Euler's formula

$$e^{j\theta} \;=\; \cos\theta \;+\; j\sin\theta$$

Basic result

Euler's formula says that the above equation is true for any real angle θ (in radians).

Cartesian and polar forms

Figure 1 shows the typical point $z = x + jy$ in the (complex) plane.

Clearly

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r = |z| &= \sqrt{x^2+y^2} \\ \theta = \arg z &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}$$

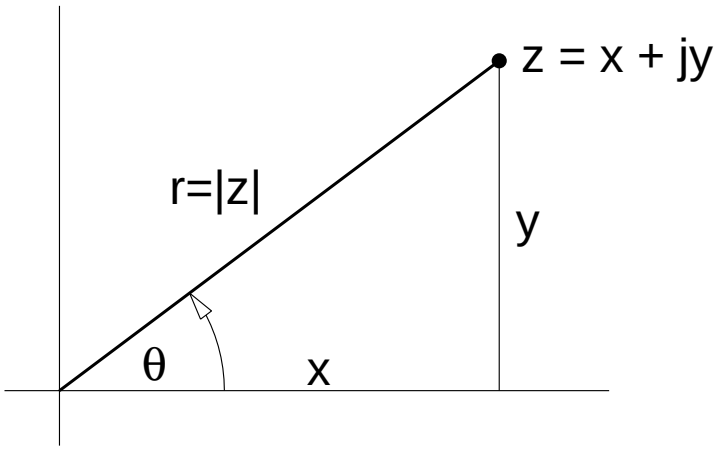

Figure 1: Cartesian and polar coordinates of a point in the (complex) plane

Euler's formula allows conversion between the cartesian and polar forms.

$$z \;=\; x \;+\; jy \;=\; r\cos\theta \;+\; jr\sin\theta \;=\; re^{j\theta}$$

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