High speed rotor noise

Michael Carley

Being to treat of the Doctrine of Sounds, I hold it convenient to premise something in the general concerning this Theory; which may serve at once to engage your attention, and excuse my pains, when I shall have recommended them, as bestow'd on a subject not altogether useless and unfruitful.

- Carley, M. 1999, Sound radiation from propellers in forward flight, *Journal of Sound and Vibration* **225**(2):353–374.
- Carley, M. 2000, Propeller noise fields, *Journal of Sound and Vibration*, **233**(2):255–277.
- Carley, M. 2001, The structure of wobbling sound fields, to appear in *Journal of Sound and Vibration*.

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Contents

- Propeller noise
- Stationary rotors:
  - acoustic integrals.
  - new computation method.
  - field structure: the acoustic orange.
- Translating propellers:
  - effect of forward flight.
  - conventional and advanced propellers.
- Rotors at incidence:
  - Green's function expansion.
  - asymmetric field.

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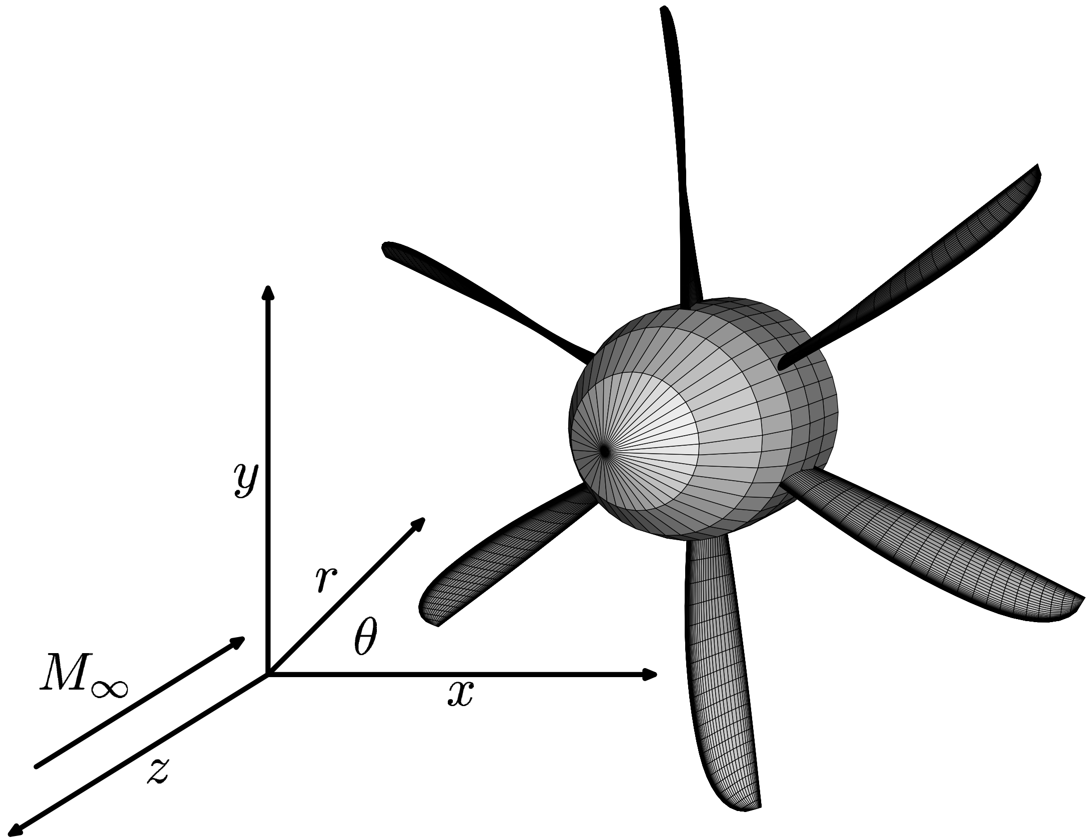
Basic problem



Given:

- geometry.
- loading.

Find:

- noise.

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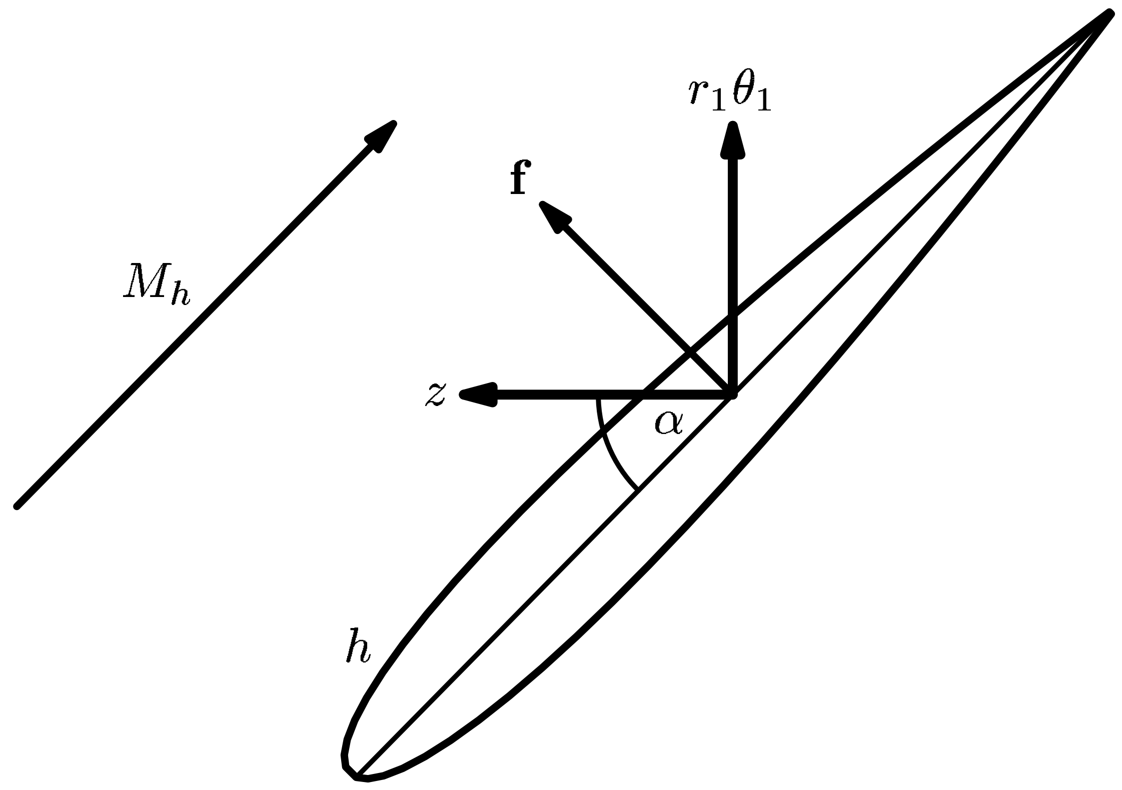
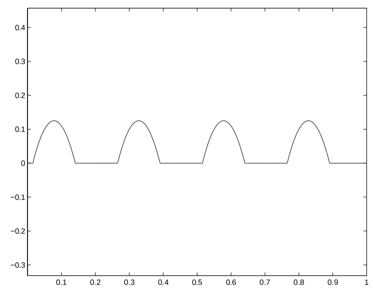
At a blade section



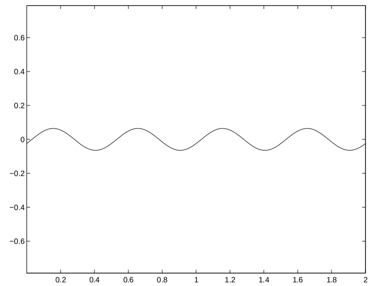
- Thickness noise: momentum injection at the surface: h, $\rho_0 v_n$.
- Loading noise: force on fluid at surface: $\mathbf{f}$.

Frequency domain decomposition

$$f(t) = \Sigma A_n e^{jnB(\Omega t - \theta_1)}$$

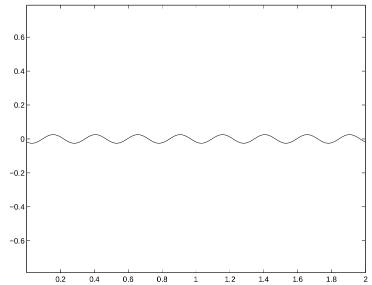


$=$



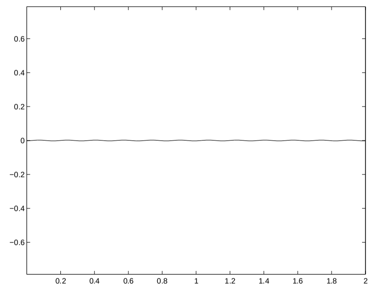
$\exp jB(\Omega t - \theta_1)$

$+$



$\exp j2B(\Omega t - \theta_1)$

$+$



$\exp j3B(\Omega t - \theta_1)$

B: number of blades.

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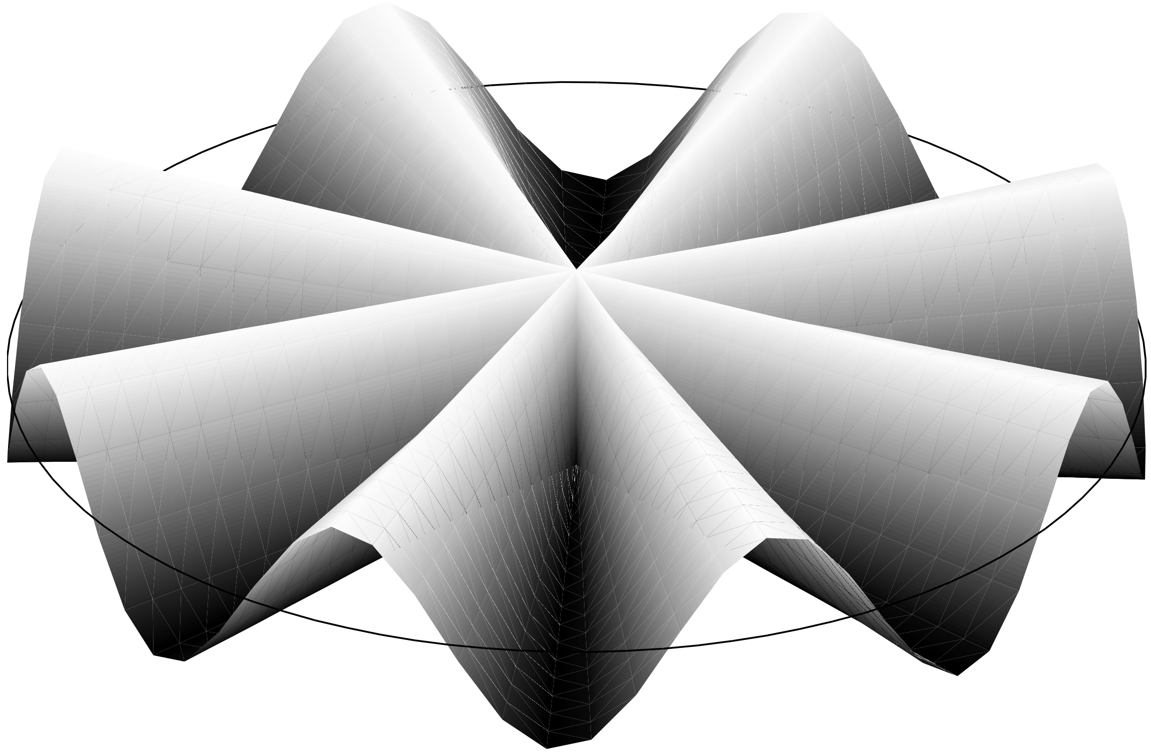
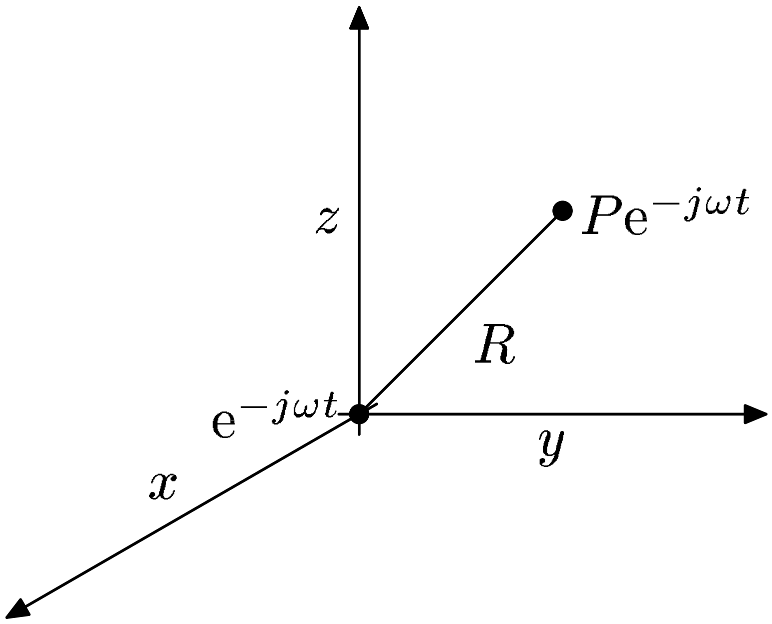
Calculate noise from a disc



