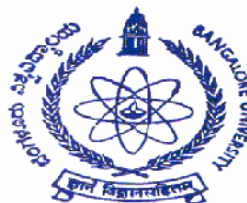# INSTABILITIES IN FERROMAGNETIC, DIELECTRIC AND OTHER COMPLEX LIQUIDS

A thesis submitted for the degree of
DOCTOR OF PHILOSOPHY
in the Faculty of Science

By

S. MARUTHAMANIKANDAN

Under the guidance of

DR. PRADEEP G. SIDDHESHWAR

**FOR QUALITY & EXCELLENCE
IN HIGHER EDUCATION**

★ ★ ★ ★ ★

ACCREDITED BY NAAC

**Department of Mathematics
Bangalore University
Central College Campus
Bangalore – 560 001
INDIA**

FEBRUARY 2005

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FEBRUARY 2005

Dedicated to

my parents

and

in loving memory of my grandparents

DECLARATION

I hereby declare that the matter embodied in this thesis is the result of investigations carried out by me at the Department of Mathematics, Bangalore University, Central College, Bangalore, under the supervision of **DR. PRADEEP G. SIDDHESHWAR**, Reader, Department of Mathematics, Bangalore University, Central College, Bangalore, and it has not been submitted for the award of any degree, diploma, associateship, fellowship etc., of any other University or Institute.

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Supervisor

ACKNOWLEDGEMENTS

CONTENTS

ABSTRACT

The thesis deals with Rayleigh-Bénard/Marangoni convection in Newtonian/non-Newtonian ferromagnetic/dielectric liquids. The effects of uniform internal heat source and thermal radiation are considered. Thermorheological and magnetorheological/electrorheological effects are given attention by treating the effective viscosity as a function of temperature and magnitude of magnetic/electric field. The non-Newtonian fluid descriptions used in the thesis subscribe to Jeffrey, Maxwell and Rivlin-Ericksen models. The linear and nonlinear convective instability problems are based respectively on the normal mode technique and a minimal Fourier series representation. The higher order Rayleigh-Ritz technique is exploited to solve the variable coefficient differential equations arising in the linear stability problems. The results of the problems investigated in the thesis may be useful in application situations with a ferromagnetic/dielectric fluid as working medium.

With the above motivation, the summary of each of the problems investigated in the thesis is given below:

(1) Linear and nonlinear ferro- and electro-convection

Linear and nonlinear analyses of Rayleigh-Bénard convection in a Boussinesq-ferromagnetic fluid are investigated. In the case of linear theory, the conditions for stationary and oscillatory modes of instabilities are obtained using the normal mode technique and the parametric perturbation method is used to analyze qualitatively the effect of various magnetic parameters on the onset of convection. The nonlinear analysis is based on a minimal representation of double Fourier series. The autonomous system of differential equations representing the generalized Lorenz model of convective process arising in the nonlinear study is solved numerically. The transient behaviour concerning the variations in the Nusselt number with time has been examined. The effect of magnetic parameters on the nonlinear evolution

of convection is analyzed by considering the time-series plots. An analogy between nonlinear ferroconvection and nonlinear electroconvection is discussed.

(2) Thermorheological and magnetorheological effects on ferroconvection with internal heat source

The effect of uniform heat source (sink) on the onset of Rayleigh-Bénard/Marangoni convection in a horizontal layer of ferromagnetic liquid is investigated by means of the classical linear stability analysis. Thermorheological and magnetorheological effects are given attention by treating the effective viscosity as a function of both temperature and magnetic field strength. The higher order Rayleigh-Ritz technique is used to determine the critical values. In the case of Rayleigh-Bénard convection, the critical values are obtained for free-free, free-rigid, rigid-rigid boundary combinations on velocity with isothermal conditions on temperature. In the case of Marangoni convection, the critical values are obtained for an upper free-adiabatic and a lower rigid-isothermal boundary. General boundary conditions on the magnetic potential are considered for both Rayleigh-Bénard and Marangoni convections. The influence of various magnetic and nonmagnetic parameters on the onset of convection has been analyzed. An analogy for Rayleigh-Bénard/Marangoni instability problems with heat source (sink) between variable-viscosity ferromagnetic and dielectric liquids is discussed.

(3) Thermal radiation effects on ferroconvection

The effect of radiative heat transfer on the onset of Rayleigh-Bénard/Marangoni convection in a horizontal layer of a ferromagnetic fluid is studied within the framework of linear stability analysis. The effective viscosity of the fluid is assumed to a function of both temperature and magnetic field strength. The fluid is assumed to absorb and emit thermal radiation and the boundaries are treated as black bodies. The Milne-Eddington approximation is employed in obtaining the initial static state expression for the radiative heat flux. The optical properties of the ferromagnetic fluid are considered to be independent of the wavelength of

radiation. Consideration is given to two asymptotic cases, *viz.*, transparent and opaque layers of fluid. The critical values marking the onset of convection are obtained using the Rayleigh-Ritz technique. The influence of radiative and magnetic parameters on the stability of the fluid layer has been analyzed. An analogy is presented for radiation-affected Rayleigh-Bénard/Marangoni convection between variable-viscosity ferromagnetic and dielectric liquids.

(4) Ferroconvection in viscoelastic liquids

The influence of thermo- and magneto-rheological effects on the threshold of Rayleigh-Benard/Marangoni convection in a Newtonian/viscoelastic ferromagnetic liquid is studied theoretically within the framework of linear stability analysis. The effective viscosity is assumed to be a quadratic function of both temperature and magnetic field strength. Consideration is given to three viscoelastic families, *viz.*, Jeffrey, Maxwell and Rivlin-Ericksen. The conditions under which overstable motions occur are discussed. The critical values marking the onset of convection are computed numerically using the Rayleigh-Ritz technique. The influence of viscoelastic, magnetic and nonmagnetic parameters on the onset of convection has been analyzed. An analogy is presented between viscoelastic ferromagnetic and viscoelastic dielectric liquids.

CHAPTER I

INTRODUCTION

1.1 OBJECTIVE AND SCOPE

In most part of the last century the engineering applications of fluid mechanics were restricted to systems in which electric and magnetic fields played no role. In recent years, the study of the interaction of electromagnetic fields with fluids started gaining attention with the promise of applications in areas like nuclear fusion, chemical engineering, medicine and high speed noiseless printing. This study can be divided into three main categories.

- *Magnetohydrodynamics* – the study of the interaction between magnetic fields and electrically conducting fluids.

- *Ferrohydrodynamics* – the study of the mechanics of fluid motions influenced by strong forces of magnetic polarization and

- *Electrohydrodynamics* – the branch of fluid mechanics concerned with electric force effects.

The investigation of convective heat transfer together with the aforementioned electrical and magnetic forces in Newtonian/non-Newtonian fluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behaviour of these fluids and also to obtain invaluable scaled up information for industrial applications. The objective of the thesis is, therefore, to study various convective instabilities in ferromagnetic/dielectric fluids affected by buoyancy, surface tension and a pondermotive force due to magnetization/polarization. The emerging areas of applications of magnetic/dielectric fluids have brought to light new thoughts and ideas for advanced level research. Most of the technologically important problems involving these fluids as working media are non-isothermal in nature. Therefore,

there is a need to study convection and heat transfer in these fluids. The variety of situations related to these application-oriented problems makes the modelling of the same intricate and complex. Since these fluids are basically suspensions which respond thermally and to electromagnetic fields, the problems are both mathematically and physically challenging. Most of the available works on convection and heat transfer in these fluids resort to linear stability theory and a Newtonian description. Moreover, they use a set of constitutive equations involving constant viscosity and a heat transport equation without heat source/radiation term. Accordingly, the scope of the thesis is to study the neglected effects on convection and to provide logical explanation on mechanisms of augmenting or suppressing convection in ferromagnetic/dielectric fluids. Many reported statements on the convection problem are revised on the basis of the investigation in the thesis and new statements are putforth. The following information is thus considered necessary to achieve the stated objective of the thesis.

