Vortex filament dynamics and noise

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- Fluid dynamics
  - Two and three dimensional vortex dynamics.
- Computation
  - Serial and parallel codes
- Acoustics
  - Aeroacoustics—Lighthill.
  - Vortex noise—Kambe.
- Some results
- Current status

Fluid mechanics

An incompressible, inviscid flow can be decomposed into two components.

- irrotational (potential).
- rotational (vorticity).

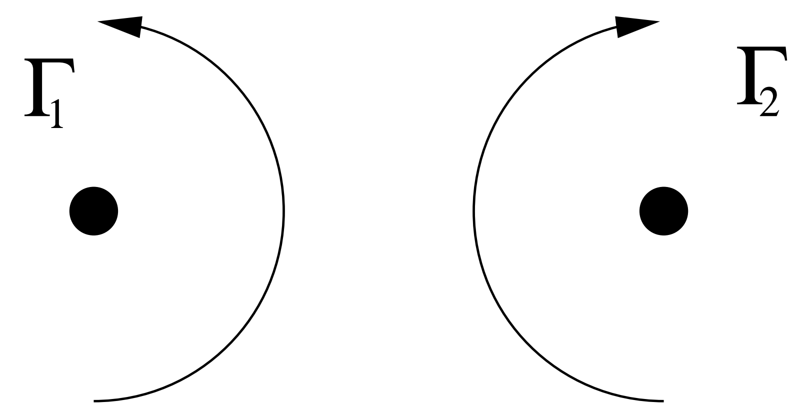
Vortices

Regions of fluid with non-zero vorticity

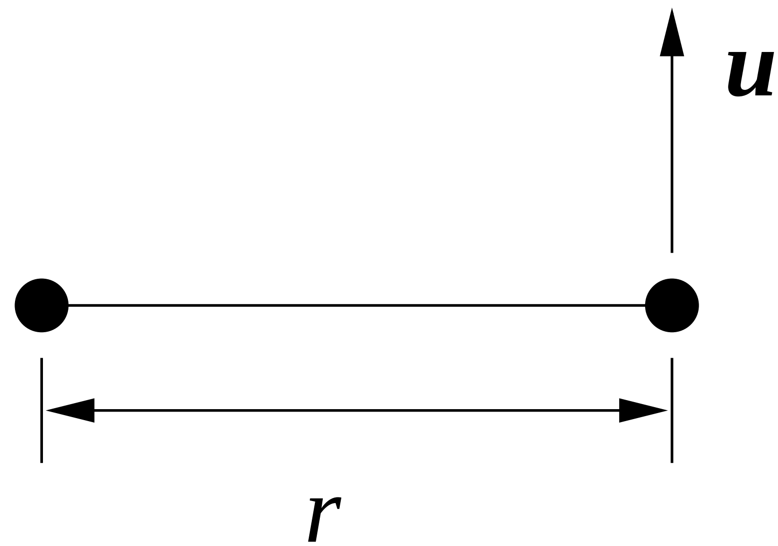
- control the development of the flow (Biot-Savart).
- retain their identity.

"the sinews and muscles of fluid motions"

Vortices in two dimensions

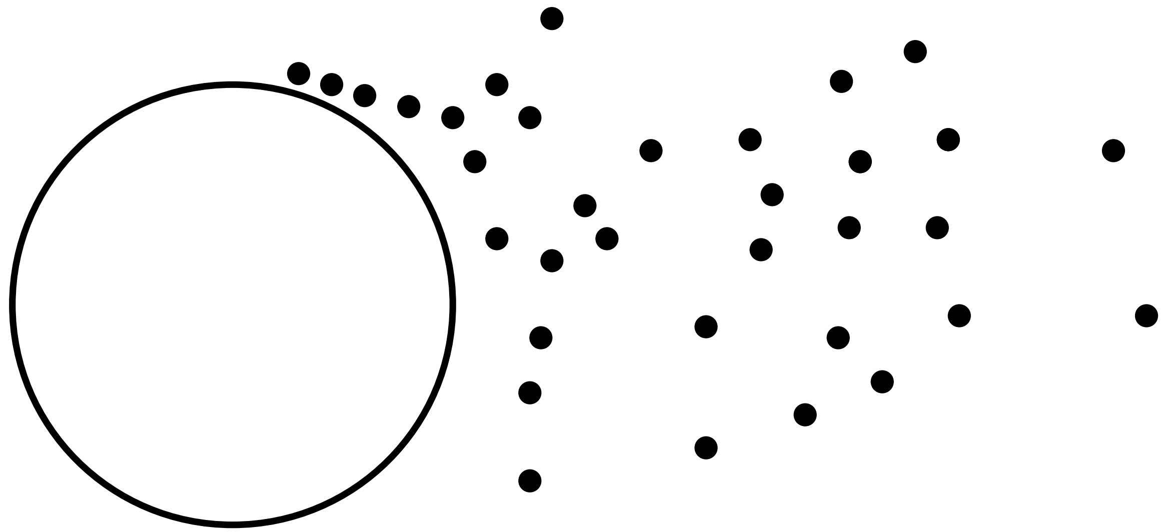


Vortices in two dimensions



Biot-Savart law: $|\mathbf{u}| = \Gamma/2\pi|\mathbf{r}|$.

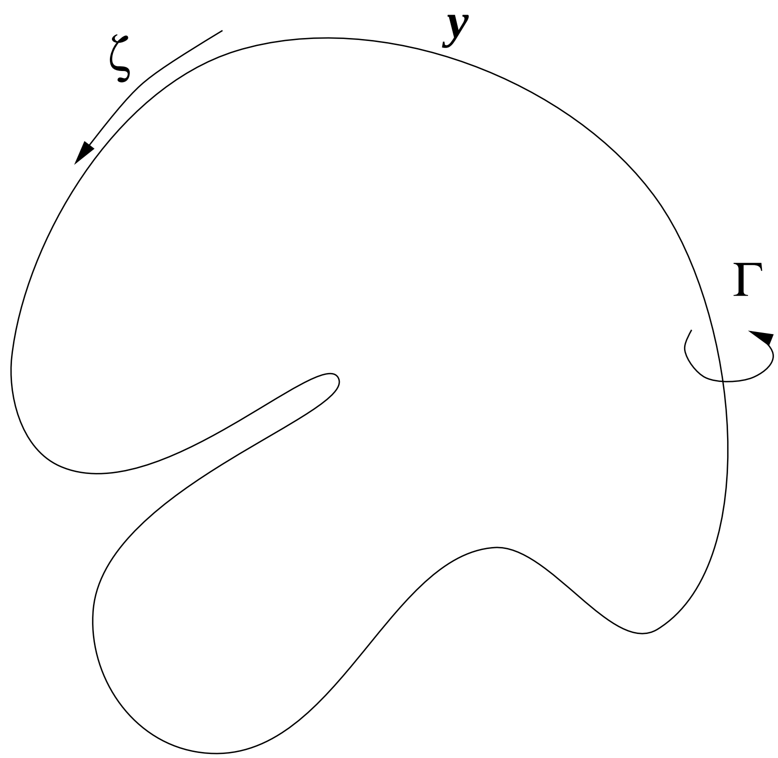
Two-dimensional aerodynamics



Generate a “cloud” of vortices and calculate their effect on each other.