Source term: $f_n(r_1)\mathrm{e}^{-\mathrm{j}nB(\Omega t+\theta_1)}$

Noise from a point source



$$p(\mathbf{x}) = P\mathrm{e}^{-\mathrm{j}\omega t},$$

$$\tau = t - R/c, \quad \text{(retarded time)}$$

$$P = \frac{\mathrm{e}^{\mathrm{j}kR}}{4\pi R}, \quad k = \omega/c.$$

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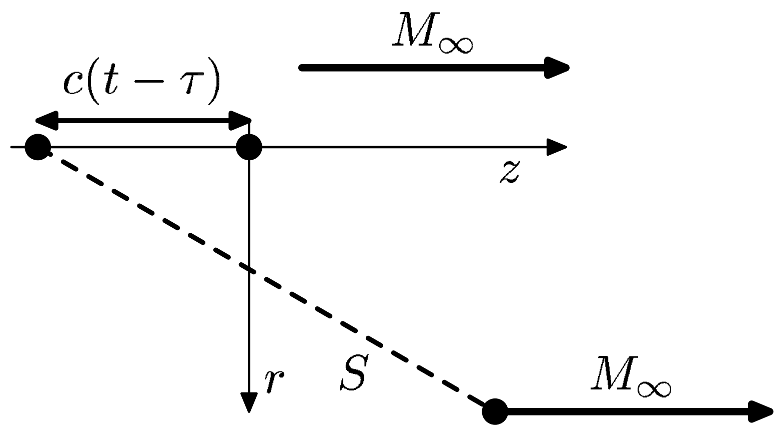
Noise from a point source in motion



$$\tau = t - \sigma/c, \quad \text{(retarded time)}$$

$$P = \frac{\mathrm{e}^{\mathrm{j}k\sigma}}{4\pi S},$$

$$S = [\beta^2 r^2 + z^2]^{1/2}, \quad \sigma = (S + M_\infty z)/\beta^2,$$

$$\beta = (1 - M_\infty^2)^{1/2}.$$

Integral formula

Thickness noise:

$$p_{\mathrm{T}} = -\mathrm{j}n\Omega\rho\frac{\mathrm{D}}{\mathrm{D}t}\int_0^a\int_0^{2\pi}\frac{\mathrm{e}^{\mathrm{j}(k\sigma-n\Omega t-n\theta_1)}}{4\pi S}h_n(r_1)r_1\,\mathrm{d}\theta_1\,\mathrm{d}r_1,$$
$$= -n^2M_{\mathrm{t}}^2\mathrm{e}^{\mathrm{j}n\theta}I(r,\theta,z,n,M_{\mathrm{t}},M_{\infty}),$$

$$I = \int_0^{2\pi}\int_0^1 h_n(r_1)\frac{\mathrm{e}^{\mathrm{j}(k\sigma-n\theta_1)}}{4\pi S^3}\left(S\sigma+\mathrm{j}\frac{M_\infty}{M_{\mathrm{t}}}\frac{z}{n}\right)r_1\,\mathrm{d}r_1\,\mathrm{d}\theta_1$$

Computation

$$
\begin{aligned}
I &= \int_0^{2\pi}\int_0^1 \boxed{h_n(r_1)\mathrm{e}^{-\mathrm{j}n\theta_1}}\ \boxed{\frac{\mathrm{e}^{\mathrm{j}k\sigma}}{4\pi S^3}\left(S\sigma + \mathrm{j}\frac{M_\infty}{M_\mathrm{t}}\frac{z}{n}\right)}\, r_1\,\mathrm{d}r_1\,\mathrm{d}\theta_1, \\
&= \iint \text{Source} \times \text{Propagation}\,\mathrm{d}(\text{Area}).
\end{aligned}
$$

A two-dimensional integral of a highly oscillatory function: $n \simeq 6$–60.

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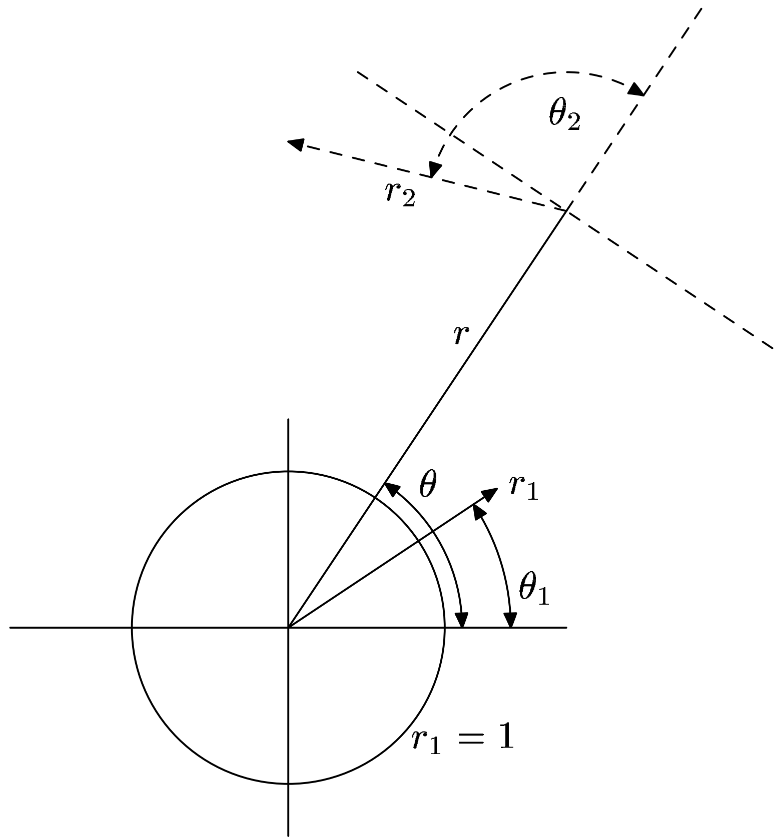
Another view



Use coordinates centred on the sideline

F. Oberhettinger 1961, *J. Res. NBS*, **65B**:1–6.

Chapman, C. J. 1993, *Proc. R. Soc. Lond.* A, **440**:257–271.

New integrals

$$
\begin{aligned}
I &= \int_{\max(0,r-1)}^{r+1} \frac{\mathrm{e}^{\mathrm{j}(k\sigma+n\theta)}}{S^3}\left(S\sigma + \mathrm{j}\frac{M_\infty}{M_\mathrm{t}}\frac{z}{n}\right) J(r,r_2) r_2 \,\mathrm{d}r_2, \\
J(r,r_2) &= \frac{1}{2\pi}\int_{\theta_2^{(0)}}^{2\pi-\theta_2^{(0)}} \mathrm{e}^{-\mathrm{j}n\theta_1}\,\mathrm{d}\theta_2 \\
S &= (\beta^2 r_2^2 + z^2)^{1/2}
\end{aligned}
$$

Set $h_n \equiv 1$; switch coordinate systems; $J(r, r_2)$ does not depend on z. J can be found analytically, we only need a one-dimensional integral to calculate the noise.

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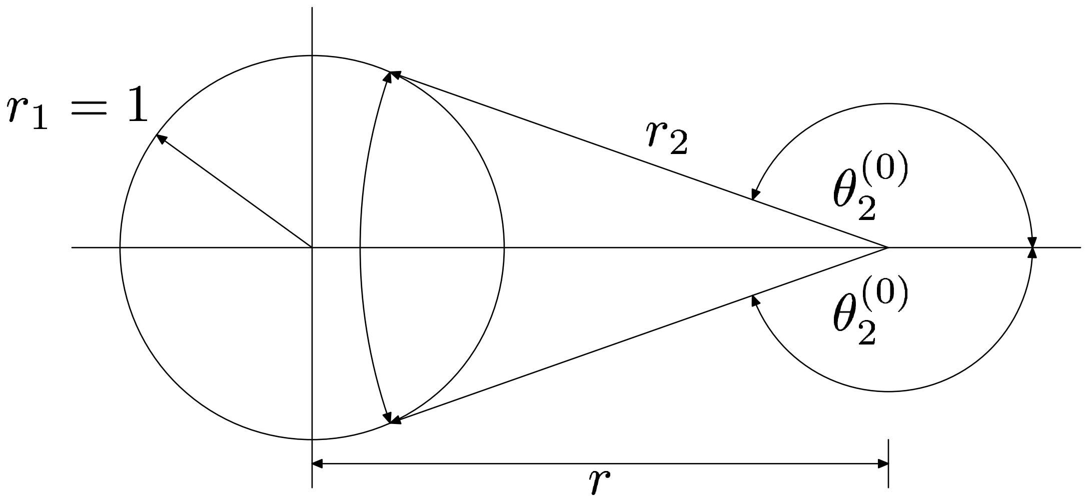
Evaluation of J



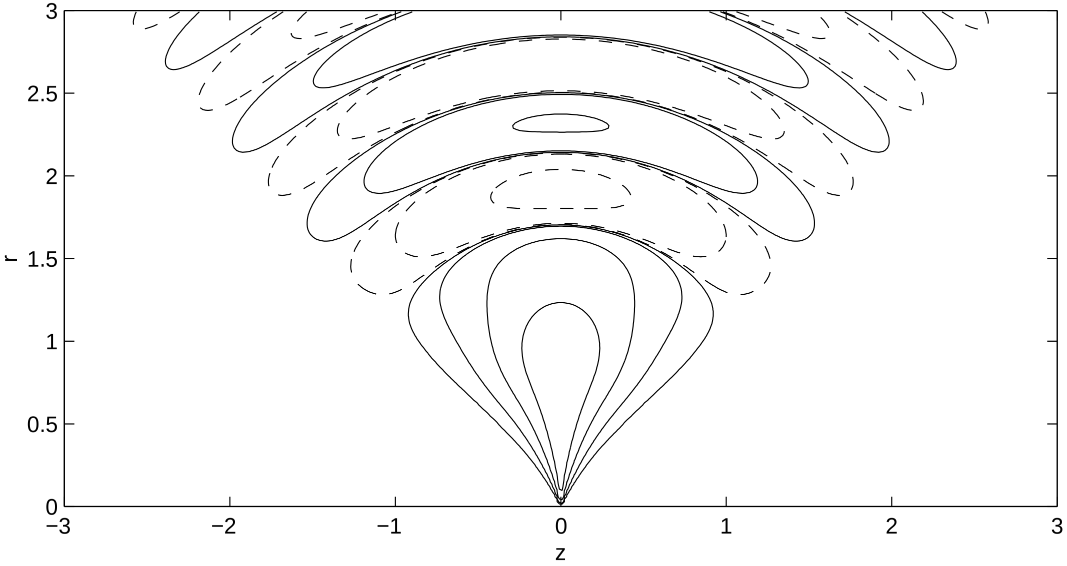
$$J(r, r_2) = \frac{1}{2\pi} \int_{\theta_2^{(0)}}^{2\pi - \theta_2^{(0)}} \mathrm{e}^{-\mathrm{j}n\theta_1} \, \mathrm{d}\theta_2.$$