1.1.1 Magnetic Fluids

Ferromagnetism is a property of iron, nickel, cobalt and some compounds and alloys of these elements. It was thought that to create a magnetic fluid one might heat the metal until it becomes molten, but this strategy did not work as ferromagnetism disappears above a certain temperature called the Curie point, which is invariably well below the melting point of the material. A magnetic fluid, better known as ferrofluid, consists of kinetically stabilized ultramicroscopic ferro- or ferrimagnetic particles coated with a monomolecular layer of surfactant and colloidally dispersed in a magnetically passive liquid. Under the influence of an external magnetic field, such a fluid exhibits a large magnetization and as soon as the field is removed, the fluid attains its zero magnetization state at once. As each particle possesses a giant magnetic moment when compared with paramagnetic particles, such a medium is called *superparamagnetic i.e.* having zero remanence and coercivity. Ferrofluids have almost the same magnetic characteristics as a solid, but in many respects behave as liquid continua. Magnetic liquids can be controlled by magnetic forces.

2

1.1.2 Composition of Magnetic Fluids

The development of many innovative applications warrants the unique combination of magnetic and fluidic property. A magnetic fluid is a two phase matter consisting of solid and liquid and a three component system comprising magnetic particles, carrier liquid and surfactant. Since randomizing Brownian energy may not be sufficient to counteract attractions owing to van der Waal and dipole-dipole forces, aggregation and sedimentation are prevented by providing suitable repulsive forces either by Coulomb or by steric repulsion. In the former case particles are either positively or negatively charged and the fluid is called *ionic ferrofluid* while in the latter case each particle is coated with an appropriate surfactant and the resulting fluid is known as *surfacted ferrofluid* (Figure 1.1).
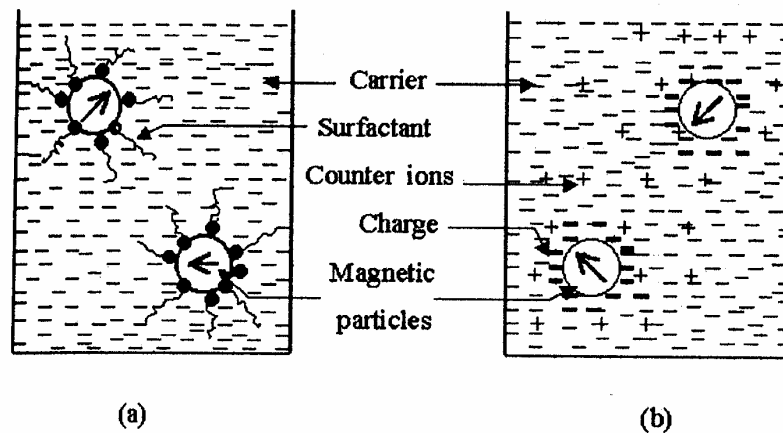
Figure 1.1: Surfacted (a) and ionic ferrofluid (b).

Figure 1.1: (a) Surfacted ferrofluid and (b) ionic ferrofluid.
(From Upadhyay, 2000)

On the other hand, the magnetic fluid should remain stable in the presence of a magnetic field, that is, there should be no agglomeration and/or phase separation. To meet this requirement, each of the three components should satisfy certain conditions. The important features of these three components are discussed in Appendix A.

To meet varied market requirements a wide range of ferrofluid-based devices have been developed and these are listed below.

Ferromagnetic fluid-based devices

1. Accelerometers – Bailey (1983).
2. Seals
   a) Exclusion seals and dynamic process seal – Raj and Moskowitz (1990).
   b) Environmental seal – Raj *et al*. (1995).
   c) Computer seals – Ferrotec Corporation Brochure.
   d) Seals for rotary shafts, RF-sputtering seal, six degrees of freedom seal and vacuum seal – Raj and Chorney (1998).
3. Fluid film bearings, NMR probe and magnetic domain detection – Raj and Moskowitz (1990).
4. Transducers, ferrofluids and plants – Anton *et al*. (1990).
5. Quenching and separation in magnetic fluids – Gogosov *et al*. (1990).
6. Ferrofluids and biological assemblies – Charles (1990).
7. High-gradient magnetic separation in blood and bone narrow processing – Roath *et al*. (1990).
8. Directed movement of radio-opaque magnetic fluid in intestine – Tsyb *et al*. (1990).
9. Magnetic fluid as a possible drug carrier for thrombosis treatment – Rusetski and Ruuge (1990).
10. Dispersion media diagnostics – Apsitis *et al*. (1990).
11. Viscous inertia dampers, loud speakers, ferrofluid sensors and specialty coating systems – Raj *et al*. (1995).
12. Magnetic fluid grinding – Umehara *et al*. (1995).
13. Transformers and ferrofluid stepper – Raj and Chorney (1998).
14. Magnetic pattern detection – Ferrotec Corporation Brochure.
15. Helicopter lag dampers – Kamath *et al*. (1999).
16. Optical switches, filters and optical lens – Horng *et al*. (2001)
17. Automotive technology – Phulé (2001)
18. Clinical applications – Jordan *et al*. (2001).
19. Nanotechnology – Zhan (2001)
20. Tribotechnical systems – Uhlmann *et al*. (2002).
21. Heat transfer applications – Nakatsuka *et al*. (2002).
22. RF-magnetic hyperthermia – Hergt *et al*. (2004).
23. Roto system vibration control – Zhu (2005).
24. Radiation therapy of cancer – Aquino *et al*. (2005).
25. Ultra-high-vacuum applications – Li and Raj (2005).
26. Magnetic drug targeting – Ganguly *et al*. (2005).

So far we have discussed about ferromagnetic fluids, their properties and their applications. We now move on to discuss about dielectric fluids.

1.1.3 Dielectric Liquids

These liquids are characterized by very slight electrical conductivity. Transformer oil (and most other organic substances) and distilled water are examples of such fluids. Compared to magnetic liquids, these liquids are easier to prepare. Under the influence of an external electric field such a fluid exhibits a large polarization and as soon as the field is removed, the fluid attains zero polarization state at once. Dielectric liquids can be controlled by electric forces (Hughes and Young, 1966; Melcher, 1981).

To meet varied market requirements a wide range of dielectric liquid-based devices have been developed and these are listed below.

Dielectric fluid-based devices

1. Electrogasdynamic pumping – Lawton *et al*. (1973).
2. EHD enhanced chemical reactors – Chang (1987).
3. Dissipative hydrodynamic oscillators – Chu *et al*. (1989).
4. Electrohydrodynamic pumps – Richter *et al*. (1991).
5. EHD electrode systems applications in power production cycles and in refrigeration – Allen and Karayiannis (1995).
6. Ion-drag dielectric pump for enhanced cooling – Yagoobi *et al*. (1995).
7. Conduction processes in highly insulating liquids and electro-optical image reproduction – Watson (1996).
8. Micropumps for microelectronic cooling – Wong *et al*. (1996).
9. Liquid driving electrostatic micromotor – Yokota *et al*. (1996).
10. Dielectric fluid motors – Otsubo and Edamura (1997).
11. Electrogasdynamics of disperse systems – Popkov *et al*. (1997).
12. Earthquake protection dampers – Kamath and Wereley (1998).
13. Aircraft EHD heat exchangers –Paschkewitz (1998).
14. Computational simulation of microfluidics, electrokinetics and particle transport in biological MEMS devices – Giridharan *et al*. (1999).
15. Nuclear power plant emergency core cooling system – Piscataway (1999).
16. Compact bubble detector –Merkel *et al*. (2000).
17. ER toners for electrophotography – Otsubo and Suda (2002).
18. ER fluid assisted polishing – Kim *et al*. (2003).
19. Thin fluid films – Shankar and Sharma (2004).
20. Electrical discharge machining – Leao and Pashby (2004).

