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Linear Acoustic Formulae for Calculation of Rotating Blade Noise with Asymmetric Inflow

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LINEAR ACOUSTIC FORMULAE FOR CALCULATION OF ROTATING BLADE NOISE WITH ASYMMETRIC INFLOW

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Abstract

In recent years a number of publications have examined the problem of the noise radiated by a propeller operating at an angle of attack. Most of these studies have used a frequency-domain formulation and posed the problem in "moving medium" (or "wind tunnel") coordinates. This paper presents a time-domain "moving medium" formulation for noise calculation which is equivalent to previously published frequency-domain methods. It also completes a set of time domain linearacoustic methods.

Nomenclature

c	speed of sound
$f(\mathbf{y}, au)$	surface definition
1	surface loading
M_{∞}	$ \mathbf{M}_{\infty} $
\mathbf{M}_{∞}	inflow Mach number
M_t	propeller tip Mach number
p'_L	loading noise
p_T'	thickness noise
R'	amplitude radius
R	$\partial R'/\partial x_i$

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observer time	
flow velocity, $c\mathbf{M}_\infty$	
surface normal velocity	
observer position	
source position	
angle of attack	
convection constant	
Dirac delta function	
azimuthal angle	
fluid density	
retarded time	

Introduction

Over the past few years, as good predictions of noise generated by propellers in axial flows have become commonplace, the problem of the noise radiated by propellers operating at an angle of attack has begun to figure more prominently in the literature. It is known that when a propeller operates at incidence its sound-field develops a marked asymmetry in azimuth. This has been ascribed to two phenomena,

- the *aerodynamic* effect of fluctuating loading on a propeller blade, which causes a variation in the sound power distribution.
- the *acoustic* effect of asymmetric convection which gives rise to an azimuthal variation in the sound power radiated by the steady loading component of the noise from a blade.

The effect of asymmetric convection has been examined by Stuff,¹ Hanson and Parzych² and Hanson³ using a frequency domain approach and Mani⁴ has pointed out that the asymmetric convection effect can have a greater influence than the fluctuating loading on the acoustic field. In each case the calculations have used a movingmedium formulation in which the propeller and observer are assumed fixed in a uniform flow. This decouples translational velocity effects on the radiated sound from the blade rotation effects and allows them to be considered separately. Wells and Han have proposed a movingmedium formulation valid for subsonic and supersonic propellers and have found that a moving-medium method can give large savings in computational time compared to a moving-observer technique. This method can be considered a moving medium equivalent to Farassat's formulation 3[°] which is valid for supersonic and subsonic bodies.

A moving-medium formulation can make the timedomain calculation of noise from a propeller at incidence easier than it would be using a moving-observer method. The propeller can be constrained to lie in a plane and a rotation of the inflow can be used to introduce an angle of attack. Then the calculation of propeller positions and velocities is quite simple as there is no forward motion and, since the propeller lies in a vertical plane, there is no need for the transformations described by Hanson and Parzych.²

Development

The formulae for the loading and thickness noise are developed in the conventional manner (see, for example, Farassat⁷). The convected wave equation is solved using the Green's function given by Lakhtakia et. al.⁸ Simple manipulations then suffice to put the solution in a familiar form. As in Farassat's work,⁷ generalised function methods are used to develop a solution in terms of quantities on a surface which is defined by the function *f* with $f(\mathbf{y}, \tau) = 0$ on the surface.

Green's function

The Green's function used in the solution is that given by Lakhtakia et. al⁸ for a source at a position $\mathbf{y} = (y_1, y_2, y_3)$ and an observer at $\mathbf{x} = (x_1, x_2, x_3)$ in a steady, uniform subsonic flow of Mach number M_{∞} in the positive x_3 direction.