Evaluation of J

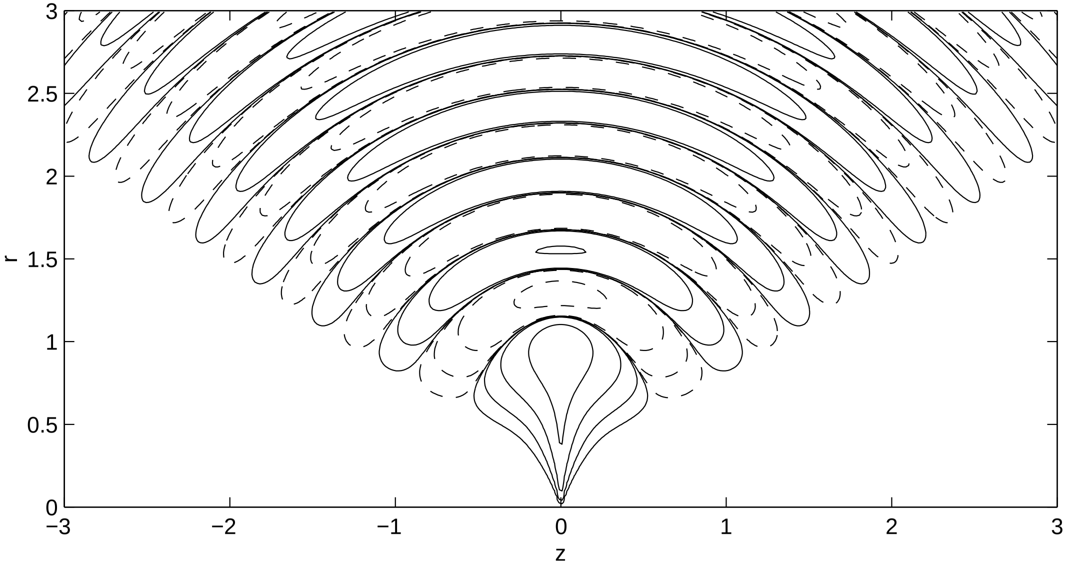
$$
\begin{aligned}
J_{2m} &= \frac{t^{-m}}{\mathrm{j}2\pi}\int_{\mu_0}^{\mu_0^*}\frac{(\mu+t)^m}{(\mu+\eta)^m}\frac{1}{\mu^{m+1}}\,\mathrm{d}\mu,\\
&= -\frac{1}{\pi}\sum_{k=0}^{m}\binom{m}{k}(-t^2)^k\sum_{i=1}^{k+1}\binom{m+k-i}{m-1}(-t)^{1-i}\frac{\sin(1-i)\theta_2^{(0)}}{1-i}\\
&+\frac{1}{\pi}\sum_{k=0}^{m}\binom{m}{k}(-t^2)^k\sum_{i=1}^{m}\binom{m+k-i}{m-i}(1/r)^{1-i}\frac{\sin(1-i)\alpha}{1-i}.
\end{aligned}
$$

$$
t = r_2/r,\ \eta = r/r_2,\ \mu_0 = \exp \mathrm{j}\theta_2^{(0)}.
$$

Thickness noise, $\theta = 0$



$M_t = 0.7$
$M_\infty = 0.0$
$p = \pm 10^{-6,-5,-4,-3}$.



$M_t = 1.05$
$M_\infty = 0.0$
$p = \pm 10^{-5,-4,-3,-2}$.

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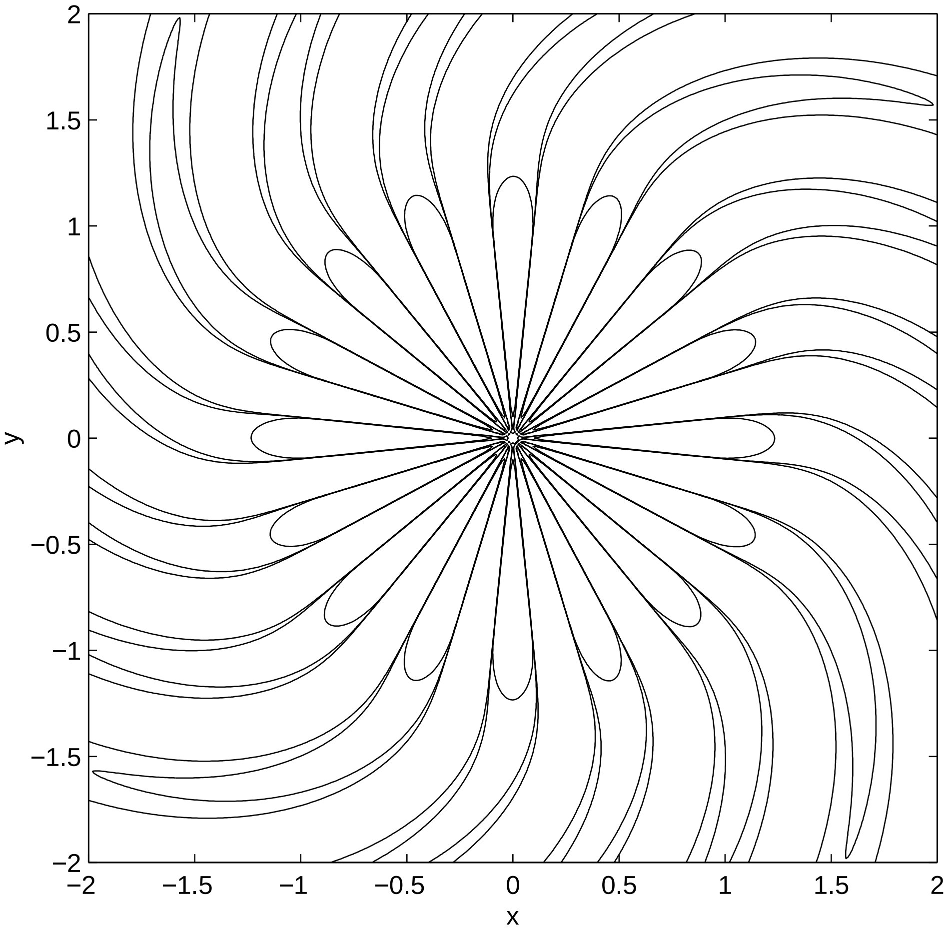
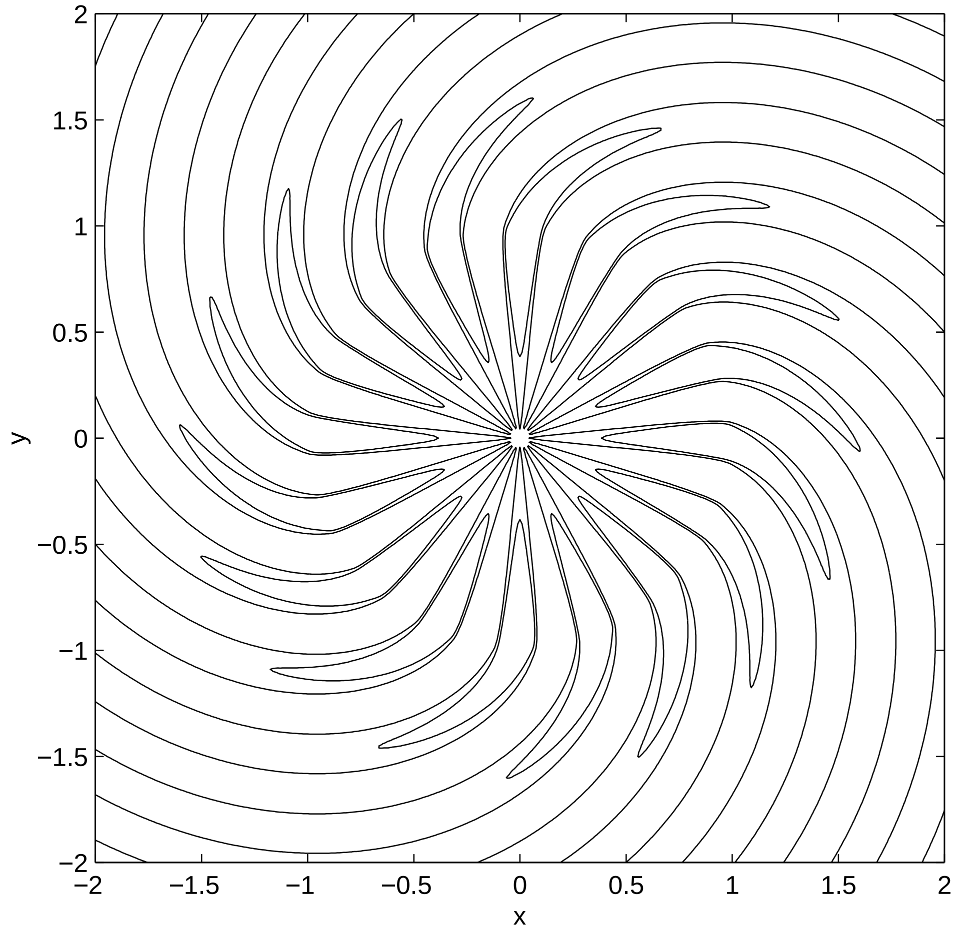
Thickness noise, $z = 0$



$M_\mathrm{t} = 0.7,\ M_\infty = 0.0$
$p = 10^{-5,-4,-3}$



$M_\mathrm{t} = 1.05,\ M_\infty = 0.0$
$p = 10^{-4,-3,-2}$

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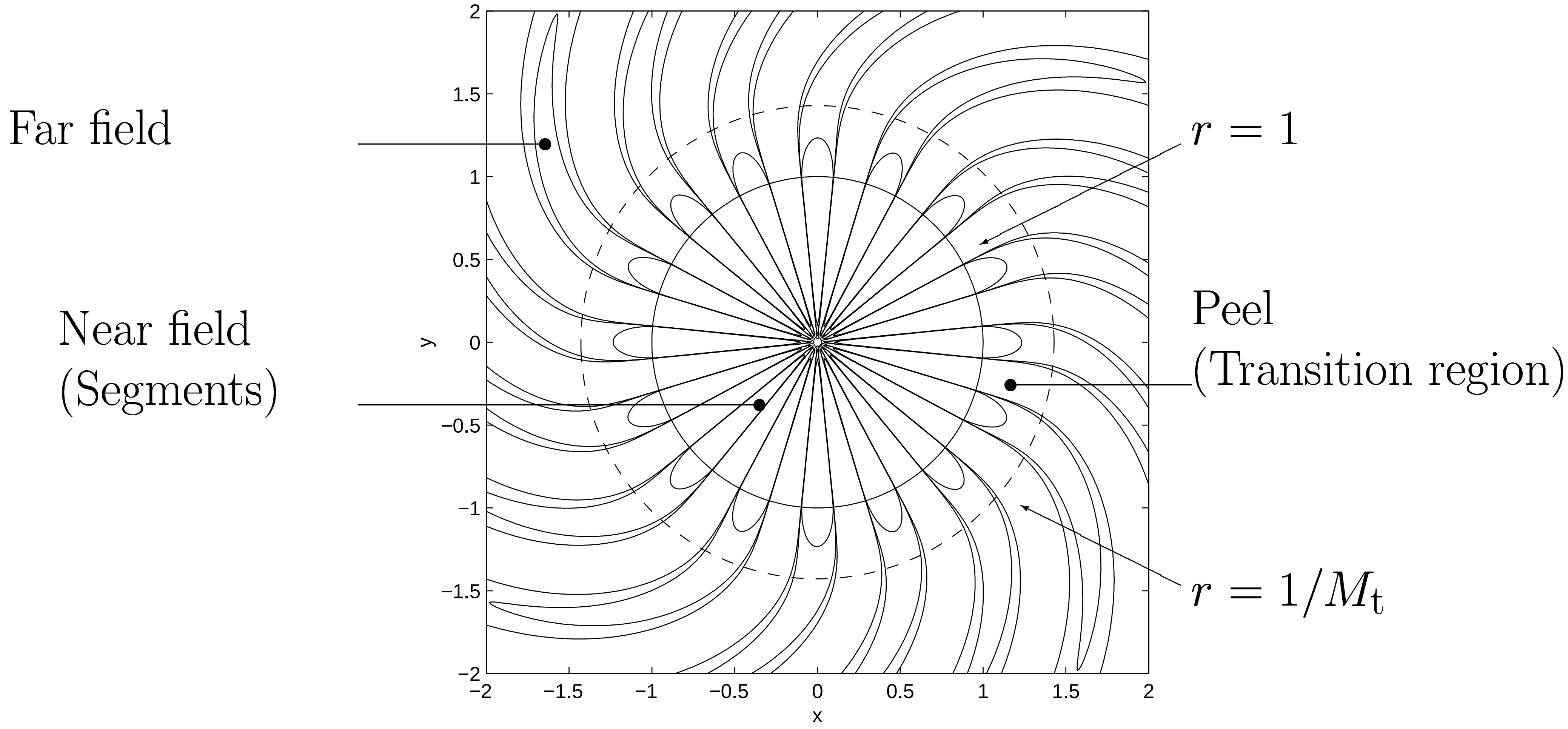
The acoustic orange