Non-isothermal application situations are relevant to the theme of the thesis. For most non-isothermal applications, the most sought-after properties of a ferrofluid/dielectric liquid are the following (Fertman, 1990; Berkovskii *et. al.*, 1993; Tao and Roy, 1994; Zhakin, 1997; Upadhyay, 2000):

❖ Long term stability within the operating temperature range of the device and within the range of electromagnetic field strengths.

❖ High saturation magnetization/polarization and large initial susceptibility.

❖ Low viscosity and low vapour pressure.

❖ Stability in gravitational fields and the magnetic field/electric field gradient.

❖ Absence of significant aggregation in the presence of a uniform magnetic field for a ferromagnetic fluid.

❖ Good thermal conductivity.

With the stated objective and scope of the thesis in mind literature review has been made and the same is presented below.

1.2 LITERATURE REVIEW

The main objective of the thesis is to deal with Rayleigh-Bénard/Marangoni convection in complex fluids like Newtonian/non-Newtonian ferromagnetic/dielectric fluids. Literature pertinent to this is classified as follows.

- Rayleigh-Bénard/Marangoni convection in Newtonian fluids.
- Rayleigh-Bénard/Marangoni convection in Newtonian ferromagnetic fluids.
- Rayleigh-Bénard/Marangoni convection in Newtonian dielectric fluids.
- Rayleigh-Bénard/Marangoni convection in viscoelastic and viscoelastic ferromagnetic/dielectric fluids.

The relevant literature for the problems posed in this thesis is briefly discussed below in keeping with the above classifications.

1.2.1 Rayleigh-Bénard/Marangoni Convection in Newtonian Fluids

Many industrial devices rely on thermal convection for the transfer of heat. Aeronautical, biomedical, civil, marine and mechanical engineers as well as astrophysicists, geophysicists, space researchers, meteorologists, physical oceanographers, physicists and mathematicians have used a little or more heat transfer theory here and there in the course of the development of their respective field. Strictly speaking, due to interactions, the three heat transfer mechanisms, *viz*., conduction, convection and radiation are inseparable. In a physical sense, the amount of heat transfer in conduction and convection depends upon the temperature difference whereas that of radiation depends upon both the temperature difference and the temperature level. Natural convection in a horizontal layer of fluid heated from below and cooled from above has been the subject of investigation for many decades owing to its implications for the control and exploitation of many physical, chemical and biological processes. We now make a quick and brief review of the linear and nonlinear analysis of the *RBC* problem in Newtonian fluids keeping in mind the objective and scope of the thesis.

The earliest experiment which called attention to the thermal instability was briefly reported by Thompson (1882). Benard (1901) later presented a much more complete description of the development of the convective flow. Lord Rayleigh (1916) was the first to study the problem theoretically and aimed at determining the conditions delineating the breakdown of the quiescent state. As a result, the thermal instability situation described in the foregoing paragraph is referred to as *Rayleigh-Bénard convection* (*RBC*). The Rayleigh theory was generalized and extended to consider several boundary combinations by Jeffreys (1926), Low (1929) and Sparrow *et al*. (1964). Chandra (1938) examined the *RBC* problem experimentally for a gas. The most complete theory of the thermal instability problem was presented by Pellew and Southwell (1940).

Malkus and Veronis (1958) investigated finite amplitude cellular convection and determined the form and amplitude of convection by expanding the nonlinear

equations describing the fields of motion and temperature in a sequence of inhomogeneous linear equations. Veronis (1959) studied finite amplitude cellular convection in a rotating fluid and showed that the fluid becomes unstable to finite amplitude disturbances before it becomes unstable to infinitesimal perturbations.

Lorenz (1963) solved a simple system of deterministic ordinary nonlinear differential equations representing cellular convection numerically. For those systems with bounded solutions, it is found that non-periodic solutions are unstable with respect to small modifications and that slightly differing initial states can evolve into considerably different states.

Veronis (1966) analyzed the two-dimensional problem of finite amplitude convection in a rotating layer of fluid by considering the boundaries to be free. Using a minimal representation of Fourier series, he showed that, for a restricted range of Taylor number, steady finite amplitude motions can exist for values of the Rayleigh number smaller than the critical value required for overstability. Veronis (1968) also examined the effect of a stabilizing gradient of solute on thermal convection using both linear and finite amplitude analysis. It is found that the onset of instability may occur as an oscillatory motion because of the stabilizing effect of the solute in the case of linear theory and that finite amplitude instability may occur first for fluids with a Prandtl number somewhat smaller than unity.

Krishnamurthy (1968a, b) presented a nonlinear theory of *RBC* problem and discussed the formation of hexagonal cells and the existence of subcritical instabilities. Busse (1975) considered the interaction between convection in a horizontal fluid layer heated from below and an ambient vertical magnetic field. It is found that finite amplitude onset of steady convection becomes possible at Rayleigh numbers considerably below the values predicted by linear theory.

The literature reviewed so far concerns *RBC* problems with constant viscosity. The classical problem of *RBC* involves the *Boussinesq approximation*. However, there are situations where the viscosity variation must be accounted for and it is

likely that studies of constant viscosity convection could have substantially overestimated the vigor of convection. In some cases of convective heat transport, in the mantle of Mercury, for example, it is unclear convection occurs at all with realistic rheologies (Platten and Legros, 1984; Gebhart *et al*., 1998).

Breaking of the top-bottom symmetry is quite generic in experimental situations. When the fluid viscosity varies with temperature, the top and bottom structures are different. We refer to this as a *non-Boussinesq effect* (Wu and Libchaber, 1991). The fluid viscosity can be a function of magnetic and electric fields as well in the case of magnetic and dielectric fluids respectively. Several mathematical models proposed for the variable viscosity and their applicability will be discussed in Chapter II. We now review the literature pertaining to variable viscosity fluids in a *RBC* situation.

Palm (1960) showed that for a certain type of temperature-dependence of viscosity, the critical Rayleigh number and the critical wavenumber are smaller than those for constant viscosity and explained the observed fact that steady hexagonal cells are formed frequently at the onset of convection.

Torrance and Turcotte (1971) investigated the influence of large variations of viscosity on convection in a layer of fluid heated from below. Solutions for the flow and temperature fields were obtained numerically assuming infinite Prandtl number, free-surface boundary conditions and two-dimensional motion. The effect of temperature-dependent and depth-dependent viscosity was studied motivated by the convective heat transport in earth's mantle.

Booker (1976) investigated experimentally the heat transport and structure of convection in a high Prandtl number fluid whose viscosity varies by up to a factor of 300 between the boundary temperatures. Horne and Sullivan (1978) examined the effect of temperature-dependent viscosity and thermal expansion coefficient on the natural convection of water through permeable formations. They found that the convective motion is unstable at even moderate values of the Rayleigh number and

exhibits a fluctuating convective state analogous to the case of a fluid with constant viscosity and coefficient of thermal expansion.

Carey and Mollendorf (1980) presented a regular perturbation analysis for several laminar natural convection flows in liquids with temperature-dependent viscosity. Several interesting variable viscosity trends on flow and transport are suggested by the results obtained. Stengel *et al.* (1982) obtained, using a linear stability theory, the viscosity-ratio dependences of the critical Rayleigh number and critical wavenumber for several types of temperature-dependence of viscosity.

Richter *et al.* (1983) showed, by an experiment with temperature-dependent viscosity ratio as large as 10^6, the existence of subcritical convection of finite amplitude near the critical Rayleigh number. Busse and Frick (1985) analyzed the problem of *RBC* with linear variation of viscosity and showed an appearance of square pattern for a viscosity ratio larger than 2.