In three dimensions

We deal with vortex lines



Biot-Savart law

Velocity at a point $\mathbf{x}$ due to a filament $\mathbf{y}(\xi)$:

$$\mathbf{u}(\mathbf{x}) = -\frac{\Gamma}{4\pi}\int_{\xi}\frac{(\mathbf{x}-\mathbf{y})\times\dfrac{\partial\mathbf{y}}{\partial\xi}}{|\mathbf{x}-\mathbf{y}|^{3}}\,\mathrm{d}\xi.$$

To calculate the motion of a vortex filament

- assign control points to the curve.
- calculate the velocity of each control point.

Some tweaking is required

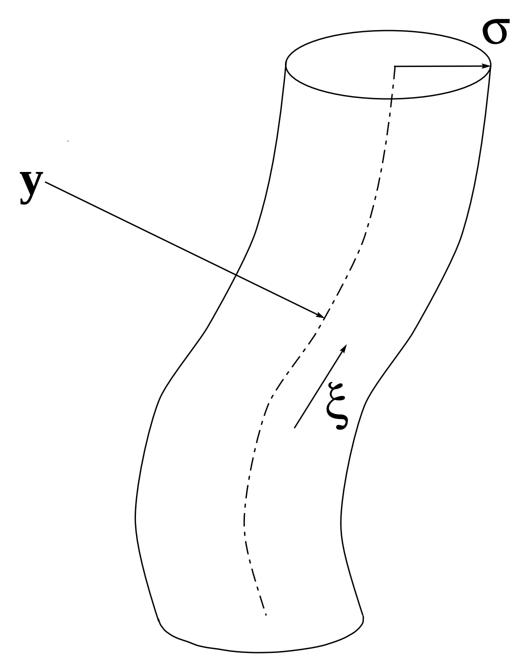
Calculating the self-induced velocity:

$$\frac{(\mathbf{x}-\mathbf{y}) \times \frac{\partial \mathbf{y}}{\partial \xi}}{|\mathbf{x}-\mathbf{y}|^3}$$

is unhappy when $\mathbf{x} \rightarrow \mathbf{y}$.

The self-induced velocity of a three-dimensional filament is not zero.

Finite core thickness



We model the finite thickness of the core, σ.

Modified Biot-Savart law

For *thin filaments*

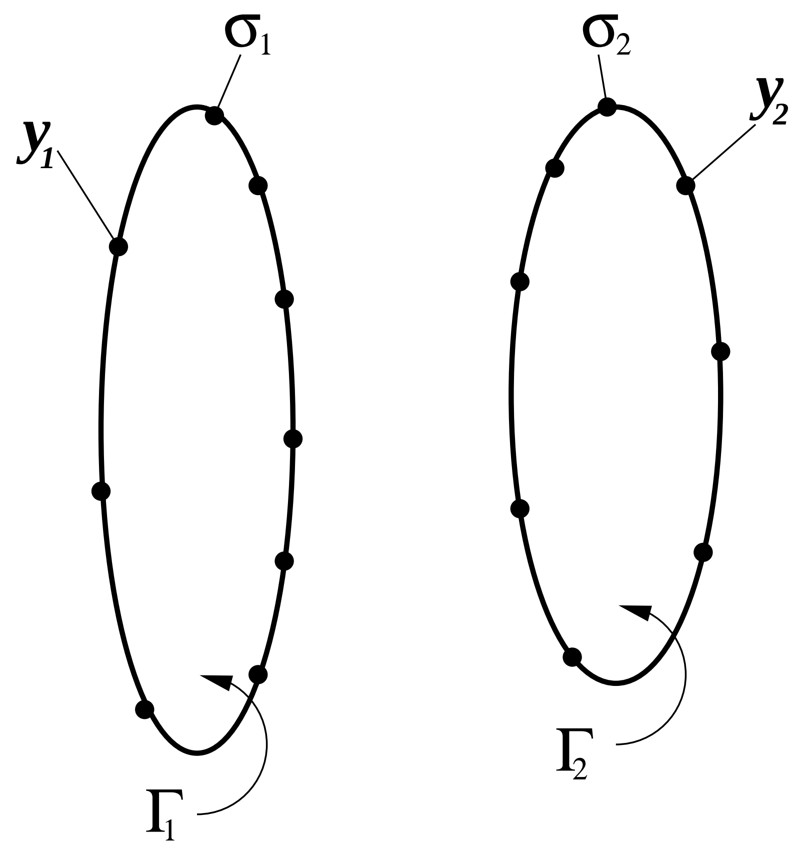
$$\frac{\partial \mathbf{y}_i}{\partial t} = -\sum_{j=1}^{N} \frac{\Gamma_j}{4\pi} \int_{C_j} \frac{\mathbf{y}_i - \mathbf{y}_j(\xi)}{(|\mathbf{y}_i - \mathbf{y}_j(\xi)|^2 + \alpha\sigma_j^2)^{3/2}} \times \frac{\partial \mathbf{y}_j}{\partial \xi}\, \mathrm{d}\xi,$$

with $\mathcal{L}_i \sigma_i^2$ constant

for $\quad i = 1, 2, \ldots, N.$

α depends on the filament thickness and internal structure.

Filament interaction



Codes

Currently two/three codes:

- Matlab (easy to code but slow).
- serial C (harder to code but quick).
- C/MPI (slightly harder to code but quick once it gets started).

Both C and Matlab codes attempt to be as “vectorized” as possible.

Matlab code

The original version:

- mature.
- used for development of methods.
- not being developed any further.

C codes

- much faster than Matlab—bigger problems.
- not as highly developed.
- ported to MPI parallel architecture.