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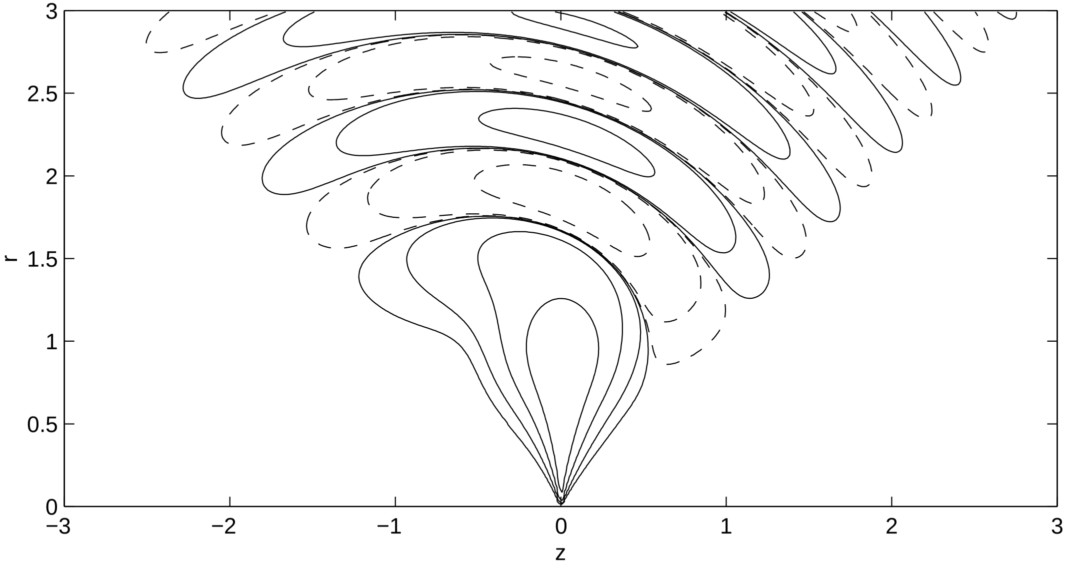
Thickness noise, with flow, $\theta = 0$



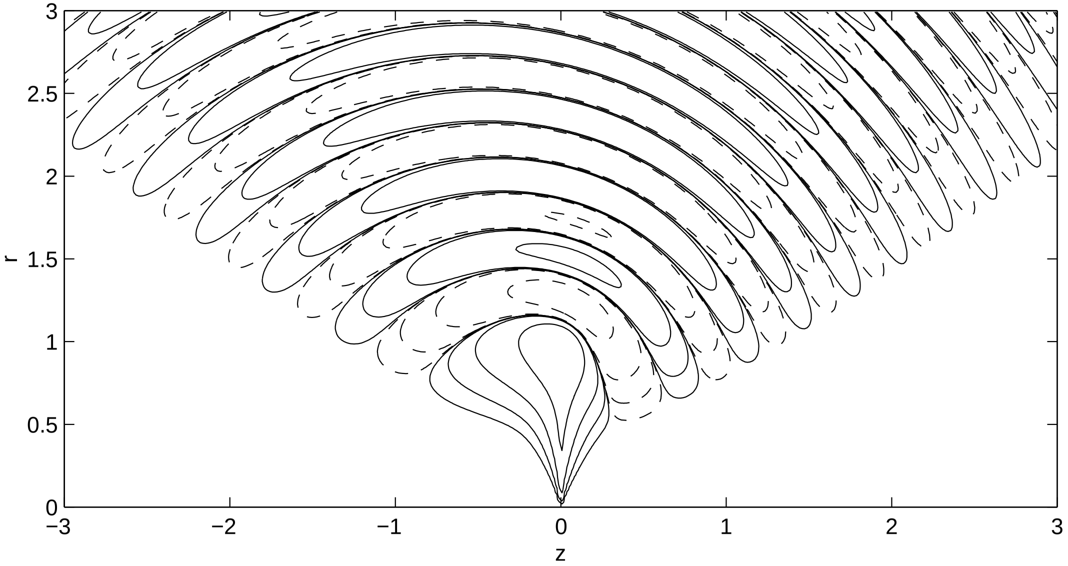
Conventional propeller

Takeoff/Landing

$M_{\mathrm{t}} = 0.7$

$M_{\infty} = 0.2$

$p = \pm 10^{-6,-5,-4,-3}$



Advanced propeller

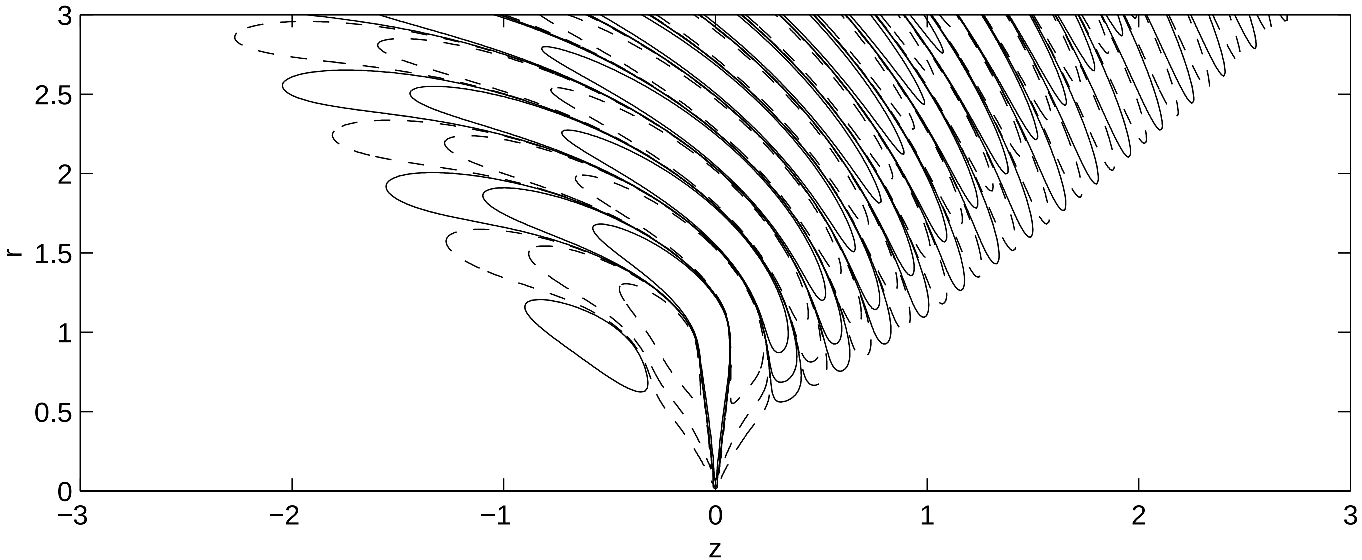
Takeoff/Landing

$M_{\mathrm{t}} = 1.05$

$M_{\infty} = 0.2$

$p = \pm 10^{-5,-4,-3,-2}$

Thickness noise, with flow, $\theta = 0$



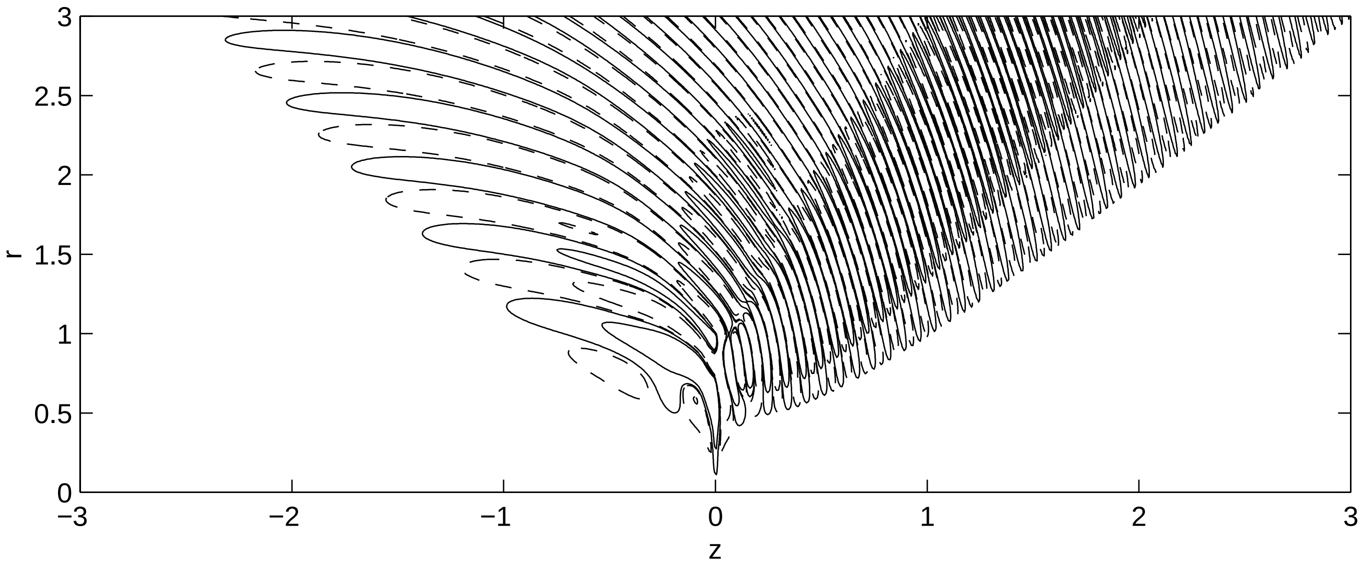
Conventional propeller

Cruise

$M_t = 0.7$

$M_\infty = 0.7$

$p = \pm 10^{-6,-5,-4,-3}$



Advanced propeller

Cruise

$M_t = 1.05$

$M_\infty = 0.8$

$p = \pm 10^{-4,-3,-2}$

Arbitrary source distributions

$$I_{\mathrm{c}}(r_1) = \int_0^{r_1}\int_0^{2\pi} \frac{\mathrm{e}^{\mathrm{j}(k\sigma - n\theta_1)}}{4\pi S^3}\left(S\sigma + \mathrm{j}\frac{M_\infty}{M_{\mathrm{t}}}\frac{z}{n}\right)\,\mathrm{d}\theta_1\, r_1'\,\mathrm{d}r_1' \quad 0 \le r_1 \le 1.$$

differentiating and reinserting

$$\frac{\mathrm{d}I_{\mathrm{c}}}{\mathrm{d}r_1} = \int_0^{2\pi} \frac{\mathrm{e}^{\mathrm{j}(k\sigma - n\theta_1)}}{4\pi S^3}\left(S\sigma + \mathrm{j}\frac{M_\infty}{M_{\mathrm{t}}}\frac{z}{n}\right)\,\mathrm{d}\theta_1\, r_1,$$

integrating by parts

$$I = h_n(1)I_{\mathrm{c}}(1) - h_n(r_0)I_{\mathrm{c}}(r_0) - \int_{r_0}^{1}\frac{\mathrm{d}h_n}{\mathrm{d}r_1}I_{\mathrm{c}}(r_1)\,\mathrm{d}r_1.$$