$$G(\mathbf{x}, t; \mathbf{y}, \tau) = \gamma \frac{\delta(\tau - t + \gamma R'/c - \gamma^2 M_{\infty} (x_3 - y_3)/c)}{4\pi R'}$$
(1)

with

$$R' = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + \gamma^2 (x_3 - y_3)^2]^{1/2}$$

and

$$\gamma^2 = \frac{1}{1 - M_\infty^2}$$

Equation 1 can easily be adapted to the case of a flow of arbitrary direction, M_{∞} ,

$$G(\mathbf{x}, t; \mathbf{y}, \tau) = \gamma \frac{\delta(g)}{4\pi R'}$$
(2)

with

$$R' = (|(\mathbf{x} - \mathbf{y})|^2 + \gamma^2 |\mathbf{M}_{\infty} \cdot (\mathbf{x} - \mathbf{y})|^2)^{1/2}$$

and

$$\gamma^{2} = \frac{1}{1 - |\mathbf{M}_{\infty}|^{2}}$$
$$g = \tau - t + \gamma R'/c - \gamma^{2} \mathbf{M}_{\infty} \cdot (\mathbf{x} - \mathbf{y})/c$$

Thickness noise

The solution of the convected wave equation for thickness noise is

$$4\pi p_T' = \gamma \left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i}\right)$$
$$\int_{\tau} \int_{\mathbf{y}} \frac{\rho_0 v_n |\nabla f| \delta(f)}{R'} \delta(g) \, d\mathbf{y} \, d\tau \qquad (3)$$

Integrating over the delta functions and y in the usual manner, see e.g. Farassat and Succi,⁹ this becomes

$$4\pi p_T'(\mathbf{x}, t) = \gamma \left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i}\right)$$
$$\int_S \left[\frac{\rho_0 v_n}{R'(1 - \mathbf{M}_s \cdot (\gamma \mathbf{R} + \gamma^2 \mathbf{M}_\infty))}\right] dS$$
(4)

where $[\cdot]$ indicates evaluation of the function inside the brackets at retarded time τ , $\mathbf{R} = \partial R' / \partial x_i$ and S is the blade surface. \mathbf{M}_s is the source velocity in the fixed reference frame.

The spatial derivative can be expanded to a time derivative and the time derivatives brought under the integral sign to give a formula for thickness noise

$$4\pi p_T' = \gamma \int_S \left[\left(\frac{1}{1 - \mathbf{M}_s \cdot \mathbf{D}} - \mathbf{M}_\infty \cdot \mathbf{D} \right) \\ \times \frac{\partial}{\partial \tau} \left(\frac{\rho_0 v_n}{R'(1 - \mathbf{M}_s \cdot \mathbf{D})} \right) \right] dS \\ -\gamma c \int_S \left[\frac{\rho_0 v_n}{R'(1 - \mathbf{M}_s \cdot \mathbf{D})} \\ \times \left(\frac{\mathbf{R} \cdot \mathbf{M}_\infty}{R'} - \frac{\gamma}{c} \frac{\dot{\mathbf{R}} \cdot \mathbf{M}_\infty}{1 - \mathbf{M}_s \cdot \mathbf{D}} \right) \right] dS$$
(5)

where $\mathbf{D} = \gamma \mathbf{R} + \gamma^2 \mathbf{M}_{\infty}$.

Loading noise

The solution of the loading noise equation is

$$4\pi p'_{L} = -\gamma \frac{\partial}{\partial x_{i}} \int_{\tau} \int_{\mathbf{y}} \frac{\delta(g)}{R'} l_{i} |\nabla f| \delta(f) \, d\mathbf{y} \, d\tau \qquad (6)$$

where l_i is the *i*th component of the force per unit area on the fluid. This can be manipulated in the same way as the thickness noise equation above to give

$$4\pi p_L' = -\gamma \frac{\partial}{\partial x_i} \int_S \left[\frac{l_i}{R'(1 - \mathbf{M}_s \cdot (\gamma \mathbf{R} + \gamma^2 \mathbf{M}_\infty))} \right] \, dS$$
(7)

Then, expanding the spatial derivative, this becomes

$$4\pi p'_{L} = \gamma \int_{S} \left[\frac{\mathbf{l}}{R'(1 - \mathbf{M}_{s} \cdot \mathbf{D})} \\ \cdot \left(\frac{\mathbf{R}}{R'} - \frac{\gamma}{c} \frac{\dot{\mathbf{R}}}{(1 - \mathbf{M}_{s} \cdot \mathbf{D})} \right) \right] dS \\ + \frac{\gamma}{c} \int_{S} \left[\mathbf{D} \cdot \frac{\partial}{\partial \tau} \left(\frac{\mathbf{l}}{R'(1 - \mathbf{M}_{s} \cdot \mathbf{D})} \right) \right] dS$$
(8)

This formulation can be considered to fit into a 'matrix' of time-domain methods.