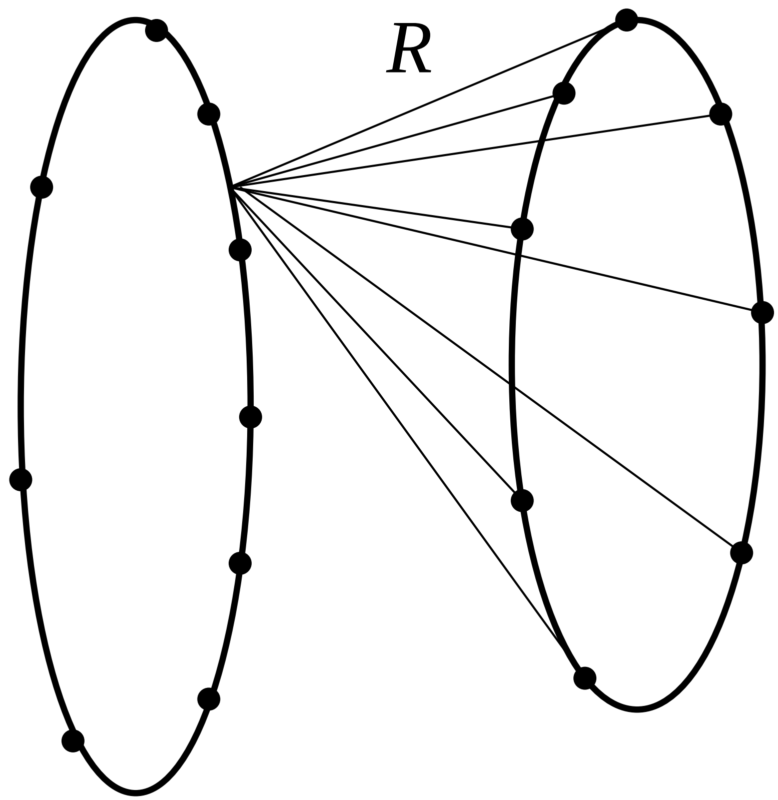
Numerical methods

- Interpolation
  - closed filaments represented by Fourier series.
- Differential equation solver
  - predictor corrector with no extrapolation.
- Numerical integration
  - "vectorized" to reduce trigonometric overhead.

Why Fourier interpolation?

- far fewer points required to represent one ring.
- curvature information is crucial.
- “velocity filtering”.

Vectorized numerical integration



Quadrature method

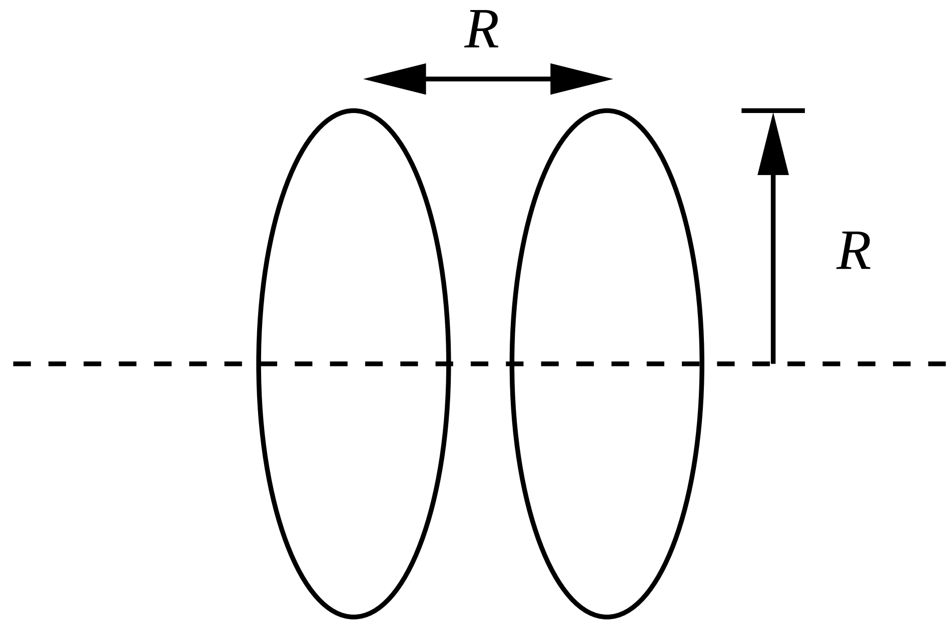
Since every point affects every other point, the velocity is calculated at every point simultaneously;

- reduces number of position evaluations required.
- requires more sophisticated quadrature code.

Some examples

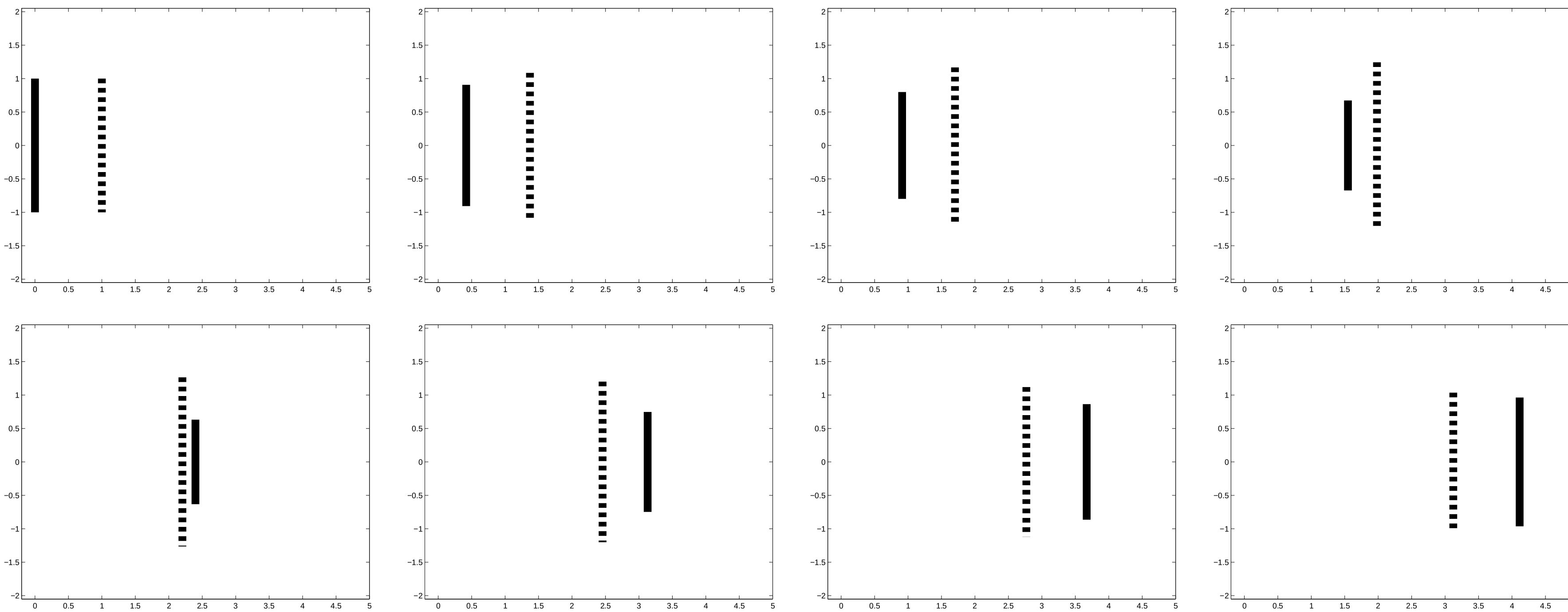
- vortex ring propagation.
- leapfrogging.
- vortex ring collision.
- jet shear layer dynamics.