White (1988) made an experiment for the fluid with Prandtl number of $o(10^5)$ and studied convective instability with several planforms for the Rayleigh number up to 63000 and the temperature-dependent viscosity ratio up to 1000. He found that if the viscosity ratio is 50 or 100 and the Rayleigh number is less than 25000, stable hexagonal and square patterns are formed in a certain range of wavenumber and that their wavenumbers increase with viscosity ratio. The possibility of multi-valued solution in the thermal convection problem with temperature-dependent viscosity has been examined numerically by Hirayama and Takaki (1993).

Kafoussias and Williams (1995) studied, using an efficient numerical technique, the effect of a temperature-dependent viscosity on an incompressible fluid in steady, laminar, free-forced convective boundary layer flow over an isothermal vertical semi-infinite flat plate. It is concluded that the flow field and other quantities of physical interest are significantly influenced by the viscosity-temperature parameter. Kafoussias *et al.* (1998) studied the combined free-forced convective laminar boundary layer flow past a vertical isothermal flat plate with

temperature-dependent viscosity. The obtained results showed that the flow field is appreciably influenced by the viscosity variation.

Severin and Herwig (1999) investigated the variable viscosity effect on the onset of instability in the *RBC* problem. An asymptotic approach is considered which provides results that are independent of specific property laws. Kozhhoukharova *et al*. (1999) examined the influence of a temperature-dependent viscosity on the axisymmetric steady thermocapillary flow and its stability with respect to non-axisymmetric perturbations by means of a linear stability analysis. The onset of oscillatory convection is studied numerically by a mixed Chebyshev-collocation finite-difference method.

You (2001) presented a simple method which can be applied to estimate the onset of natural convection in a fluid with a temperature-dependent viscosity. Straughan (2002a) developed an unconditional nonlinear energy stability analysis for thermal convection with temperature-dependent viscosity. The nonlinear stability boundaries are shown to be sharp when compared with the instability thresholds of linear theory.

Hossain *et al*. (2002) analyzed the effect of temperature-dependent viscosity on natural convection flow from a vertical wavy surface using an implicit finite difference method. They have focused their attention on the evaluation of local skin-friction and the local Nusselt number. Chakraborty and Borkakati (2002) studied the flow of a viscous incompressible electrically conducting fluid on a continuously moving flat plate in the presence of uniform transverse magnetic field. Assuming the fluid viscosity to be an inverse linear function of temperature, the nature of fluid velocity and temperature is analyzed.

Siddheshwar (2004) studied numerically the thermorheological effect on magnetoconvection in fluids with weak electrical conductivity under 1g and μg conditions using Rayleigh-Ritz method. The possibility of an over-prediction of the critical eigenvalue in the classical approach with constant viscosity is proved.

The literature mentioned above so far dealt with *RBC* problems where the basic temperature gradient is uniform across the layer. In many situations the stability or instability of a fluid in the presence of a nonlinear basic temperature profile is of practical importance and such a profile arises due to

    (i)   heat sources in the fluid

    (ii)   radiation

    (iii) sudden heating or cooling at the boundaries,

    (iv) throughflow at the boundaries

and so on. In this thesis, we have sought to focus on the effects of heat source and radiation on the onset of instability. We now a make a concise review of the *RBC* problem with internal heat source.

Interest in natural convection in fluids with internal heat sources has been stimulated by the demands of nuclear power engineering (Bolshov *et al.* 2001). Joule heating and magnetocaloric effect are responsible for the internal heat sources in dielectric and magnetic fluids respectively. Sparrow *et al.* (1964) investigated the problem of *RBC* with internal heat generation. With increasing departures from the linear temperature profile, it is found that the fluid layer becomes more prone to instability. It was corroborated by Watson (1968) that the *RBC* problems with heat source and heat sink are identical and the effect of both heat source and heat sink is to destabilize the system.

Yu and Shih (1980) considered the onset of thermal instability of an electrically conducting fluid layer subjected to volumetric heating and bounded between two rigid surfaces in the presence of a magnetic field. The stability conditions are found for different thermal boundary conditions.

Riahi (1984) studied the problem of nonlinear *RBC* with an internal heat source. The presence of internal heating is found to be able to affect strongly the cell size, the stability of the convective motion and the internal motion of the hexagonal cells. Riahi (1986) also studied the problem of nonlinear thermal convection in a

low Prandtl number fluid with internal heating. It is found that subcritical instability associated with the hexagons can occur for a range of the amplitude of convection and non-uniform internal heating can affect various flow features and the stability of the convective motion.

Krishnamurti (1997) showed, assuming the heat source to be a function of the species concentration, that convection can occur in a fluid layer even when there is stable stratification by a species field. Straughan (2002b) studied the linear instability and nonlinear stability of a model due to Krishnamurti (1997).

Kim *et al*. (2002) examined the time-dependent buoyant convection in an enclosure in the presence of internal heat generation under a time-periodic thermal boundary condition. Estimations of the resonance frequencies are made and physical explanations are offered.

In what follows we review the literature relating to the problem of *RBC* in a radiating fluid.

There exist situations in which thermal radiation is important even though the temperature may not be high. It is a fact that, even under some of the most unexpected situations, the radiation heat transfer could account for a non-negligible amount of total heat transfer. Earlier works on heat transfer in Newtonian fluids considered convection and conduction and overlooked the effect of thermal radiation (Siegel and Howell, 1992; Modest, 1993; Howell and Menguc, 1998). The available literature barely delineates the part played by convection in a fluid combined with radiation. The formulation of heat transfer by conduction and convection leads to differential equations while that by radiation leads to integral equations. Thus the complexity involved in the solution of the integro-differential equations resulting from the combined convection and radiation problem warrants the use of several simplifying assumptions. In this thesis we restrict our attention to the case in which the absorption coefficient of the fluid is the same at all wavelengths and is independent of the physical state (the so-called *gray medium*

approximation). The equation of radiative transfer is developed in optically thick and thin approximations and the effect of scattering is ignored.

Goody (1956) investigated the *RBC* problem subject to radiative transfer using a variational technique and a gray, two-stream radiative model to find the critical conditions for linear stability. He considered the limits of optically thin and thick fluids, and free-slip, optically black boundaries. Goody noted that radiative damping tends to diminish temperature perturbations and it causes the basic temperature profile in the interior of the domain to have a more stable lapse rate.

Spiegel (1960) considered the *RBC* problem in a radiating fluid layer for rigid boundaries and for the entire range of optical thickness but neglected the effect of conduction. The principle of exchange of stabilities is proved and the critical value for instability is given as a function of the optical thickness of the layer. Following Goody's approach, Murgai and Khosla (1962) and Khosla and Murgai (1963), respectively, included the effects of magnetic field and rotation. The principle of exchange of stabilities and the concept of overstability have been discussed.

Christophorides and Davis (1970) added thermal conduction and Goody's static temperature profile to Spiegel's integral formulation when limited to optically thin media. An estimate for the convective heat transport in a transparent medium is made using the shape factor assumption and compared with non-radiative convection.

Arpaci and Gozum (1973) were the first to introduce the effects of fluid non-grayness and boundary emissivities. Using the analysis of Arpaci and Gozum, Onyegegbu (1980) added the rotation effect, and Yang (1990) introduced external convective boundary conditions for rigid boundaries, which is suitable for solar collector applications. Bdeoui and Soufiani (1997) provided a sophisticated treatment of nongray fluids and also a short review of prior work. Mansour and Gorla (1999) presented a regular perturbation analysis for the radiative effects on laminar natural convection with temperature-dependent viscosity.

Larson (2001) studied linear and nonlinear stability properties of Goody's model analytically. When thermal diffusivity is zero, the energy method is used to rule out subcritical instabilities. When thermal diffusivity is nonzero, the energy method is used to find a critical threshold below which all infinitesimal and finite amplitude perturbations are stable.

