as if the two inequalities above bind then it must be that $1 = \frac{1-\lambda}{\lambda}$, which is false in general. This leaves us with two possibilities, either only one participation constraint binds, or neither do.

Interest group l's maximization problem is

$$\left.\begin{array}{ll} \max_{(y_l,t_l)} & -v_l\left(y_l P_L + y_R(1-P_L)\right) - t_l \\ \text{subject to:} & \frac{y_l}{y_R - y_l} \geq -\frac{1-\lambda}{\lambda}\frac{t_l}{t_l+t_R} \end{array}\right\}$$

The maximization problem of interest group r can be obtained in a similar fashion. If we consider the case where no participation constraints binds, it can be shown that at the optimum $x_L = \frac{1-\gamma}{2(1-\lambda)}$ and, similarly, $x_R = \frac{1-\gamma}{2(1-\lambda)}$. However, these two equations imply that $\gamma = \lambda$, which is false.

Thus, we have only one case left to consider, i.e. one participation constraint binds and the other one does not. Assume without loss of generality that interest group l's participation constraint binds. However, in this case, it can be shown that in order to maximize both interest groups' profit it must be that $v_l \frac{1-2\lambda+\gamma}{2\gamma} = v_r \frac{2\lambda-1+\gamma}{4\lambda-1-\gamma}$, which again is false.

Thus, when both components of the utility function are proportional, there is no equilibrium in pure strategies where both interest groups offer a contract. Intuitively, the main reason for this is that given that both components are proportional in the utility function of the voter, both interest groups maximize their profit by equating the relative ratios of y and t in a certain way. This is only possible if the parameter values have specific values as otherwise one interest group fixing the ratios of y and t means that for the other interest group its optimal ratios are not satisfied.

Notice that there cannot be an equilibrium where one interest group offers a contract and the other does not (or offers a contract that is rejected). This is because if one interest group does not offer a contract, then the other interest group's best response is to offer an infinitesimal amount of campaign funding. Since this quantity is not defined, there cannot be an equilibrium such that only one interest group offers a contract.

Both Terms are Linear

In this case, the utility function is given by

$$u(p) \quad = \quad -\lambda|y_p| + (1-\lambda)t_p - \varepsilon\mathbb{1}_{p=L}.$$

The probability that the voter votes for party L is

$$P_L \quad = \quad \frac{\lambda(y_L + y_R) + (1-\lambda)(t_L - t_R) + \gamma}{2\gamma}.$$

The participation constraints are:

$$\begin{aligned} y_l &\geq -\frac{1-\lambda}{\lambda} t_l, \\ y_r &\leq \frac{1-\lambda}{\lambda} t_r. \end{aligned}$$

Interest group l's maximization problem is, therefore,

$$\left.\begin{array}{ll} \max_{(y_l,t_l)} & -v_l\left(y_l P_L + y_R(1-P_L)\right) - t_l \\ \text{subject to:} & y_l \geq -\frac{1-\lambda}{\lambda} t_l \end{array}\right\}$$

If the participation constraint binds, then we have that the first order conditions for interest group l imply $\frac{1-\lambda}{\lambda} v_l P_L = 1$. Moreover, if the participation constraint does not bind, then the first order conditions also lead to $\frac{1-\lambda}{\lambda} v_l P_L = 1$. Similarly, for interest group r, whether the participation constraint binds or not, the first order conditions lead to $\frac{1-\lambda}{\lambda} v_r(1-P_L) = 1$. This implies that $\frac{\lambda}{1-\lambda}\left(\frac{1}{v_l} + \frac{1}{v_r}\right) = 1$, which is false in general.

Therefore, there is no equilibrium in pure strategies where both interest groups offer a contract that is accepted by the parties. An equilibrium where an interest group does not offer a contract (or offers one that is rejected) is also not possible. If the other interest group offers a contract where its party's participation constraint binds, then $P_L = \frac{1}{2}$, and equation $\frac{1-\lambda}{\lambda} v_r(1-P_L) = 1$ or $\frac{1-\lambda}{\lambda} v_r(1-P_L) = 1$, depending on which interest group offers a contract, cannot be satisfied. If the interest group offering a contract does not offer a contract where the participation constraint binds, the first order condition of this interest group with respect to its position in the policy space is such that the interest group will offer negative campaign spending, which violates the party's participation constraint.