Source	Moving observer	Moving fluid
Subsonic	Farassat $1A^7$	Equations 5, 8
Supersonic	Farassat 3 ⁶	Wells and Han [°]

This shows the relationship between "movingmedium" and "moving-observer" solutions for the noise generated by a solid body.

Comparison with previous formulae

The formulation of equations 8 and 5 is precisely equivalent to previously published frequency domain methods, e.g. Hanson,³ as would be expected. It can be used for angle-of-attack calculations in the same way as a moving medium method has been used by Mani⁴ and Hanson³ for frequency domain calculations. Both authors have emphasised the importance of the acoustic effect of asymmetric convection on the sound radiated by the steady blade loading, an effect separate from the aerodynamic effect of fluctuating load introduced by incidence. The aerodynamic effect on the sound level asymmetry has been examined in earlier work, such as that of Wright.¹⁰ These equations provide a way of examining acoustic angle-of-attack effects in the time domain and of explaining how those effects come about.

Computational and experimental results

A computer code called SCRUMPI¹¹ has been written to implement the formulation of equations 5 and 8. This code takes account of both of the incidence effects identified above, the unsteady loading and the convective effect.

Results

Some sample numerical predictions from this code are presented and compared to experimental measurements made in the near field. Data presented are for tests carried out on a six-bladed model propeller at Mach numbers of 0.6, 0.65 and 0.7. Results presented include axial and circumferential directivity plots for the first harmonic, spectra and time records for the in-plane noise.



Figure 1: Measured data and numerical predictions, axial directivity, $\alpha=0^\circ$

Figure 2: Measured data and numerical predictions, in-plane spectra, $\alpha=0^\circ$

Axial directivity

The first set of axial directivity plots are for tests carried out at three inflow Mach numbers, 0.6, 0.65 and 0.7 and constant advance ratio in the Aircraft Research Association transonic wind tunnel (figures 1 and 2). The second set (figures 3 and 4) show more detailed results for tests carried out at a different advance ratio and blade pitch setting, an inflow Mach number of 0.7 and at two angles of attack, 0° and 3°. In each case, measured blade pressures were interpolated to generate a pressure distribution for input to the prediction code, with unsteady pressures being included in the input to the incidence cases. The directivities shown are for a sideline at 1.22 blade radii from the propeller centre of rotation, parallel to the inflow. In each case the axial coordinate is positive in the upstream direction.

The directivity of the first harmonic of the propeller signal is shown for three Mach numbers in figure 1, while the strength of those harmonics of the in-plane spectrum which could be distinguished above the background noise is shown in figure 2. In the main the predicted directivities match quite well. The predicted levels match well in the first case ($M_t = 0.83$) and deteriorate progressively as the tip Mach number increases with the inplane predictions being affected first (at $M_t = 0.9$) and more distant points suffering at the higher tip Mach number ($M_t = 0.98$). Even for the highest tip Mach number, the shape of the directivity curve is quite well predicted. It is thought that the underprediction of the noise in this case may be due to neglecting quadrupole noise in the analysis. The predicted spectrum of the in-plane signal at $M_t = 0.83$ matches the experimental results very well and in the other cases, the general trend has been accurately picked up except for the higher harmonics of the $M_t = 0.98$ case, where the prediction does not show the fifth harmonic higher than the fourth.

In the two cases for $M_{\infty} = 0.7$ presented in figures 3 and 4, the comparison of experiment and prediction is also quite good. In figure 3, showing results for the zero incidence case, the predicted first harmonic directivity is a good match, within 3 or 4 dB of the measured value over most of the range of axial displacement. The comparison of the in-plane time records and spectra is very good. The amplitude of the predicted time series is quite close to that of the measured one and the negative pressure peak has been accurately calculated. The most obvious error is in the positioning of the positive pressure peak which gives a waveform which is more rounded than the measured shape, without the sharp fall from the positive to the negative peak. As might be expected from the close match in amplitudes in the time domain, the spectra also compare favourably.

In the angle of attack case, the predicted results are also quite acceptable. While the error is large for the first harmonic measured in-plane, it is very small in the upstream direction and does not increase markedly in the way that it does in the zero angle of attack case. The comparison of time records is also quite good and, as in the axial inflow case, the negative peak has been accurately calculated with the positive peak being wrongly positioned. The predicted harmonic strengths fall off too fast but are still reasonable for the first two harmonics.