Lan *et al*. (2003) analyzed the stability of a fluid subject to combined natural convection and radiation using a spectral method. Black boundaries and a gray medium are prescribed. The influences of conduction-radiation parameter, Rayleigh number and optical thickness on flow instabilities and bifurcations are discussed.

We now briefly review the literature on Marangoni convection (*MC*) in Newtonian fluids.

Block (1956) was of the view that Bénard cells in shallow pools are actually produced by variations in surface tension rather than due to buoyancy force which are in turn due to non-uniformities in temperature over the free surface, which would account for the surface depressions over upwelling hot liquid. This mechanism is called the *Marangoni effect*.

Pearson (1958) theoretically demonstrated that surface tension force is sufficient to cause hydrodynamic instability in a liquid layer with a free surface, provided there is a temperature or concentration gradient of proper sense and of sufficient magnitude across the layer. Thus, Pearson (1958) showed that, variation of surface tension with temperature would drive steady Marangoni convection in a fluid layer provided the non-dimensional Marangoni number is sufficiently large and positive. The most significant limitation of Pearson's (1958) work is that it considers only the case of a non-deformable free surface corresponding to the limit of strong surface tension.

Nield (1964) studied the effect of non-uniform temperature gradient on the onset of Marangoni convection subject to the constant heat flux on the upper free surface. Using a single-term Galerkin technique, he obtained critical Marangoni

number for different temperature profiles. Nield (1966) also studied the effect of vertical magnetic field in an electrically conducting fluid on the onset of combined *RBC* and *MC* problems. It is shown that, as the magnetic field strength increases, the coupling between the two agencies causing instability becomes weaker so that the values of Rayleigh number at which convection begins are independent of surface-tension effect and similarly the critical Marangoni number is unaffected by the buoyancy forces provided the Rayleigh number is less than critical.

Vidal and Acrivos (1966) and Takashima (1970) found that, when surface deformation was neglected, the principle of exchange of stabilities is valid in the case of Marangoni instability.

Davis (1969) examined the linear and nonlinear Rayleigh-Bénard-Marangoni convection using energy method. The subcritical instability was found in a small range of the Marangoni convection. It is shown that the equations governing the energy theory are independent of the linear theory problem and that the surface tension behaves like a bounded perturbation to the Bénard problem. The effects of surface tension and buoyancy on the convective instability of a fluid layer with a mean parabolic temperature distribution are examined by Debler and Wolf (1970).

Sarma (1979, 1981 and 1985) analyzed the effect of both uniform rotation and magnetic field on the onset of steady Marangoni convection in a horizontal fluid layer with a deformable free surface for a variety of combinations of thermal and magnetic boundary conditions. He demonstrated the stabilizing effect of rotation/magnetic field on the Marangoni instability of a fluid layer with a deformable free surface. Takashima (1981a, 1981b) examined the effect of a free surface deformation on the onset of stationary and oscillatory surface tension driven instability using linear stability theory.

Lebon and Cloot (1984) studied the nonlinear analysis of the combined *RBC* and *MC* in a horizontal fluid layer of infinite extent. They used the Gorkov-Malkus-Veronis technique, which consists of developing the steady solution in terms of a

small parameter measuring the deviation from the marginal state and solved the nonlinear equations describing the fields of temperature and velocity.

Lam and Bayazitoglu (1987) solved the problem of *MC* in a horizontal fluid layer with internal heat generation using the sequential gradient-restoration algorithm (*SGRA*) developed for optimal control problems. It is shown that the temperature-dependent viscosity plays a larger role than the surface tension in determining the critical conditions.

Maekawa and Tanasawa (1988) studied theoretically the onset of *MC* in an electrically conducting fluid subject to a vertical magnetic field. It is found that convection always sets in the form of longitudinal rolls whose axes are aligned with the horizontal component of the magnetic field and that only the vertical component of the magnetic field has any effect on the critical Marangoni number.

Benguria and Depassier (1989) studied linear stability of a fluid with a free deformable upper surface. They found that when the heat flux on the upper and lower surface is plane and isothermal, oscillatory instability occurs at lower values of the Rayleigh number than the critical value for the onset of steady convection.

Gouesbet *et al*. (1990) investigated the overstability for the combined *RBC* and *MC* problem by means of small disturbance analysis. The influence of Prandtl, Bond and Crispation numbers, the modification induced by interfacial viscosities, heat transfer at the free surface, buoyancy with respect to the pure Marangoni mechanism and different thermal conditions at the rigid wall are discussed in the analysis. Perez-Garcia and Carneiro (1991) analyzed the effects of surface tension and buoyancy on the convective instability in a layer of fluid with a deformable free surface. Their analysis is restricted to fixed values of a Prandtl number and Biot number in order to determine the role of the Crispation number on convection.

Wilson (1993a, 1993b) investigated the effect of a uniform magnetic field on the onset of Marangoni instability in a horizontal layer of quiescent electrically conducting fluid in the presence as well as in the absence of buoyancy force using a

combination of analytical and numerical techniques. The vertical magnetic field was found to have a stabilizing effect .Wilson (1994) studied the problem of *MC* in an electrically conducting fluid with a uniform vertical temperature gradient subjected to a prescribed heat flux at its rigid lower boundary. The critical Marangoni number obtained was different from that of the isothermal case.

Char and Chiang (1994a) studied the problem of *MC* in fluids with internal heat generation. Thess and Nitschke (1995) derived asymptotic expressions for the first unstable mode of surface tension driven instability in an electrically conducting fluid subjected to a strong magnetic field. The spatial structure of the velocity, temperature and electric current density is characterized in terms of Hartmann boundary layers.

Parmentier *et al*. (1996) performed the first weakly nonlinear analysis of the combined *RBC* and *MC* problem without surface deformation in the case of finite Prandtl number. They concluded that hexagons are preferred near the onset of convection and found that the direction of motion in the cells depends on the value of the Prandtl number.

Wilson (1997) used a combination of analytical and numerical techniques to analyze the effect of uniform internal heat generation on the onset of steady Marangoni convection. He obtained for the first time a closed form analytical solution for the onset of steady Marangoni convection and presented asymptotically- and numerically-calculated results for the linear growth rates of the steady modes. Char *et al*. (1997) investigated the onset of oscillatory instability of *MC* in a horizontal fluid layer subject to the Coriolis force and internal heat generation. The upper surface is assumed to be deformably free and the lower surface is rigid. The Crispation number is found to be significant for the occurrence of oscillatory modes.

Selak and Lebon (1997) studied coupled surface tension and gravity driven instabilities in fluids with variable thermophysical properties. Viscosity is assumed

to vary exponentially with temperature, while linear laws are assumed for the heat capacity and thermal conductivity. Computations for glycerol and liquid potassium show that temperature dependence of the thermophysical properties may have a significant effect on the onset of convection.

Hashim and Wilson (1999a, b) have analyzed the effect of a uniform vertical magnetic field on the onset of oscillatory *MC* and on the linear growth rates of steady *MC* in a horizontal layer of electrically conducting fluid heated from below. Their investigations showed that the presence of a magnetic field could cause the preferred mode of instability to be oscillatory rather than steady and that the effect of increasing the magnetic field strength was always to stabilize the layer by decreasing the growth rate of the unstable modes. Hashim and Wilson (1999c) have also investigated the effects of surface tension and buoyancy on the convective instability in a planar horizontal layer of fluid in the most physically relevant case when the non-dimensional Rayleigh and Marangoni numbers are linearly dependent. The comprehensive asymptotic analysis of the marginal curves in the limit of both long and short wavelength disturbances are studied.

Kozhoukharova and Roze (1999) studied stationary and oscillatory Marangoni instability in a fluid layer with a deformable upper surface. The viscosity is assumed to be temperature-dependent and the problem has been solved numerically by Taylor series expansion method. The results of the study reveal that oscillatory convection is possible only when the surface deformability is considered. Bau (1999) demonstrated for the first time the critical Marangoni number for transition from the conduction state to the motion state can be increased through the use of feedback control strategies.