Leapfrogging

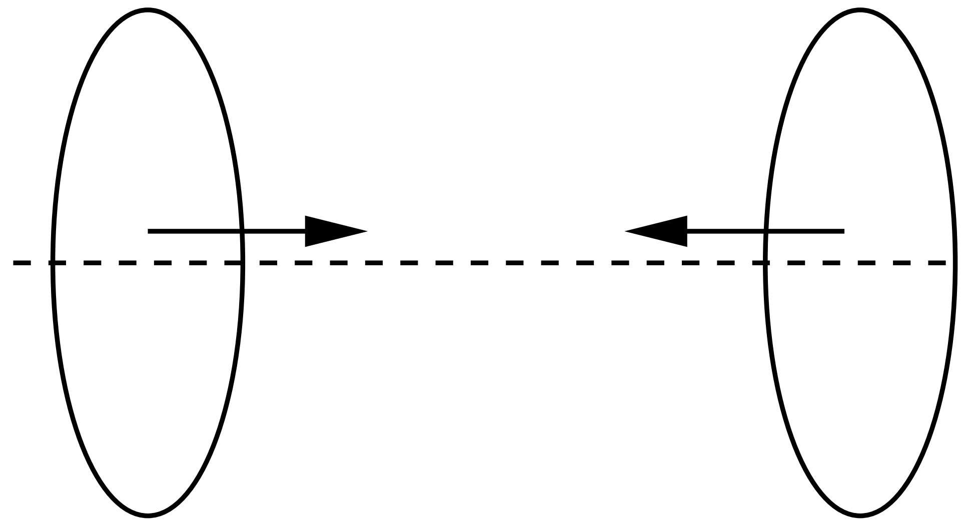


Two vortex rings of identical circulation

Leapfrogging

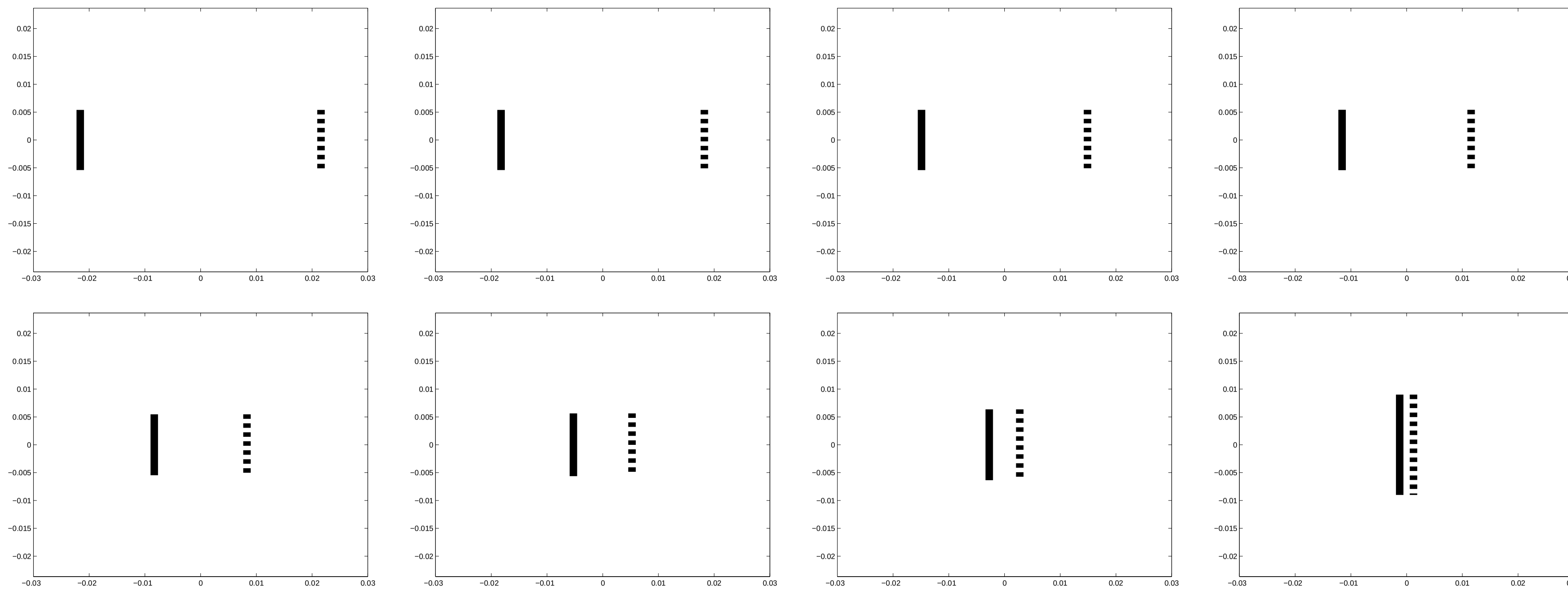


Vortex ring collision

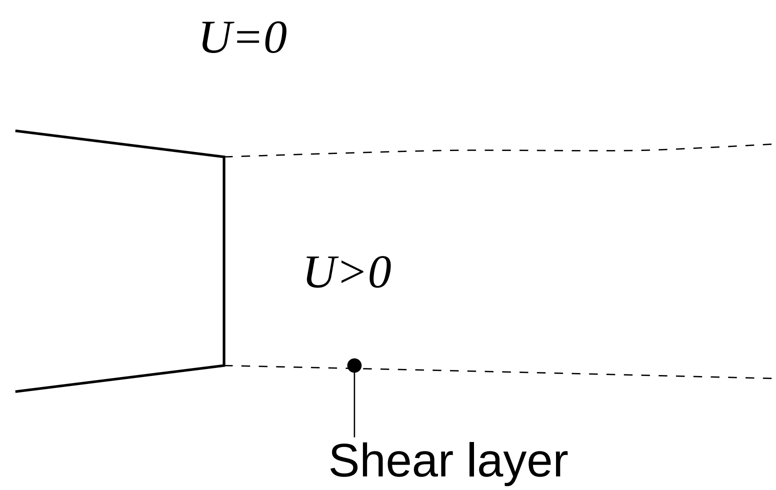


Two vortex rings of equal and opposite circulation

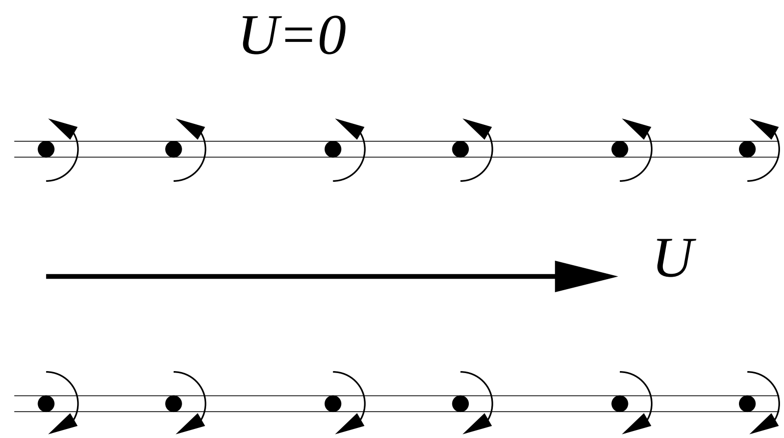
Vortex ring collision



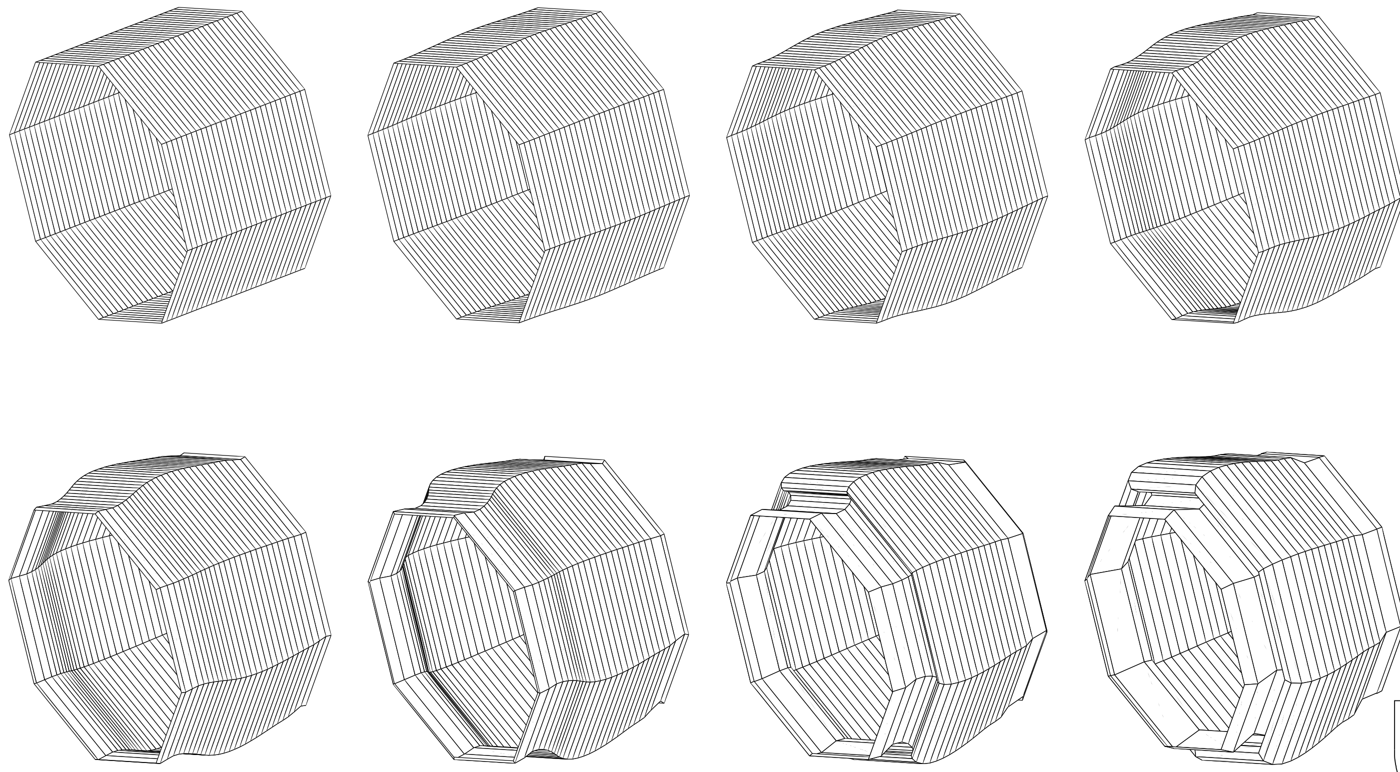