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Propeller noise

Subsonic propeller noise is

- tip dominated;
- depends on ν with $s \sim S(1 - r_1)^{\nu}$ near the tip

Supersonic propeller noise is

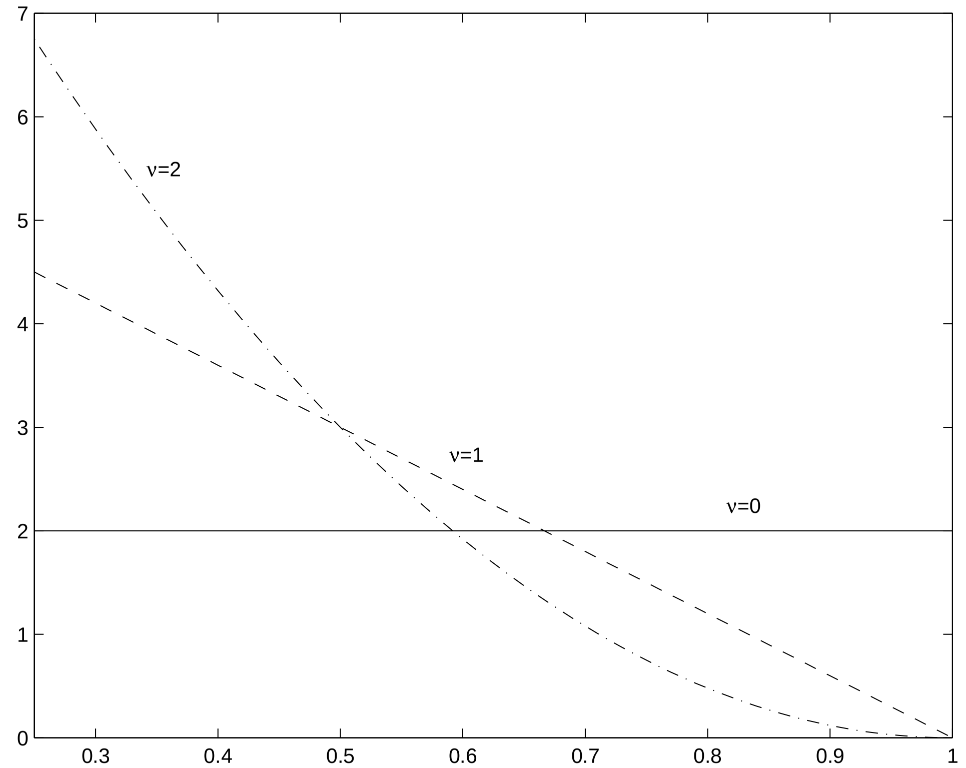
- Mach radius dominated;
- depends on source at Mach radius.

Parametric study

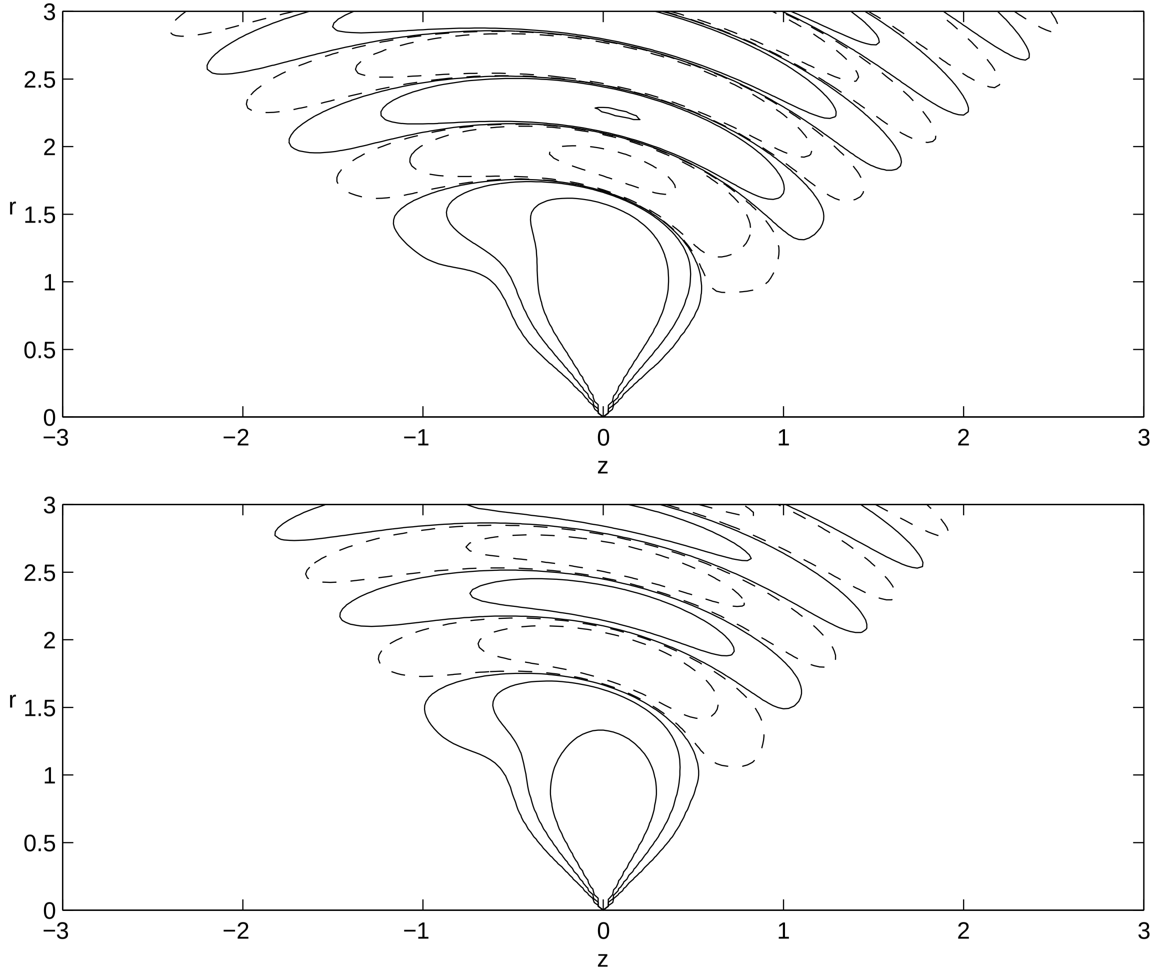
Set source

$$s = (\nu + 1)(\nu + 2)(1 - r_1)^{\nu},$$

s is normalized to give a constant area-weighted source.



Thickness noise: $\nu = 1, 2,\ M_\mathrm{t} = 0.7,\ M_\infty = 0.2.$



$$p = \pm 10^{-6,-5,-4}$$

Asymptotic theory

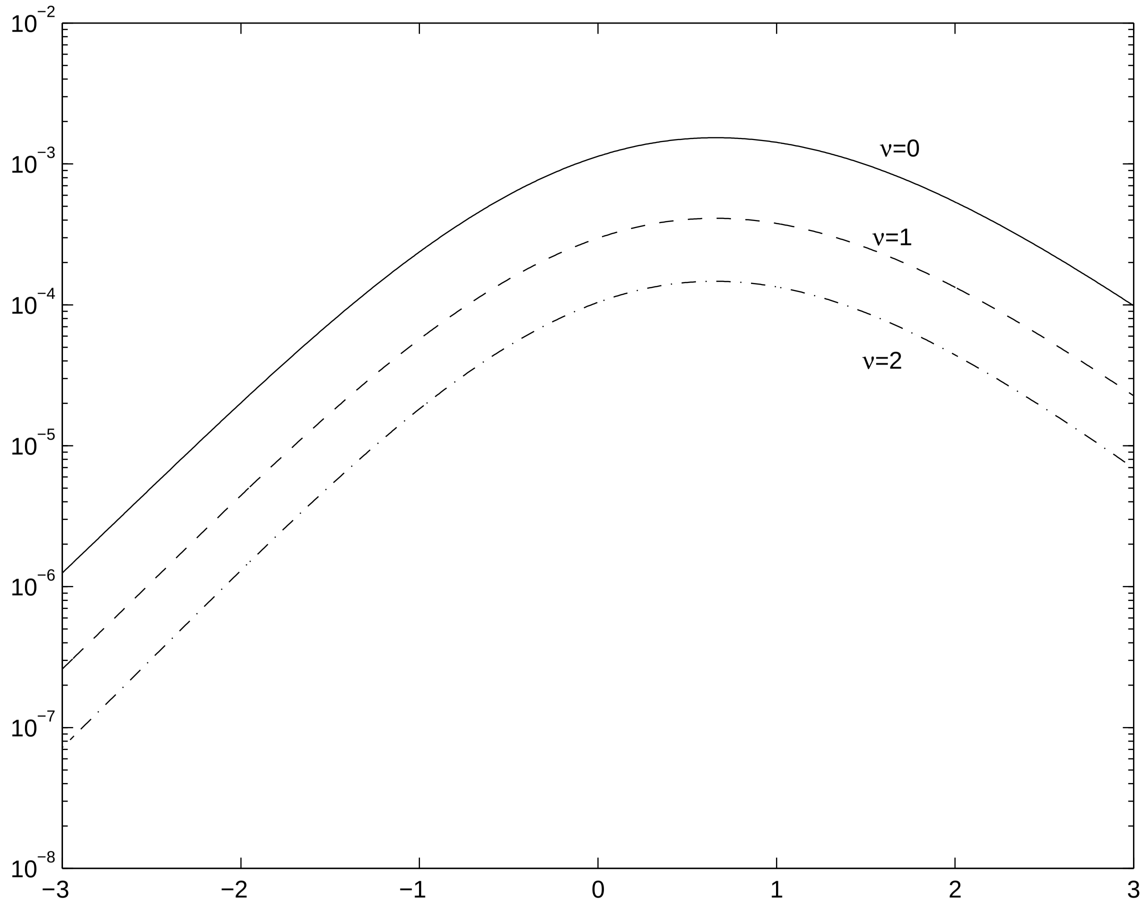
The asymptotic theory for the far-field amplitude predicts:

$$p_n \sim \frac{1}{1 - M_\infty \cos\phi} \frac{(\nu+2)!}{(n \tanh\gamma)^{\nu+1} (n \tanh\gamma)^{1/2}} \mathrm{e}^{n(\tanh\gamma - \gamma)},$$

$$\text{where} \quad \gamma = \operatorname{sech}^{-1}\left[\frac{M_\mathrm{t} \sin\phi}{1 - M_\infty \cos\phi}\right],$$

$$\text{and} \quad \phi = \tan^{-1}\frac{r}{z}.$$

On the outer sideline, $r = 3$:



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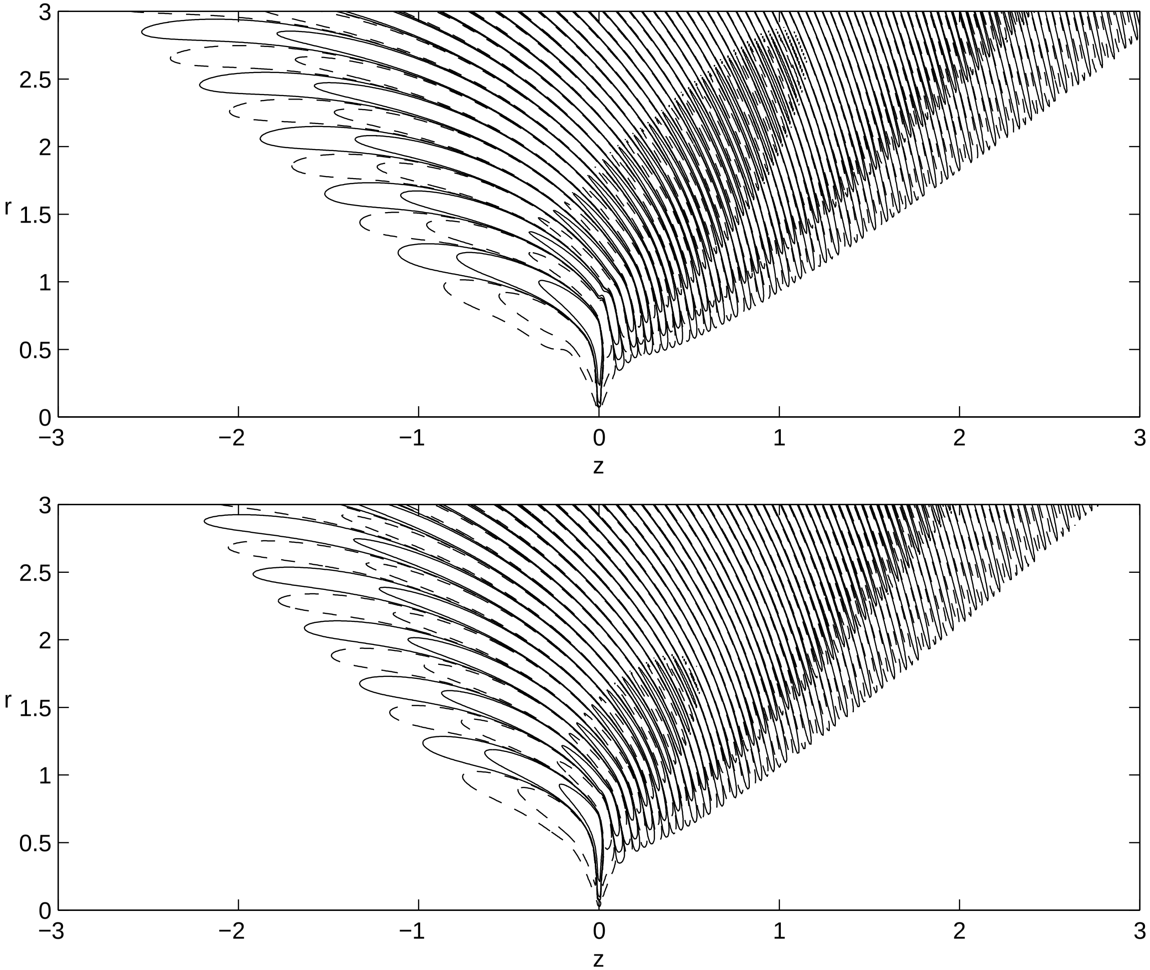
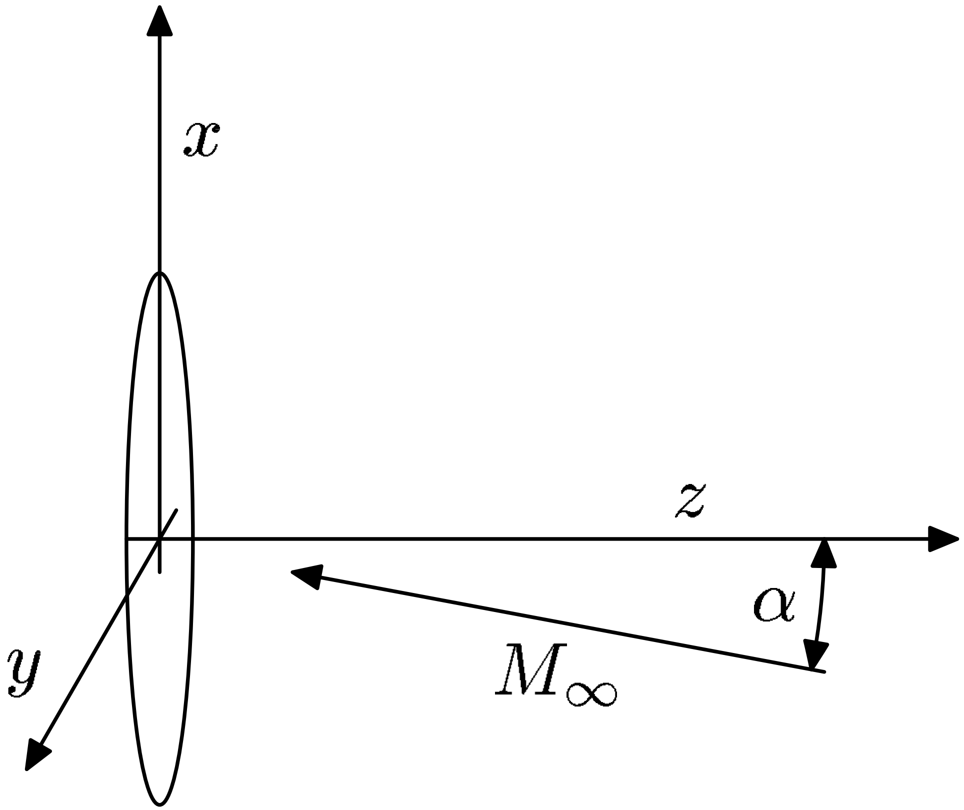
Thickness noise: $\nu = 1, 2$, $M_{\mathrm{t}} = 1.05$, $M_\infty = 0.8$.



$$p = \pm 10^{-3,-2,-1}.$$

Incidence effects



Aerodynamics Unsteady loading and thickness terms.

Acoustics Asymmetric convection of sound.

At incidence

$$\begin{aligned}
P &= \frac{e^{jk\sigma}}{4\pi S}, \\
S &= [\beta^2 R^2 + (M_x r_2 \boxed{\cos(\theta + \theta_2)} + M_z z)^2]^{1/2}, \\
\sigma &= S + M_x r_2 \boxed{\cos(\theta + \theta_2)} + M_z z, \\
R^2 &= r_2^2 + z^2.
\end{aligned}$$

Propagation is not axially symmetric.

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Green's function expansion

$$\frac{\mathrm{e}^{\mathrm{j}kS}}{4\pi S} = \frac{\mathrm{e}^{\mathrm{j}kS_0}}{4\pi S_0}\sum_{p=-\infty}^{\infty} g_p \mathrm{e}^{\mathrm{j}p\theta_2},$$

$$\frac{\mathrm{e}^{\mathrm{j}k\sigma}}{4\pi S} \simeq \frac{\mathrm{e}^{\mathrm{j}k(S_0+M_z z)}}{4\pi S_0}\sum_{p=-P}^{P}\sum_{m=-M}^{M} \mathrm{j}^m g_p J_m(kM_x r_2)\mathrm{e}^{\mathrm{j}(p+m)\theta}\mathrm{e}^{\mathrm{j}(p+m)\theta_2}$$

g_p can be evaluated from power series expansion of $\exp(\mathrm{j}kS)/S$.

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Power series expansion of Green's function

$$\frac{\mathrm{d}^u}{\mathrm{d}C^u}\left(\mathrm{e}^{\mathrm{j}k(S-S_0)}\frac{S_0}{S}\right) = \sum_{q=0}^{u}\binom{u}{v}\frac{\mathrm{d}^u}{\mathrm{d}C^u}\left(\mathrm{e}^{\mathrm{j}k(S-S_0)}\right)\frac{\mathrm{d}^{u-v}}{\mathrm{d}C^{u-v}}\left(\frac{S_0}{S}\right),$$

$$C = \cos(\theta + \theta_2).$$

All the required derivatives can be evaluated exactly using recursion relations or in closed form.

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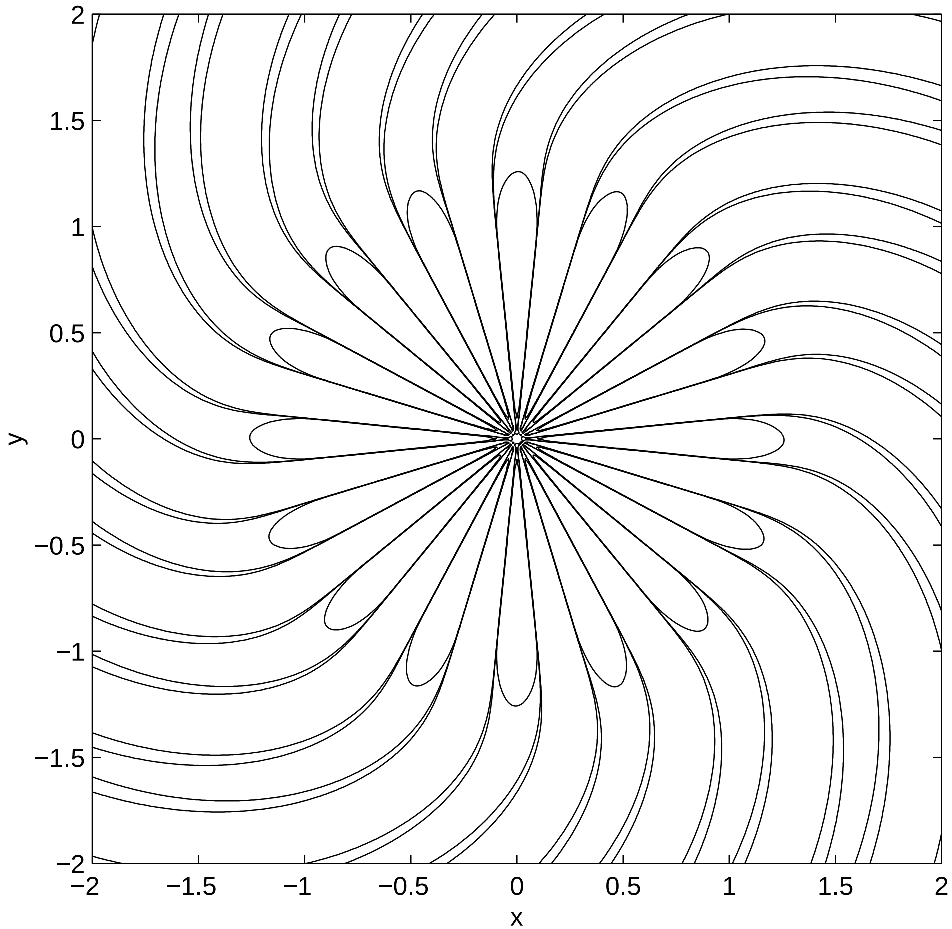
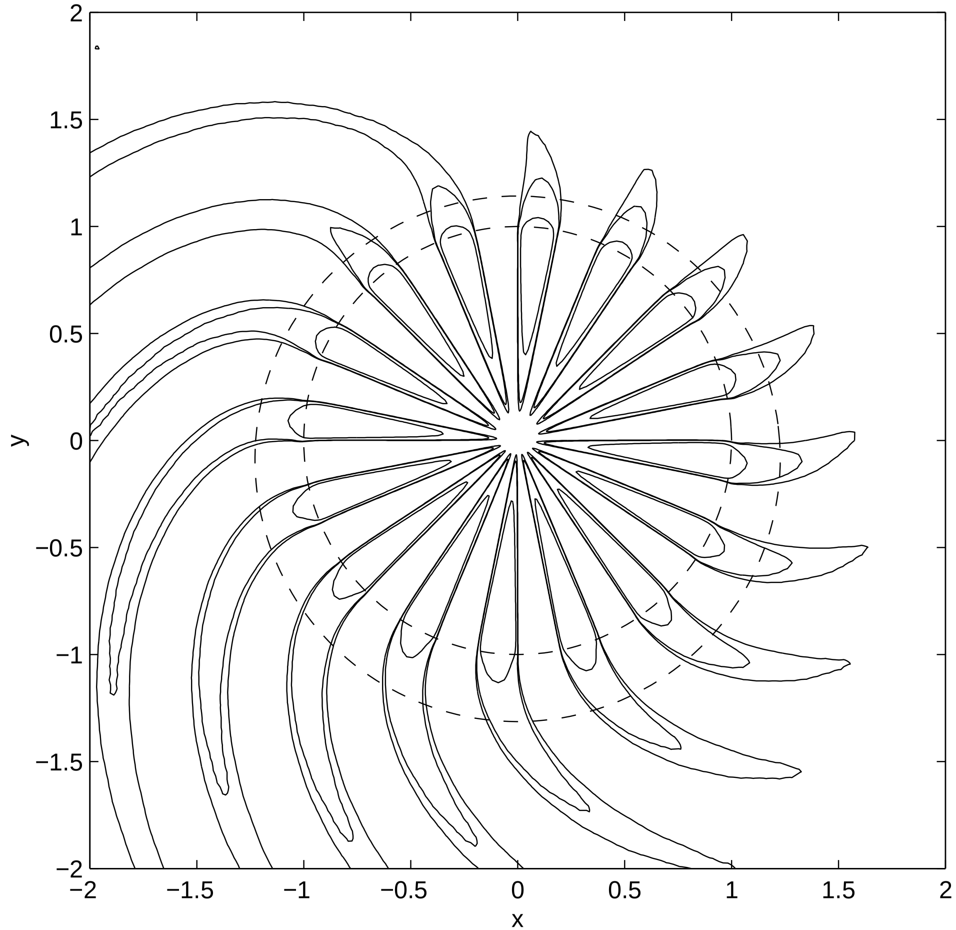
Low speed takeoff