Hashim (2001) obtained for the first time an analytical description of the marginal mode for the onset of oscillatory *MC* in the presence of uniform heat sources in the asymptotic limit of short waves. Hashim and Arifin (2003) presented numerically a necessary and sufficient condition for oscillatory *MC* in a horizontal layer of electrically conducting fluid in the presence of a vertical magnetic field.

For detailed descriptions of linear and nonlinear problems of both *RBC* and *MC*, one may refer to the books of Chandrasekhar (1961), Gershuni and Zhukhovitsky (1976), Kays and Crawford (1980), Zierep and Oertel (1982), Platten and Legros (1984), Gebhart *et al*. (1988), Getling (1998), Colinet *et al*. (2001) and Straughan (2004). Chapters on thermal convection are included in the books by Turner (1973), Joseph (1976a, b), Tritton (1979) and Drazin and Reid (1981). Reviews of recent research on convective instability have been given by Normand *et al*. (1977), Davis (1987) and Bodenschatz *et al*. (2000).

We have so far reviewed the literature relating to Newtonian fluids in both *RBC* and *MC* situations. In what follows we review the literature pertaining to a Newtonian ferrofluid.

1.2.2   Rayleigh-Bénard/Marangoni Convection in Newtonian Ferromagnetic Fluids

The problem of convection in a ferromagnetic fluid is different from magnetoconvection even though the influence of the magnetic field exists in both the problems. In the case of magnetoconvection, the fluid is electrically conducting and we see the influence of a body force, known as, *Lorentz force*. Magnetic fluids are not electrically conducting and hence the Lorentz force does not appear. As a result of the magnetization of the micron-sized suspended ferrite particles a pondermotive force, analogous to the Lorentz force, appears and gives rise to a dynamically different situation than the type that occurs in the magnetoconvection problem.

Finlayson (1970) made a detailed study of convective instability in a ferromagnetic fluid. He showed that convection is caused by a spatial variation in the magnetization which is induced when the magnetization is a function of temperature and a temperature gradient is established across the fluid layer. He also predicted the critical temperature gradient for the onset of convection when only the magnetic mechanism is important as well as when both the magnetic and buoyancy mechanisms are operative. The magnetic mechanism is shown to

predominate over the buoyancy mechanism in fluid layers which are about 1 mm thick. For fluid layers contained between two free boundaries, which are constrained flat, the exact solution has been obtained for some parameter values and oscillatory stability is ruled out. For rigid boundaries, an approximate solution for stationary instability using a higher order Galerkin method is obtained. It is shown that, the Galerkin method yields an eigenvalue which is stationary to small changes in the trial functions because the Galerkin method is equivalent to an adjoint variational principle.

Lalas and Carmi (1971) investigated a nonlinear analysis of the convective stability problem in magnetic fluids using the energy method. They showed that the linear and energy theories give identical results for stationary ferromagnetic flow under the assumption that the magnetization is independent of the magnetic field intensity. Subcritical instabilities were ruled out.

Berkovskii and Bashtovoi (1971) investigated the problem of gravitational convection in an incompressible non-conducting ferromagnetic fluid resulting from the magnetocaloric effect. This problem is shown to be equivalent to the problem of natural convection with a vertical temperature gradient. Closed form solutions for both velocity and temperature are obtained in this study and numerical estimates of the critical magnetic field gradients are given. Kamiyama *et al*. (1988) investigated an analogous problem both numerically and analytically using a perturbation procedure. Elaborate comments on Oberbeck convection in magnetic fluids have been made.

Shilomis (1973) studied the conditions under which instability arises in the equilibrium of a non-uniformly heated ferrofluid in a gravitational field and a non-uniform magnetic field. Shulman *et al*. (1976) experimentally investigated the effect of a constant magnetic field on the heat transfer process in ferromagnetic suspensions by varying the type and concentration of the disperse phase, the strength of the magnetic field and the orientation of the field relative to the direction of the temperature gradient. They observed that the thermal resistance of

disperse systems depends on the size, shape, nature and surface purity of the particles of the disperse phase. The effective thermal conductivity of ferromagnetic suspensions has been shown to be anisotropic in character.

Berkovsky *et al*. (1976) presented numerical and experimental study of convective heat transfer in a vertical layer of a ferromagnetic fluid. A critical relationship is given between heat transfer and characteristic parameters.

Nogotov and Polevikov (1977) studied Oberbeck convection in a vertical layer of a magnetic liquid in a magnetic field of current carrying sheet. The dependence of heat transfer on Rayleigh number, Prandtl number and aspect ratio were clearly exhibited. The convective stability of a vertical layer of magnetic fluid in a uniform longitudinal magnetic field was studied by Bashtovoi and Pavlinov (1978). Rosensweig *et al*. (1978) established experimentally the penetration of ferrofluids in the Heleshaw cell.

Gupta and Gupta (1979) investigated thermal instability in a layer of ferromagnetic fluid subject to Coriolis force and permeated by a vertical magnetic field. It is substantiated that overstability cannot occur if the Prandtl number is greater than unity. Gotoh and Yamada (1982) investigated the linear convective instability of a ferromagnetic fluid layer heated from below and confined between two horizontal ferromagnetic boundaries. The Galerkin technique is used and the Legendre polynomials are taken as the trial functions. It is shown that the magnetization of the boundaries and the nonlinearity of fluid magnetization reduce the critical Rayleigh number and the effects of magnetization and buoyancy forces are shown to compensate each other.

Schwab *et al*. (1983) performed an experiment to examine the influence of a homogeneous vertical magnetic field on the Rayleigh-Bénard convection in a ferrofluid layer. The results agreed with theoretical predictions. Schwab and Stierstadt (1987) demonstrated the preparation and visualization of distinct wavevectors for thermal convection in ferrofluids.

Blums (1987) examined the possibility of having convection in ferromagnetic fluids as a result of magneto-diffusion of colloidal particles which give rise to non-uniform magnetization. Kamiyama *et al.* (1988) studied both analytically and numerically the effect of combined forced and free steady convection in a vertical slot of ferromagnetic fluid in the presence of a transverse magnetic field taking into account the magnetocaloric effect. The relative magnitudes of the magnetization parameter and thermal Rayleigh number along with the uniform pressure gradient are shown to significantly influence the dynamics of the ferrofluid in a vertical slot.

Ageev *et al.* (1990) studied magnetic fluid convection in a non-uniform magnetic field. Results from both numerical and experimental studies are presented. Nakatsuka *et al.* (1990) studied the effect of thermomagnetic convection, which arises when a temperature sensitive magnetic fluid is heated in a vessel under a non-uniform magnetic field.

Stiles and Kagan (1990) examined the thermoconvective instability of a horizontal layer of ferrofluid in a strong vertical magnetic field. Their paper also questioned the satisfactory agreement claimed to exist between the experiments and the theoretical model of Finlayson which has been generalized by them. Schwab (1990) investigated the stability of flat layers of ferrofluid subject to a vertical temperature gradient and a vertical magnetic field experimentally. It is shown that magnetostatic stresses reinforce the surface deformation of Marangoni convection but they work against the surface deformation of Rayleigh-Bénard convection.

Abdullah and Lindsay (1991) examined convection in a nonlinear magnetic fluid under the influence of a non-vertical magnetic field. It is found that both stationary and overstable instabilities can be expected to be realizable possibilities. Sekhar and Rudraiah (1991) studied convective instability in magnetic fluids bounded by isothermal non-magnetic boundaries with internal heat generation. Oscillatory convection is ruled out by proving the validity of the principle of exchange of stabilities. The solutions are obtained using a higher order Galerkin expansion technique.