Jet dynamics



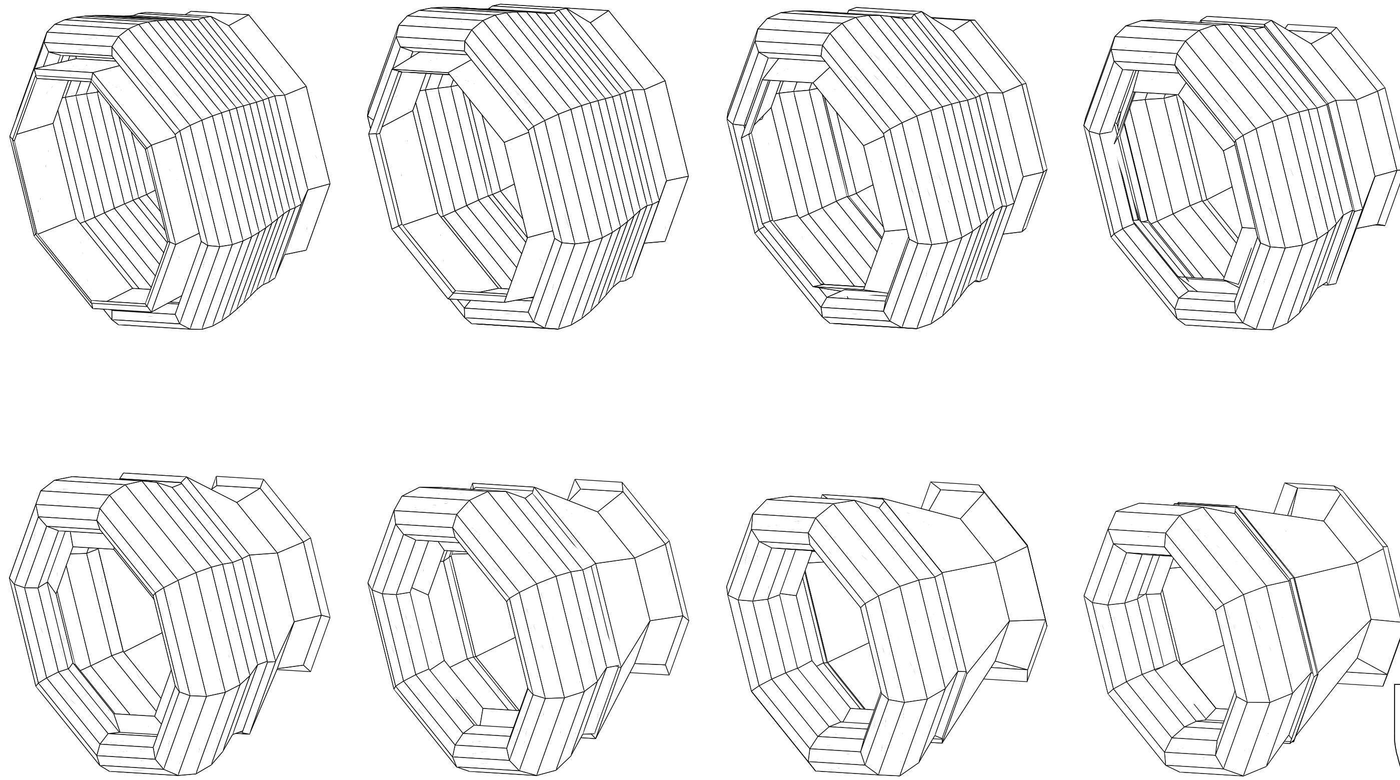
Shear layer dynamics



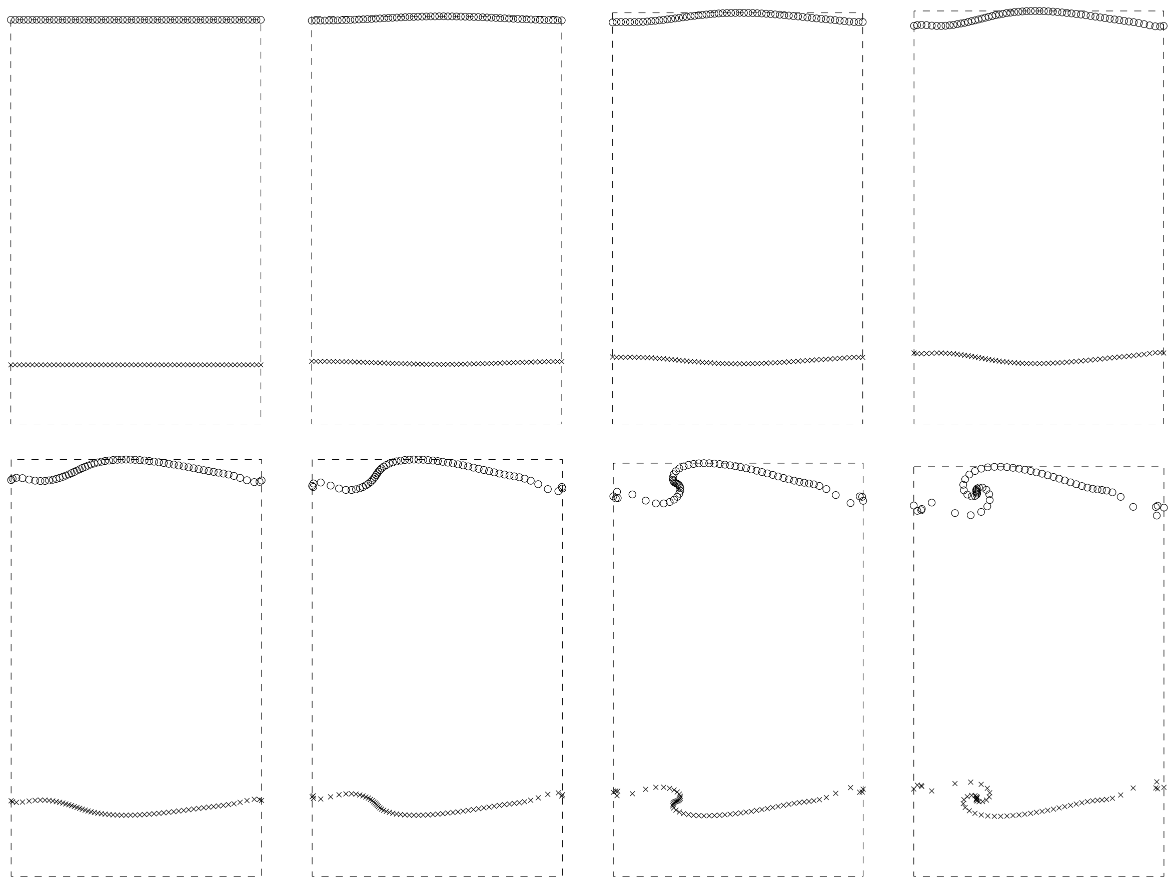
Periodic jet: I



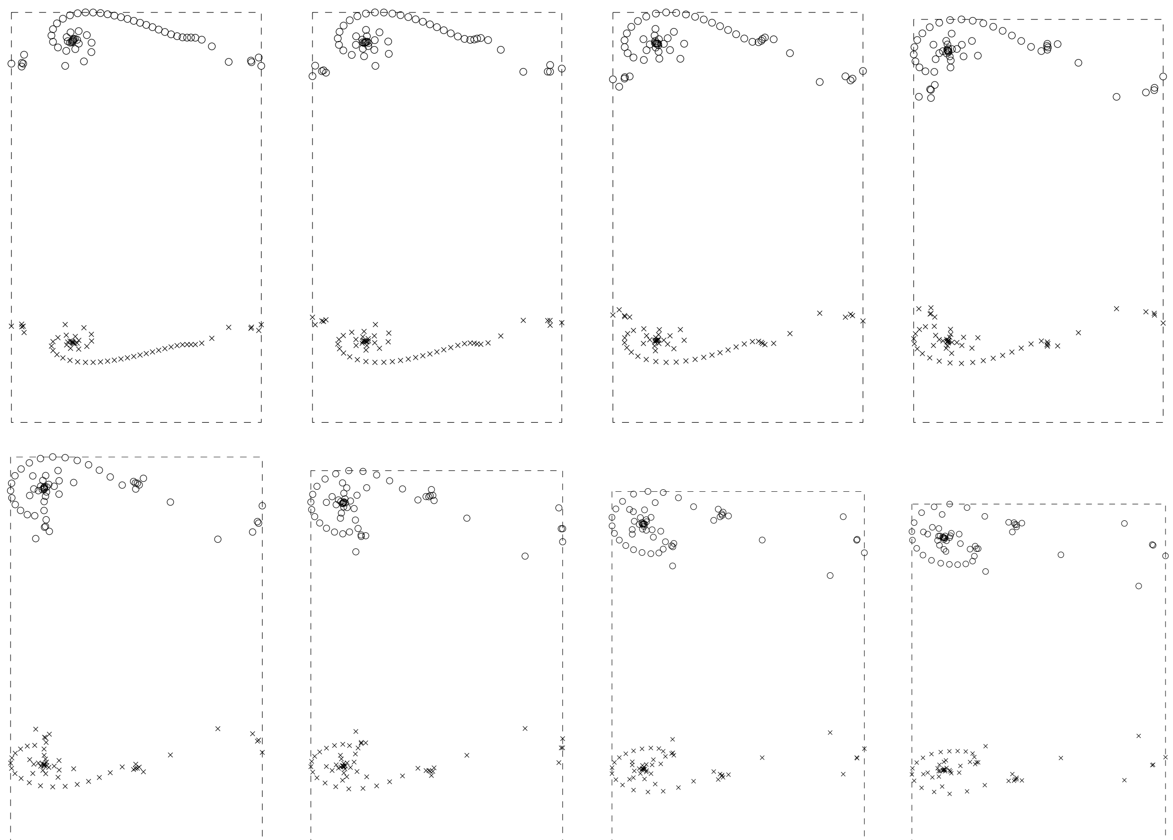
Periodic jet: II



Vorticity dynamics: I



Vorticity dynamics: II



Acoustics

Lighthill says:

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