$\alpha = 0^\circ$, $p = 10^{-5,-4,-3}$ $\qquad$ $\alpha = 20^\circ$, $p = 0.01, 0.02, 0.1.$

$$M_{\mathrm{t}} = 0.7,\ M_\infty = 0.2$$

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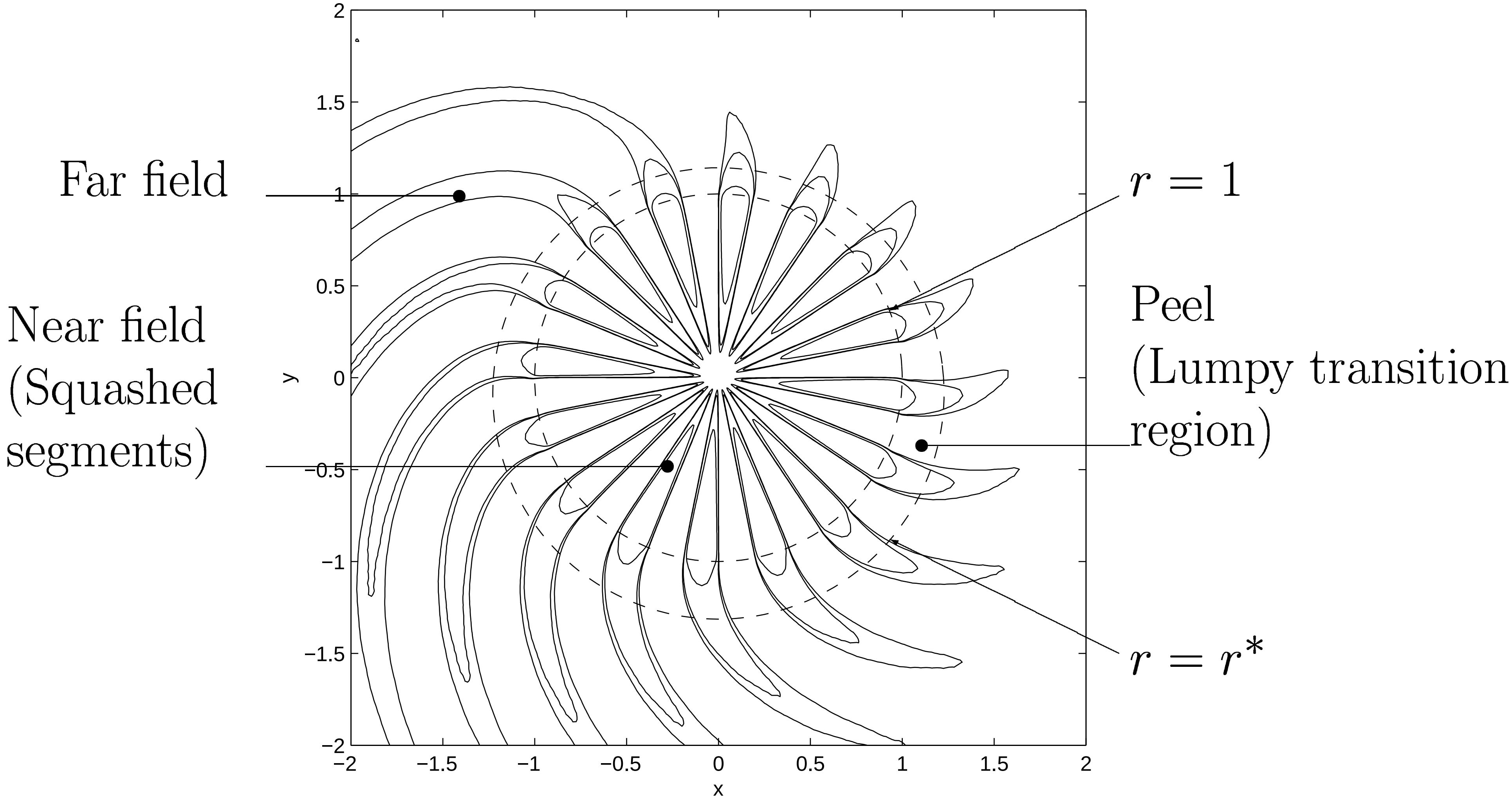
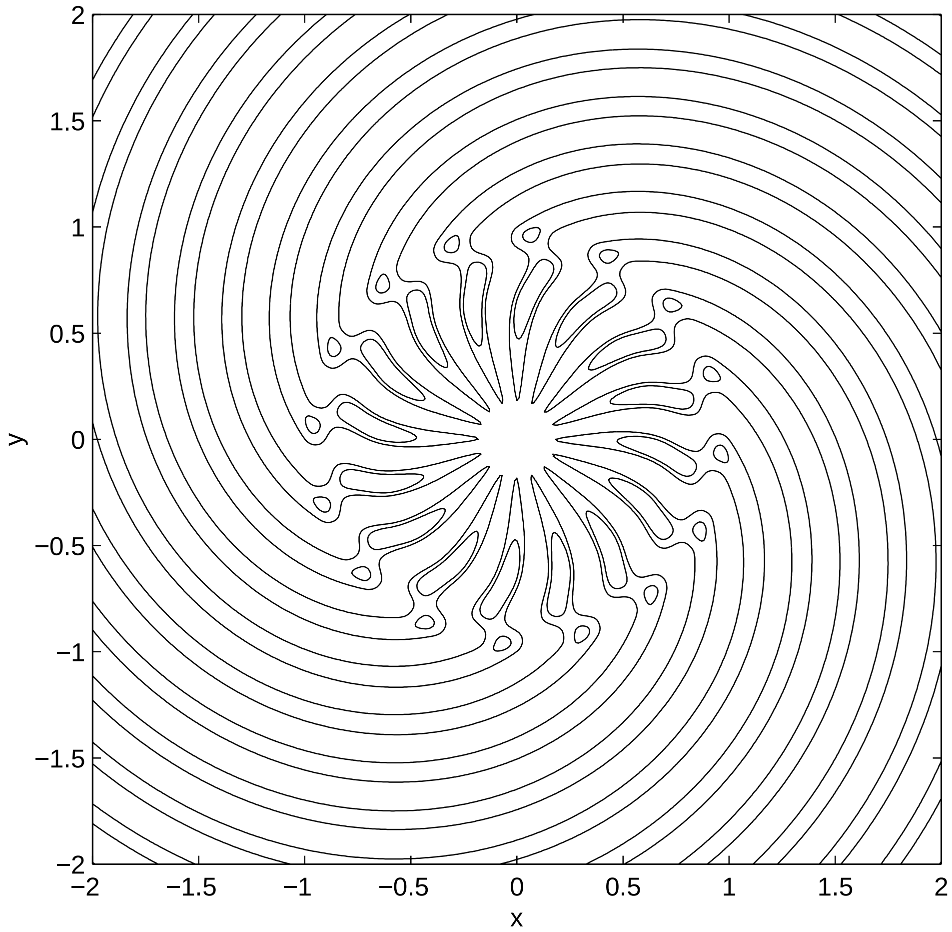
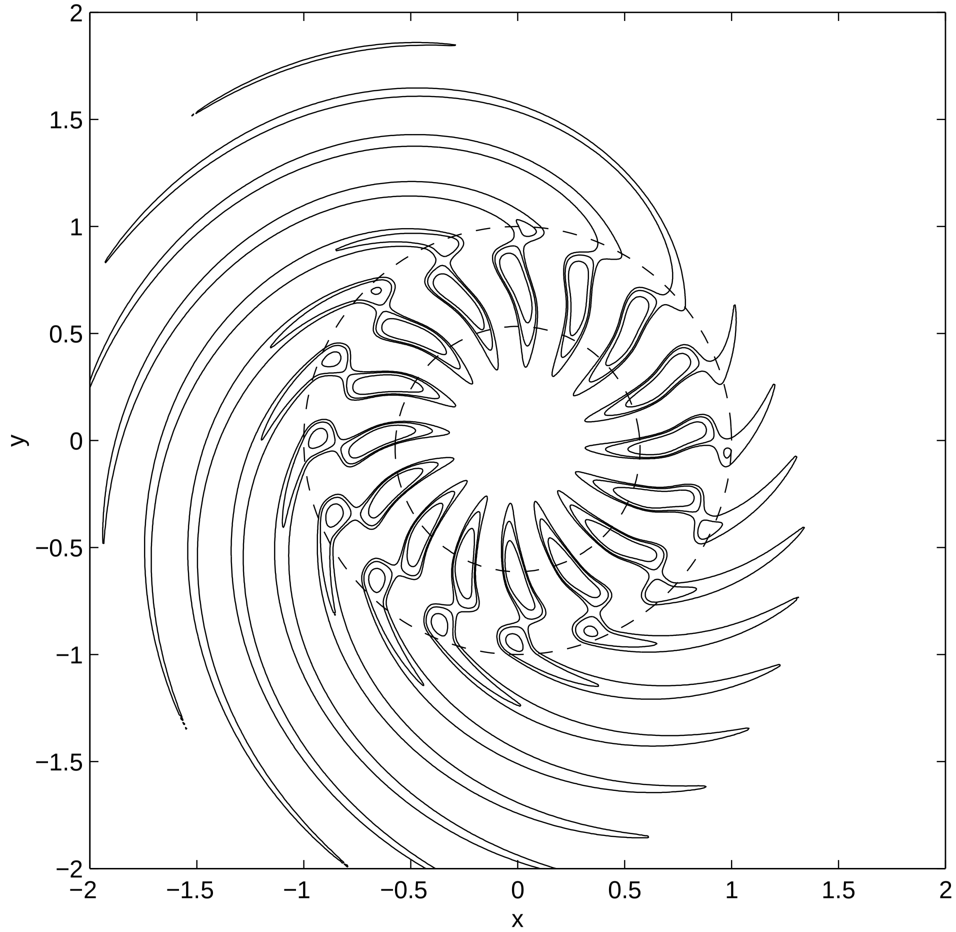
Lumpy orange



$$r^* = -(M_x/M_\mathrm{t})\sin\theta + (M_x^2\sin^2\theta + \beta^2)^{1/2}/M_\mathrm{t}$$