Blennerhassett *et al*. (1991) analyzed the linear and weakly nonlinear thermoconvective stability of a ferrofluid, confined between rigid horizontal plates at different temperatures and subjected to a strong uniform external magnetostatic field in the vertical direction. When the ferrofluid is heated from above and when convection is due to magnetic forces, the Nusselt numbers for a given supercritical temperature gradient are significantly higher than when the ferrofluid is heated from below. Following the analysis of Blennerhassett *et al*. (1991), Stiles *et al*. (1992) analyzed linear and weakly nonlinear thermoconvective stability in weakly magnetized ferrofluids. They showed that if the ferrofluid is heated from above, the magnitudes of the critical horizontal wavenumbers are substantially higher than those when the ferrofluid is heated from below.

Rudraiah and Sekhar (1992) analyzed the thermohaline convection in a Boussinesq-ferrofluid layer confined between rigid-rigid boundaries using the Galerkin method. The conditions for direct and oscillatory modes are established. It is shown that the concentration gradient and the diffusivity ratio significantly influence the stability of the system.

Siddheshwar (1993) investigated the *RBC* problem of a Newtonian ferromagnetic fluid with second sound. It is shown that oscillatory convection is possible for heating from above. He further showed that the critical eigenvalue for stationary convection, when heated from below, is significantly influenced by second sound effects. Aniss *et al*. (1993) made an experimental investigation of the *RBC* problem in a magnetic fluid contained in an annular Hele-Shaw cell.

Qin and Kaloni (1994) developed a nonlinear stability analysis based on energy method to study the effects of buoyancy and surface tension in a ferromagnetic fluid layer which is heated from below. The free surface is assumed to be flat and non-deformable. The possibility of the existence of subcritical instabilities is pointed out. Venkatasubramanian and Kaloni (1994) studied the effects of rotation on the thermoconvective instability in a horizontal layer of ferrofluid heated from below in the presence of a uniform vertical magnetic field.

Aniss *et al*. (1995) made a theoretical investigation of Rayleigh-Bénard convection in a magnetic liquid enclosed in a Hele-Shaw cell. It is shown that the Hele-Shaw approximation leads to two nonlinear problems; each one depending on the order of magnitude of the Prandtl number. Results of linear and weakly nonlinear analysis of stability near the onset of convection are presented.

Odenbach (1995a) investigated the convective flow generated by the interaction of a magnetic field gradient with a gradient in magnetization in a magnetic fluid. This gradient was caused by the diffusion of the magnetic particles in the field gradient. Odenbach (1995b) investigated the onset and the flow profile of thermomagnetic convection in a cylindrical fluid layer experimentally. Under microgravity conditions and with periodic boundary conditions, he established counter-rotating vortices.

Russell *et al*. (1995) examined heat transfer in strongly magnetized ferrofluids in the case of strong heating from above. The convective patterns at critical conditions have a large wave number and this is used to derive simplified equations for the temperature field in the ferrofluid. The results show that the heat transfer depends nonlinearly on the temperature difference.

Siddheshwar (1995) studied convective instability of a ferromagnetic fluid in the Rayleigh-Bénard situation between fluid-permeable, magnetic boundaries and subject to a uniform, transverse magnetic field. The Galerkin method is used to predict the critical eigenvalue for free-free and rigid-rigid boundaries. This paper reaffirmed the qualitative findings of earlier investigations which are in fact limiting cases of the present study.

Weilepp and Brand (1996) presented a linear stability analysis of a layer of a magnetic fluid with a deformable free surface, which is heated from below and exposed to a uniform, vertically applied magnetic field. In this configuration the temperature dependence of the surface tension, the buoyancy and the focusing of the magnetic field due to surface fluctuations act as destabilizing effects. It is

demonstrated that there is no oscillatory instability in the regions of the parameter space considered in this problem.

Odenbach (1996) investigated the behaviour of a magnetic fluid under the influence of an inhomogeneous magnetic field gradient. The onset of the convective flow is described by a model based on a time-dependent dimensionless parameter. Zebib (1996) performed a theoretical study of the character and stability of thermomagnetic flow in a microgravity environment. Convection is driven owing to imposed radial magnetic and temperature gradients in a cylindrical shell containing a ferrofluid. It is shown that convection sets in as a stable supercritical bifurcation.

Bajaj and Malik (1997) have investigated a nonlinear convective instability in a layer of magnetic fluid in the presence of an applied magnetic field and temperature gradient. The stability of steady state patterns resulting from the convective instability has been discussed using bifurcation theory. Rolls are found to be stable on both the square and hexagonal lattices.

Morimoto *et al*. (1998) investigated the dissipative structure of thermomagnetic convection by microgravity experiments through linear and nonlinear numerical simulations. The effect of the aspect ratio of the magnetic fluid layer on the pattern formations is investigated. In the case of linear theory, the critical magnetic Rayleigh number and the critical wave number have been obtained by solving the eigenvalue equations using harmonic analysis and the finite difference method. Linear stability theory results agree with the microgravity experiments. The nonlinear equations have been solved by the control volume finite difference method. The flow patterns obtained by the nonlinear calculation coincide with those obtained by the microgravity experiments. It is found that the critical magnetic Rayleigh number obtained by the nonlinear analysis agrees with that obtained by the linear stability analysis and the bifurcations from one pattern to another are clearly demonstrated as a problem of probability.

Bajaj and Malik (1998) studied pattern formation due to double-diffusive convection in ferrofluids in the presence of an externally applied transverse magnetic field. The critical value of the Rayleigh number for steady state bifurcation is found to be different from that for Hopf-bifurcation in contrast to ordinary fluids where the two critical values are the same.

Siddheshwar and Abraham (1998) considered the problem of convection in ferromagnetic fluids occupying a rectangular vertical slot with uniform heat flux along the vertical walls. A closed form solution based on the Oseen-linearization technique is obtained. It is found that the effect of the magnetization is to increase the Nusselt number. Rudraiah *et al*. (1998) examined the effect of non-uniform concentration distribution on double diffusive convection in a Boussinesq-magnetic fluid layer confined between two rigid boundaries analytically using the Galerkin method. The conditions for direct and oscillatory modes for different nonlinear basic concentration distributions have been established.

Russell *et al*. (1999) examined the structure of two-dimensional vortices in a thin layer of magnetized ferrofluid heated from above in the limit as the critical wave number of the roll cells become large. They present a nonlinear asymptotic description of the vortex pattern that occurs directly above the critical point in the parameter space where instability first sets in. Tangthieng *et al*. (1999) investigated heat transfer enhancement in ferrofluids subjected to steady magnetic fields. Luo *et al*. (1999) examined novel convective instabilities in a magnetic fluid.

Yamaguchi *et al*. (1999) studied experimentally and numerically the natural convection of a magnetic fluid in a two dimensional cell whose aspect ratio is one. Results obtained reveal that the vertically imposed magnetic field has a destabilizing influence and at the supercritical state the flow mode becomes quite different from that without the magnetic field.

Sekar *et al*. (2000) studied the effect of ferrothermohaline convection in a rotating medium heated from below and salted from above. The effect of salinity is

included in the magnetization and density of the ferrofluid. The conditions for both stationary and oscillatory modes have been obtained using linear stability analysis and it is found that the stationary mode is favored in comparison with oscillatory mode. Auernhammer and Brand (2000) investigated the effect of rotation on *RBC* in a ferrofluid using both a linear and a weakly nonlinear analysis of the governing hydrodynamic equations in the Boussinesq approximation.

Aniss *et al*. (2001) investigated the effect of a time-sinusoidal magnetic field on the onset of convection in a horizontal magnetic fluid layer heated from above. The Floquet theory is used to determine the convective threshold for free-free and rigid-rigid cases. The possibility to produce a competition between the harmonic and sub-harmonic modes at the onset of convection is discussed.

