THE RADIAL INJECTION OF A HOT FLUID INTO A COLD POROUS MEDIUM: THE EFFECTS OF LOCAL THERMAL NON-EQUILIBRIUM

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ABSTRACT
We consider the manner in which Local Thermal Non-Equilibrium effects affect the development of the thermal field when a hot fluid is injected radially into a cold porous medium. A purely forced convection situation is considered, and the evolving thermal fields depend on four non-dimensional parameters including the Péclet number and the nondimensional inter-phase heat transfer coefficient, $H$. In this primarily numerical study we find firstly that Local Thermal Equilibrium is always attained eventually, but after a time which depends strongly on the value of $H$. When the Péclet number is large a thermal shock wave is formed within the fluid phase which degrades slowly by imparting heat to the solid phase.

INTRODUCTION
A saturated porous medium occupies the region $r \geq r_0$.

For $t < 0$, cold fluid with temperature $T_i$ enters the medium with speed $U_0$ at $t = r_1$ and flows radially.

At $t = 0$ the temperature of this injected fluid is raised suddenly to the new level, $T_f$.

Forced convection is assumed, i.e. buoyancy forces are negligible.

Local thermal non-equilibrium (LTNE) effects are significant. We use the Anzelius-Schumann model: Anzelius (1926) and Schumann (1929).

Application to: geothermal energy extraction from hot dry rock formations

Extension of Rees, Bassom and Siddheshwar (2008) who considered the uniform unidirectional infiltration of a hot fluid into a cold porous medium.

GOVERNING EQUATIONS
The governing equations are,

\[ \frac{\partial T_1}{\partial t} = \nabla \cdot (\kappa_1 \nabla T_1) + h(t, T_1 - T_i), \]

\[ \frac{\partial T_2}{\partial t} = \nabla \cdot (\kappa_2 \nabla T_2) + h(t, T_2 - T_i), \]

where the fluid velocity is given by $\mathbf{u} = \nabla \phi (t) / \sqrt{\kappa_1}$.

On assuming that the temperatures are dependent only upon $r$ and $t$, the non-dimensional equations become,

\[ \frac{1}{1 + \epsilon} \nabla \cdot (\kappa_1 \nabla \eta) = \frac{1}{1 + \epsilon} \left( \frac{\partial}{\partial t} + H(\phi - \theta) \right), \]

\[ \frac{1}{1 + \epsilon} \nabla \cdot (\kappa_2 \nabla \eta) = \frac{1}{1 + \epsilon} \left( \frac{\partial}{\partial t} + H(\phi - \theta) \right), \]

\[ \frac{\partial}{\partial t} \eta = \nabla \cdot \left( \frac{\kappa_2}{1 + \epsilon} \nabla \eta \right) + \frac{1}{1 + \epsilon} H(\phi - \theta), \]

\[ \frac{\partial}{\partial t} \alpha = \frac{\kappa_2}{\kappa_1} \frac{\partial}{\partial t} \eta + \frac{1}{1 + \epsilon} H(\phi - \theta), \]

where $\eta$ and $\alpha$ are the temperatures of the fluid and solid phases, respectively, and the four non-dimensional parameters are given by $H = \frac{H_f}{H_i}$ (inter-phase heat transfer coefficient), $\alpha = \frac{\kappa_2}{\kappa_1}$ (porosity-modified conductivity ratio), $\gamma = \frac{1 + \epsilon}{\epsilon}$ (diffusivity ratio), $P_e = \frac{T_1}{T_2}$ (Péclet number).

As the mathematical problem consists of a sudden change in temperature of the fluid phase, it is natural to introduce the transformation:

\[ \eta = \frac{1}{2\sqrt{T}} \tau = \sqrt{T}. \]

The governing equations now become,

\[ \frac{1}{1 + \epsilon} \nabla \cdot (\kappa_1 \nabla \tau) = \frac{1}{1 + \epsilon} \left( \frac{\partial}{\partial t} \tau + H(\phi - \theta) \right), \]

\[ \frac{1}{1 + \epsilon} \nabla \cdot (\kappa_2 \nabla \tau) = \frac{1}{1 + \epsilon} \left( \frac{\partial}{\partial t} \tau + H(\phi - \theta) \right), \]

Under the Keller-box method — implicit time-stepping (Keller 1978).

Results

Figure 1. Showing the evolution with time of the temperature fields of the fluid (continuous lines) and solid (dashed lines) phases for three different values of the Péclet number, with $H = 1$, $\gamma = 1$ and $\alpha = 1$. Left hand figures display isotherms in ($\eta$, log$r$)-coordinates, while the right hand figures use ($r$, log$\eta$)-coordinates.

Figure 2. Showing the evolution with time of the temperature fields of the fluid (continuous lines) and solid (dashed lines) phases for four different values of $H$ with $P_e = 10$, $H = 1$ and $\alpha = 1$.

Figure 3. Numerical scheme

Figure 4. Showing the evolution with time of the temperature fields of the fluid (continuous lines) and solid (dashed lines) phases for four different values of $\alpha$ with $P_e = 10$, $H = 1$ and $\gamma = 1$.

ASYMPTOTIC ANALYSES
When $\gamma \ll 1$, then

\[ \theta = \frac{\phi - \phi_m}{\phi_m} \approx 0, \]

\[ \phi = \frac{H(\phi - \phi_m)}{1 - \phi_m} \approx 1, \]

\[ \phi_m \approx \frac{1}{1 + \epsilon} \eta, \]

\[ \alpha \approx \frac{\kappa_2}{\kappa_1} \frac{\partial}{\partial t} \eta + \frac{1}{1 + \epsilon} H(\phi - \theta), \]

Hence the thickness of the early-time boundary layer depends only on $\gamma$ and $\alpha$, but not on $H$ or $P_e$.

When $\gamma > 1$, then

\[ \theta = \frac{\phi - \phi_m}{\phi_m} \approx \int \frac{1}{1 + \epsilon} \left( \frac{\partial}{\partial t} \eta + H(\phi - \theta) \right) \]

and Local Thermal Equilibrium is established at leading order.

CONCLUSIONS
In the paper we have extended the numerical analysis of Rees et al. (2008) on the combined effect of LTNE and a uniform unidirectional infiltration of a hot fluid into a cold porous medium on the evolution of the respective temperature fields of the fluid and solid phases to the more realistic scenario of radial injection. Some of the qualitative features found in Rees et al. (2008) also arise for the radial injection problem, namely the formation of a gradually decaying thermal shock at large values of the Péclet number, the effects of different values of $H$ and $\gamma$ on the time at which the solid phase begins to heat up, the peculiar behaviour of the solid phase isotherms when $\alpha$ is small, as shown in Figure 4, and the eventual attainment of LTE at large times. The main qualitative difference arises at large times where the thickness of the advancing thermal front depends only on the value of $\phi_m$, i.e. $H(\phi - \eta) / (1 + \gamma)$, the work of Rees et al. (2008) shows that this thickness depends strongly on $H$, $\gamma$ and $\alpha$.

REFERENCES