High speed cruise





$\alpha = 0^\circ$, $p = 10^{-2,-1}$

$\alpha = 3^\circ$, $p = 2, 1, 0.5$

$M_\mathrm{t} = 1.05$, $M_\infty = 0.8$

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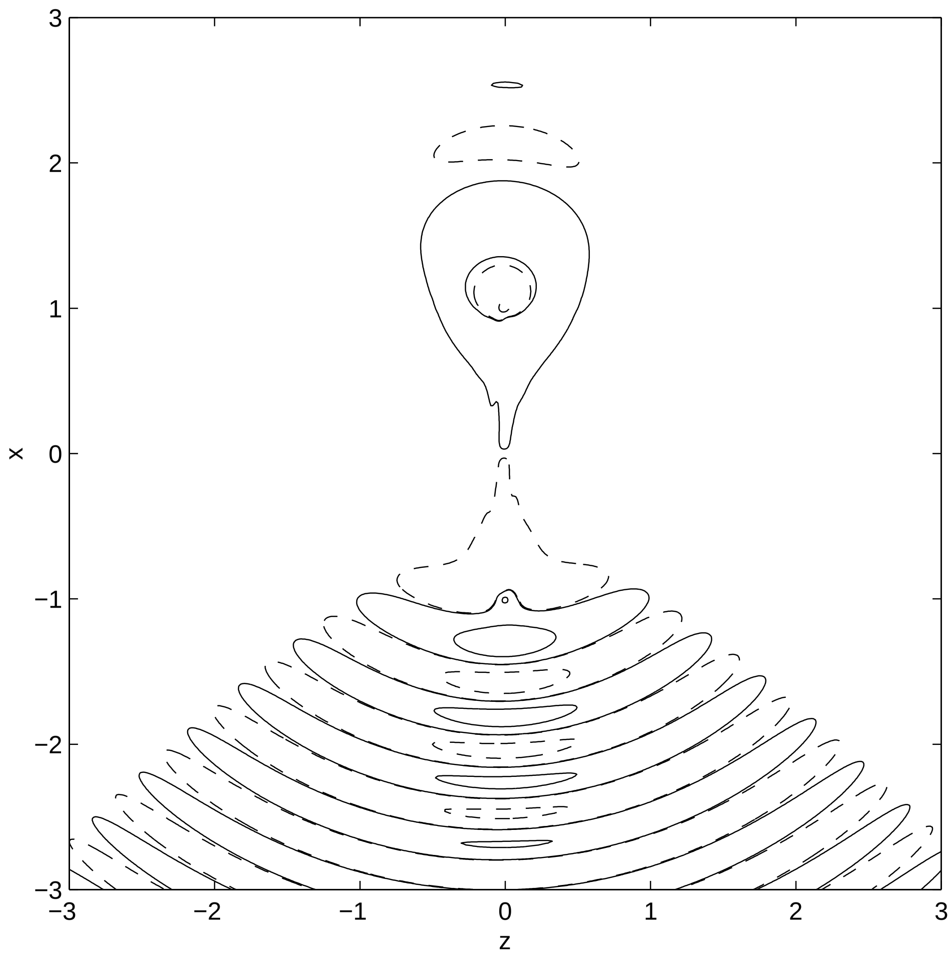
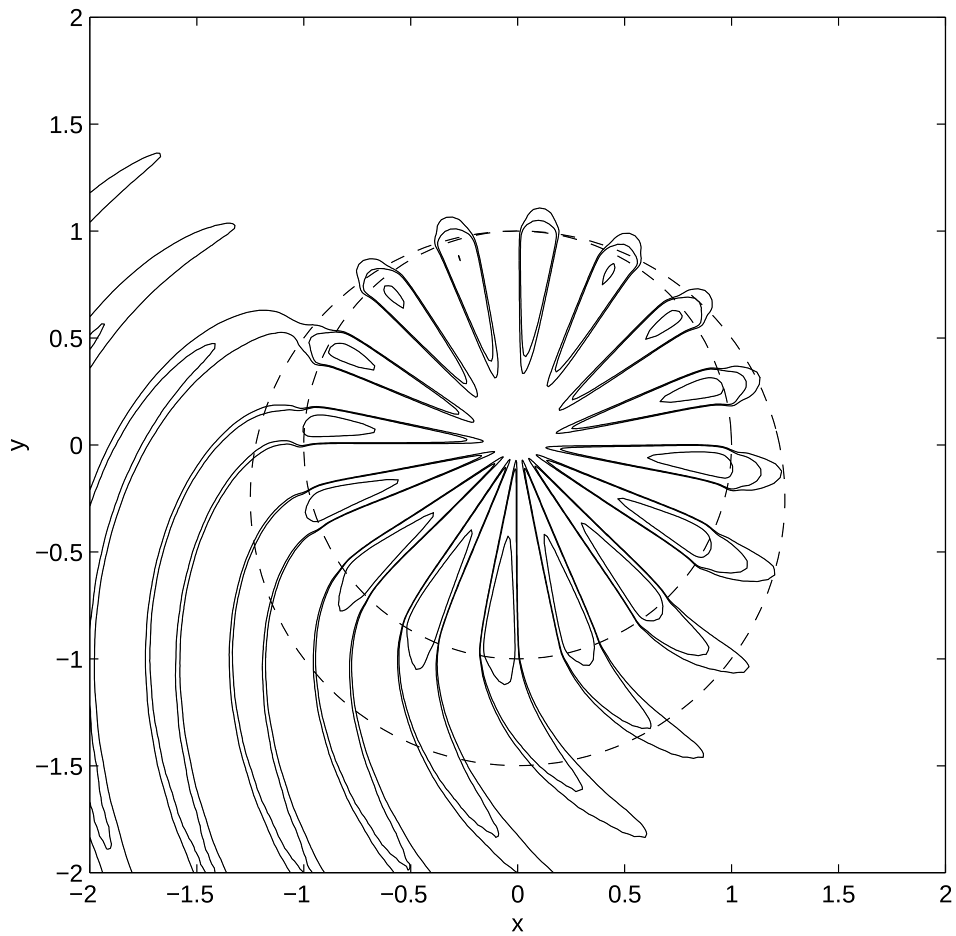
Helicopter rotor





$y = 0,\ p = \pm 0.1, \pm 0.05, \pm 0.001$ $\qquad z = 0,\ p = 0.25, 0.05, 0.025$

$M_t = 0.8,\ M_\infty = 0.2,\ \alpha = 85^\circ.$

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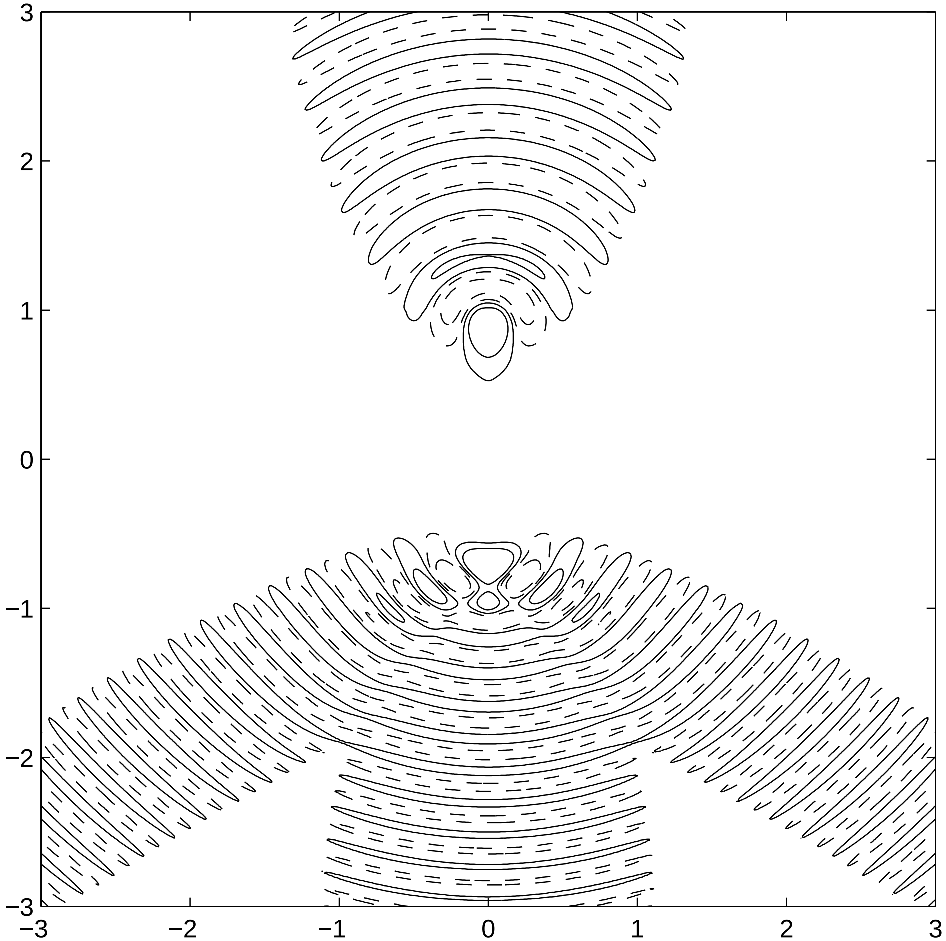
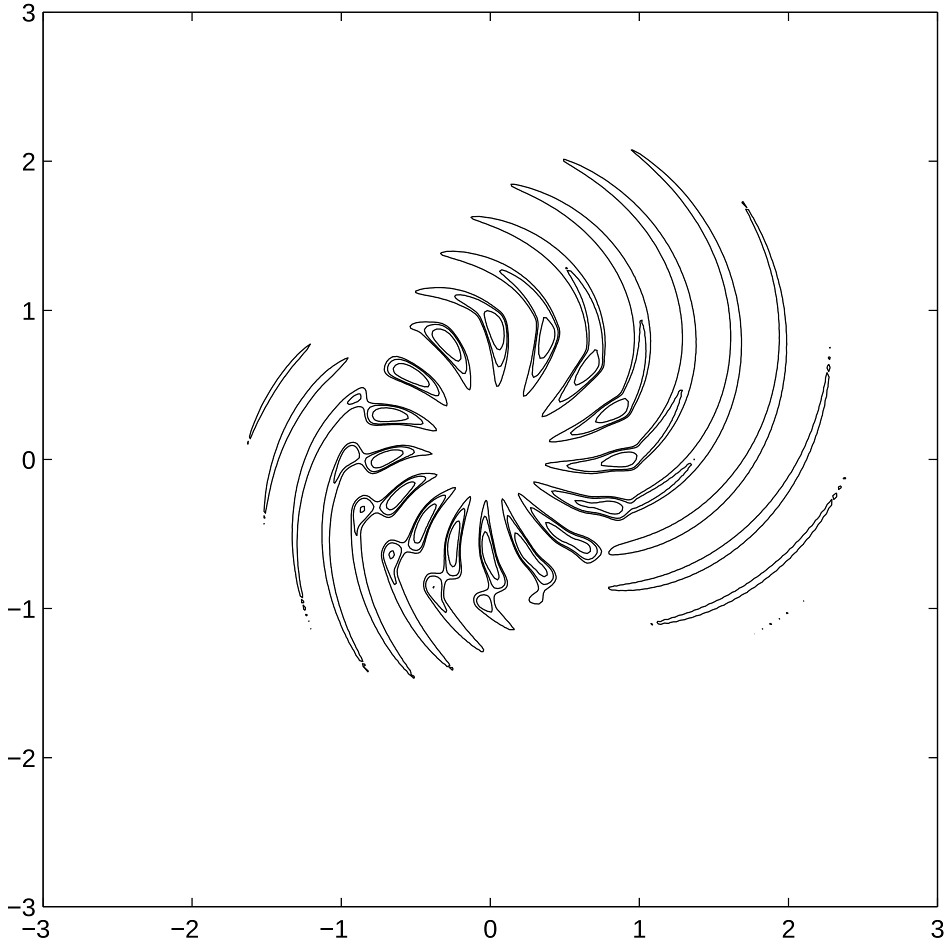
Very high speed rotor





$y = 0,\ p = \pm 10^{-3,-2,-1}$ $z = 0,\ p = 1, 0.5, 0.25$

$M_{\mathrm{t}} = 1.5,\ M_\infty = 0.2,\ \alpha = 90^\circ.$

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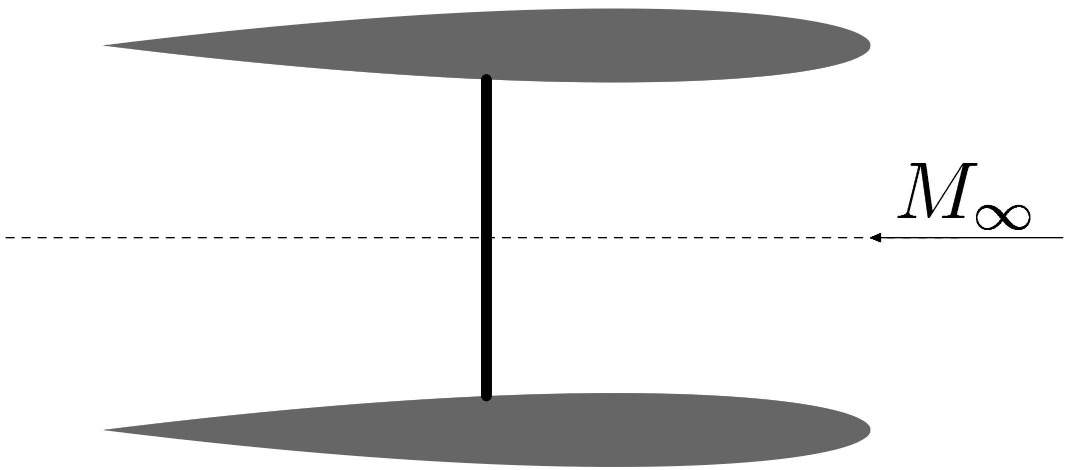
Current work: Ducted fan



- Boundary element method for acoustics in mean flow.
- Wake shear layer to be modelled.

Next . . .

- Extend the approach to ducted rotors (current work).
- Other applications:
  - vortex dynamics in axisymmetric domains.
  - vibrating systems.
- Improve calculation of J:
  - asymptotic methods (hard).
  - power series (easy?).
- Inversion?
  - bit of a disaster . . .
  - but has potential.