$$T_{ij} = \rho v_i v_j + (p - c^2 \rho)\delta_{ij}$$

T_{ij} depends on the whole flow field:

- difficult to calculate numerically.
- even more difficult to integrate numerically to calculate sound.

Kambe's theory

Low Mach number incompressible flow depends only on the vorticity so

- the acoustics should depend only on the vorticity distribution.
- the source should depend only on the filaments.

A theory for *far-field* noise from *incompressible, low Mach number flows.*

Kambe's theory

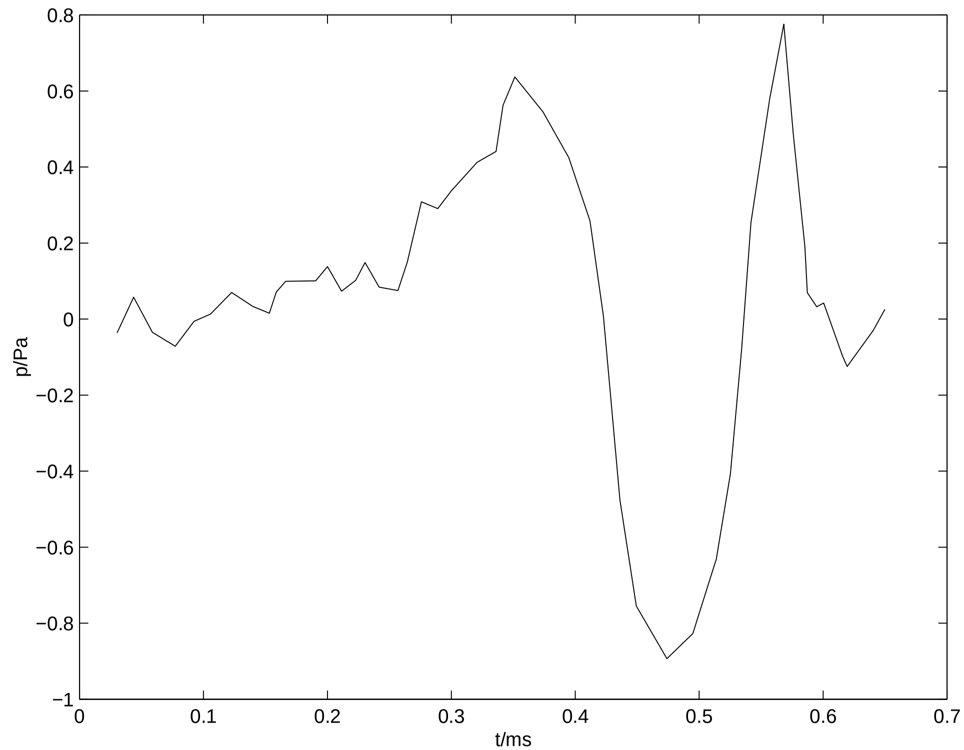
$$p = \frac{\rho_0}{c^2}\frac{x_i x_j}{x^3}\dddot{Q}_{ij}\left(t - \frac{x}{c}\right),$$

$$Q_{ij} = \frac{1}{12\pi}\iiint y_i(\mathbf{y} \times \boldsymbol{\omega})_j \, \mathrm{d}^3\mathbf{y},$$