Circumferential directivity

Figures 5 and 6 show spectra and circumferential directivity plots for the same operating conditions as in figures 3 and 4 respectively. As there is an angle-of-attack effect on the azimuthal directivity, these results are perhaps the most important. The data were taken over an arc of 30° in the propeller plane 1.22 blade radii from the propeller axis.

Figure 5 shows results for the zero incidence case. The slight variation of the numerically predicted SPL with azimuth is due to the inclusion of fluctuating loading in the pressure distribution. As might be expected from the in-plane results in figure 3, the predictions here are quite good, with an error no greater than 2dB. The prediction of the variation in SPL with azimuth is of mixed quality, very good near the end of the arc and not so good near the start.

Figure 6, showing the angle of attack data, is not as good as figure 5 but is still in reasonable agreement with experiment. The error in the prediction is about 5dB, but the general trend over the whole traverse is quite close to the measured results.

Discussion

Errors in the predicted results could come from a number of possible causes.

Background noise

This does not seem to have affected the results very much. The predictions for the lower Mach number cases are quite good (see figure 1 for example). If there were serious errors due to background noise, they would be



Figure 3: Numerical predictions and experimental results, $\alpha = 0^{\circ}$, $M_{\infty} = 0.7$, $M_t = 0.89$.

Figure 4: Numerical predictions and experimental results, $\alpha = 3^{\circ}$, $M_{\infty} = 0.7$, $M_t = 0.89$.



Figure 5: Numerical predictions and experimental results, $\alpha = 0^{\circ}$, $M_{\infty} = 0.7$, $M_t = 0.89$

Figure 6: Numerical predictions and experimental results, $\alpha = 3^{\circ}$, $M_{\infty} = 0.7$, $M_t = 0.89$

most serious in the lower tip Mach number cases, where the noise levels are lower and more prone to contamination. The only effect of background noise here is that fewer harmonics of the propeller signal could be measured.

Quadrupole noise

Noise from quadrupole sources is not included in these results. It is known (see Magliozzi et. al.¹²) that for blade tip Mach numbers higher than 0.9 there is a large increase in quadrupole noise. Indeed in figure 1, showing the axial directivity of the first harmonic for three tip Mach numbers, there is quite a good match between theory and experiment at $M_t = 0.83$, an error of about 3dB for the in-plane measurement at a tip Mach number of 0.90 (the beginning of the range where quadrupole noise is important) and a larger error (about 6dB) for the $M_t = 0.98$ case, although even in this case the directivity has been quite accurately predicted.

Interpolated pressures

This is probably the main cause of errors. As pressures interpolated from static tappings were used to generate a loading distribution in each case presented, there is always the problem that the interpolation will not detect features which lie between tappings or, if it does include them, positions them incorrectly or assigns them the wrong strength. It should also be noted that some degree of *extrapolation* is inevitable in estimating the loading near a blade tip. As is known from the work of Parry and Crighton,¹³ for example, the blade tip is exponentially dominant over inboard regions in determining the noise radiated by a subsonic propeller. This makes noise calculations quite sensitive to any error in the tip pressure distribution.

In the angle of attack case, measured dynamic pressures were used. To obtain a fluctuating pressure distribution for the whole blade, it was assumed that the ratio of unsteady to steady pressure at a given point on a chord was independent of radius. This assumption can really only be justified on the grounds that a more sophisticated model would have been based more on guesswork than on a knowledge of the aerodynamics.

On balance it would appear that the main cause of error is whatever inaccuracy is introduced by interpolation. Errors are quite small in the zero-incidence cases, where the pressure distribution is very accurately known, while it is larger in the angle-of-attack cases where the unsteady pressures are important. At higher tip Mach numbers, neglecting quadrupole noise seems to be important, although it is hard to be sure of this without first knowing what the effect of interpolation errors is.

Conclusions

It can be concluded that

- a subsonic moving medium formulation can be used for angle of attack calculations of propeller noise in the time domain, in the same way that a uniformly valid formulation suitable for supersonic rotor calculations has been shown to be useful by Wells and Han,⁵
- such a formulation gives results in reasonable agreement with experimental results.

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