A3 - Low Uncertainty about the Voter

We now study other potential equilibria that may arise in the model when the assumption $\gamma > 3(1-\lambda)$ is not satisfied. Note that below we do not prove when other equilibria can exist, as unfortunately the complexity of the model is such that we cannot carry out such analysis. Instead, we do a comparative statics analysis on the 2 candidates for equilibria that were ruled out as a result of our assumption on the parameter γ.

Candidate 2

In the main text, we deal with what we refer to as Candidate 1, which deals with cases 1 and 3 in the maximization problem of the interest groups. Next, we focus on the comparative statics of Candidate 2, which deals with cases 2 and 4. Notice that this candidate has the unrealistic feature that both parties have a 50% chances of winning the election regardless of the value of all parameters of the model. The equilibrium value for y_L, y_R, t_R and t_L follows from equations (22), (23), (24) and (25) in the proof of lemma 1. An inspection on these terms leads to the conclusion that the result in Remark 1 holds true. Furthermore, all comparative statics in Proposition 1 hold true except for the Salience Effect for the low-valuation interest group only if interest groups have different enough valuations and $\lambda < \frac{1}{2}$ (in this case the low-valuation interest group could become more polarized when increasing salience), and the Uncertainty Effect for the low-valuation party (which also becomes more polarized in this candidate for equilibrium).

The comparative statics for expected polarization follow those of welfare and as such we refer the the paragraph on welfare below. In terms of the comparative statics in Proposition 3, all hold true except for the fact that the Salience Effect does not affect relative spending, that the high-valuation party unequivocally reduces absolute campaign spending, and that the Uncertainty Effect does not affect spending at all, neither relative nor absolute.

In terms of welfare, given equations (22), (23), (24) and (25) and the fact that in this equilibrium each party wins the election with a probability 50%, we have that welfare (expected utility of the voter) is given by $W = -\frac{\gamma}{2} + \frac{1-\lambda}{2}$. Thus, we have that the interest groups' valuations do not affect welfare at all, that the more salience the election the lower the welfare, and that uncertainty reduces welfare. We believe that the fact that valuations do not affect welfare and that more salience decreases welfare are not desirable properties. First, it seems clear that valuations should have an effect on welfare, given how in the real world interest groups that can potentially make significant profit from the right policies do affect policy outcomes in a way that does not reflect the voter preferences. Second, as discussed in the main text, more salient elections tend to be better protected from the influence of interest groups, and the polices implemented are more aligned with the voter's preferences than in low salience elections.

In terms of the profit of the interest groups, given equations (22), (23), (24) and (25), and the fact that each party wins the election with probability 50% we have that

$$\pi_l = \frac{1-\lambda}{\lambda} \frac{1}{v_l + v_r} \left(\frac{v_l - v_r}{2} - \frac{v_l^2 v_r}{v_l + v_r} \right).$$

An inspection of this term confirms that as v_l increases, the profit of interest group l increases for low values of v_l and decreases for high values of v_l. This contradicts intuition and is the opposite of what we found for candidate 1 in the main text. In terms of v_r, we have that increasing this parameter increases π_l if v_r is large enough and $v_l > 1$. Again this seems to contradict common sense, as it implies that the higher a interest group's valuation, the higher the profit of the other interest group. From the mathematical point of view, the reason why this happens is that increasing a interest group's valuation, decreases the campaign spending of the other interest group, which has a positive effect on its profit.

The way policy salience λ affects the profit of the interest group is the same as in candidate 1, i.e. higher λ decreases the profit of the high-valuation interest group while it increases the profit of the low-valuation interest group as in the benchmark model. Finally, we find that uncertainty about the voter γ does not affect the interest groups' profit at all.

From the discussion above, we conclude that the comparative statics for this candidate for equilibrium are largely the same as in the candidate considered in the main text. In the situations where the two candidates for equilibrium do not deliver the same comparative static results, we find that the comparative statics for candidate 1, the one considered in the main text, are more realistic than the ones in candidate 2.

Candidate 3

Candidate 3 deals with cases 1 and 4 in the maximization problem of the interest groups. This means that this candidate is a mixture of candidates 1 and 2 and, thus, as it follows from the proof of lemma 1, the comparative statics for this candidate are between those of the other two candidates. Therefore, we omit the analysis of Candidate 3.