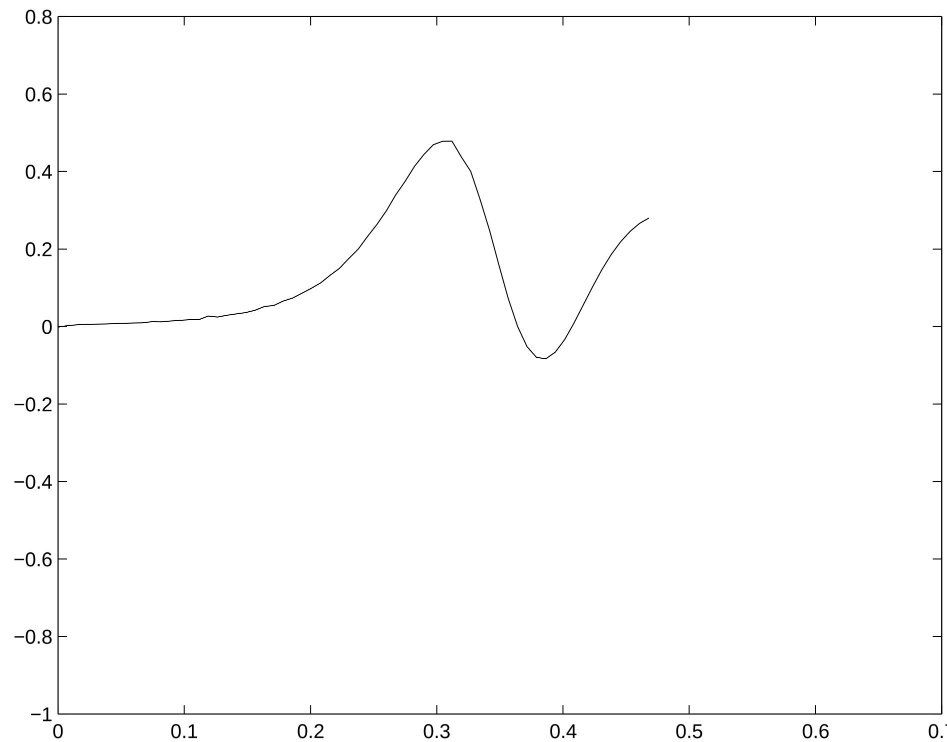
and for filaments

$$Q_{ij} = \frac{\Gamma}{12\pi}\int_C \mathbf{y}_i(\xi)\left(\mathbf{y}(\xi) \times \frac{\partial \mathbf{y}}{\partial \xi}\right) \mathrm{d}\xi.$$

Head-on collision noise: Experimental result



Head-on collision noise: Prediction

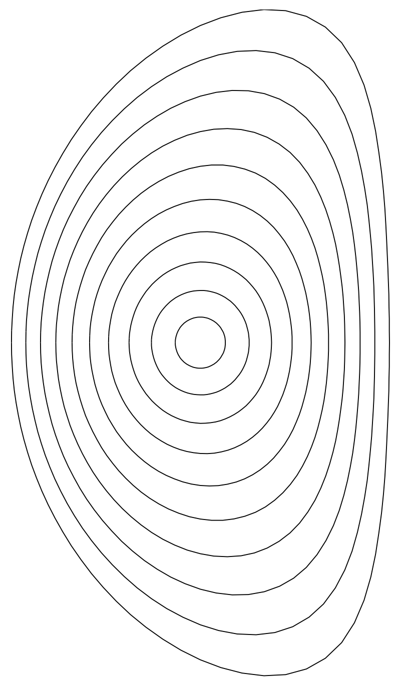


Comparison with experiment

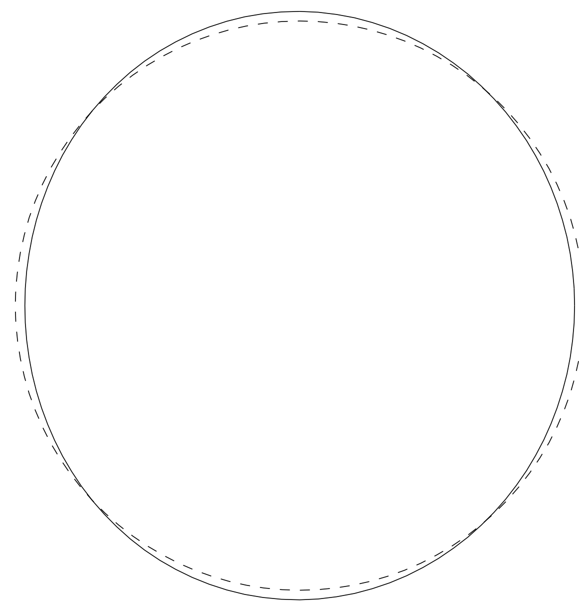
Discrepancies due to

- lack of detail in core model.
- require non-circular core (Norbury ring).
- modelling of core deformation.

Norbury rings

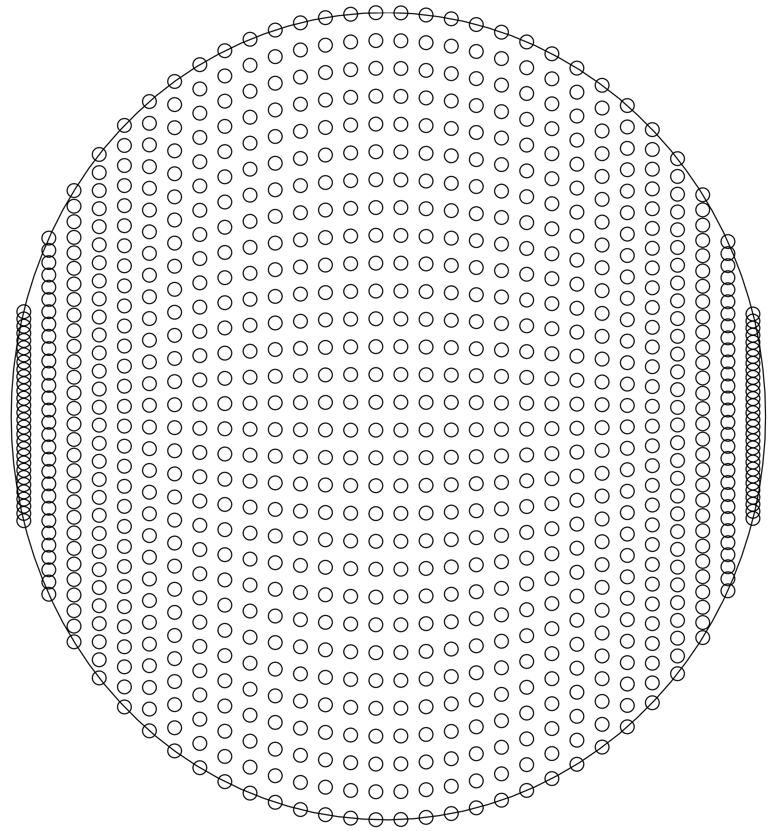


$\alpha = 0.1, 0.1, 1$



$\alpha = 0.2$

Modelling a finite ring



State of play

- One completed code (Matlab)
- Two codes in need of bells and whistles
- Accurate predictions of dynamics and noise