Rudraiah *et al*. (2002) and Shivakumara *et al*. (2002) investigated the effect of different basic temperature gradients on the onset of *MC*, and on the onset of the combined *RBC* and *MC* in ferrofluids respectively in the presence of a vertical uniform magnetic field. The mechanism of suppressing or augmenting the ferroconvection is discussed.

Abraham (2002a) investigated the *RBC* problem in a micropolar ferromagnetic fluid layer in the presence of a vertical uniform magnetic field analytically. It is shown that the micropolar ferromagnetic fluid layer heated from below is more stable as compared with the classical Newtonian ferromagnetic fluid. Lange (2002) studied the thermomagnetic convection of magnetic fluids in a cylindrical geometry subject to a homogeneous magnetic field. The general condition for the existence of a potentially unstable stratification in the magnetic fluid is derived.

Siddheshwar and Abraham (2003) examined the thermal instability in a layer of a ferromagnetic fluid when the boundaries of the layer are subjected to synchronous/asynchronous imposed time-periodic boundary temperatures (*ITBT*) and time-periodic body force (*TBF*). It is shown that the stability or instability of ferrofluids can be controlled with the help of *ITBT* and *TBF*.

Kaloni and Lou (2005) presented linear and weakly nonlinear analysis of thermal instability in a layer of ferromagnetic fluid rotating about a vertical axis and permeated by a vertical magnetic field. The amplitude equation is developed by multiscale perturbation method and it is found that the ratio of heat transfer by convection to that by conduction decreases as magnetic field increases.

Bajaj (2005) considered thermosolutal convection in magnetic fluids in the presence of a vertical magnetic field and bifrequency vertical vibrations. The regions of parametric instability have been obtained using the Floquet theory. Vaidyanathan *et al*. (2005) obtained the condition for the onset of thermoconevctive instability in ferrofluids due to the Soret effect. Both stationary and oscillatory instabilities have been investigated.

We have so far reviewed the works related to both *RBC* and *MC* problems in a Newtonian ferrofluid. We now review the literature pertaining to Newtonian dielectric fluids.

1.2.3 Rayleigh-Bénard/Marangoni Convection in Newtonian Dielectric Fluids

Turnbull (1968a, b) investigated both theoretically and experimentally the electroconvective instability with a temperature gradient. The analysis shows that for liquids with short or moderate electrical relaxation times, the electric field causes the internal gravity wave propagating downward to become stable. Turnbull (1969) examined the effect of dielectrophoretic forces on the Bénard instability. The principle of exchange of stabilities is shown to hold for a certain set of boundary conditions. Approximate solutions for the critical temperature gradient as a function of the wavelength and the electric field are found using the variational principles and the Galerkin method.

The effect of uniform rotation on the onset of convective instability in a dielectric fluid under the simultaneous action of a vertical *ac* electric field and a vertical temperature gradient was considered by Takashima (1976). It is shown that

the principle of the exchange of stabilities is valid for most dielectric fluids. It is shown that, even when the electrical effects are taken into account, the Coriolis force has an inhibiting effect on the onset of instability and as the speed of rotation increases the coupling between the two agencies causing instability (electrical and buoyancy force) becomes tighter.

Bradley (1978) studied overstable electroconvective instabilities. It is found that overstable modes can be excited by an electric field of sufficient strength.

Takashima and Ghosh (1979) analyzed the problem of the onset of instability in a horizontal layer of viscoelastic dielectric liquid under the simultaneous action of a vertical *ac* electric field and a vertical temperature gradient. It is shown that oscillatory modes of instability exist only when the thickness of the liquid layer is smaller than about 0.5 mm and in this case the force of electrical origin is much more important than the buoyancy force.

Castellanos and Velarde (1981) analyzed the effect of a temperature-dependent dielectric constant in the stability analysis of a liquid layer subjected to an electric field, weak unipolar injection and temperature gradient.

Takashima and Hamabata (1984) examined the effect of a horizontal *ac* electric field on the stability of natural convection which occurs in a dielectric fluid between two parallel vertical plates maintained at different temperatures. The linear stability theory is considered. Using the power series method, the eigenvalue equation is obtained which is then solved numerically. It is shown that when the electrical Rayleigh number is less than about 2130, the electrical field has no effect on the stability of natural convection and that when it exceeds this value the electric field and the natural convection flow are coupled strongly.

Oliveri and Atten (1985) studied electroconvection between nonparallel electrodes. The linear critical conditions are determined for various values of the different parameters. Using this analysis it is possible to model general atmospheric circulation. Atten *et al*. (1988) examined the basic properties of electroconvection

resulting from unipolar injection into an insulating liquid. It is shown that for very weak injection the induced motion of the liquid has only a negligible influence on the total current across the layer.

Ko and Kim (1988) studied electrohydrodynamic convective instability in a horizontal fluid layer with temperature gradient. Nonlinear evolution of disturbances near the onset of convection is also considered. It is found that subcritical instabilities are also possible for small or high enough values of the Prandtl number.

Stiles (1991) investigated the problem of an electrically insulating liquid layer confined between horizontal conducting electrodes, the upper of which is warmer. It is found that the system becomes unstable with respect to the onset of steady convection when the electric field strength reaches a critical value, which in a rapidly varying *ac* field is due to the polarization body force.

Maekawa *et al*. (1992) considered the convective instability problem in *ac* and *dc* electric fields. Linearized perturbation equations are solved by the Galerkin method. Stiles and Kagan (1993), using a linear stability analysis, predicted the onset of convective instability in an annulus of a pure dielectric liquid between two long coaxial cylinders where the liquid experiences a radial temperature gradient and a strong radial *ac* electric field. When the two cylinders are stationary, the results obtained have been in good agreement with experimental data on silicone oils.

Stiles *et al*. (1993) studied the problem of convective heat transfer through polarized dielectric liquids. It is shown that for a critical voltage, as the gravitational Rayleigh number becomes increasingly negative, the critical wavenumber at the onset of convection becomes very large. As the temperature drop between the plates increases the fraction of the heat transfer associated with convection is found to pass through a maximum value when the critical horizontal wavenumber is close to 4 times its value when gravity is absent.

Haque *et al.* (1993) studied the effect of non-uniform electric field on convective heat transfer in a colloidal fluid. The electroconvective heat transfer coefficient exhibits a 'timing' effect as well as an 'aging' effect. An *ac* field always enhances the heat transfer, whereas a *dc* field produces an enhancement that is almost vertical in the vicinity of the origin. An increase in particle concentration increases the heat transfer coefficient, while a sharp rise in heat transfer coefficient is observed when the surface charge of the colloidal particle is increased. When the inclination of the cylinder is changed from the horizontal to the vertical position the convective heat transfer coefficient increases.

Char and Chiang (1994b) presented a theoretical study of the Bénard-Marangoni instability problem for a liquid layer with a free upper surface, which is heated from below by a heating coil through a solid plate in an *ac* electric field. The boundary effects of the solid plate, which include its thermal conductivity, electric conductivity and thickness, have great influence on the onset of convective instability in the liquid layer. The problem is analyzed using the linear stability theory and the eigenvalue equations obtained solved by using the fourth order Runge-Kutta-Gill's method with the shooting technique. The results indicate that the solid plate with a higher thermal or electric conductivity and a bigger thickness tends to stabilize the system.

Haque and Arajs (1995a, b) examined convective specific heat transfer in liquids in the presence of non-uniform electric fields. The heat transfer coefficient has been evaluated under the influence of *ac* and *dc* electric fields, and the efficiency obtained in a *dc* field is found to be higher than in the *ac* field. A similar trend is also noticed for the electric Nusselt number.

El Adawi *et al.* (1996) examined the problem of natural convection in an inclined fluid layer with uniform heat source in the presence of a normal *ac* electric field. The power series method is used to obtain the stable and unstable solutions. El Adawi *et al.* (1997) using a linear stability theory examined the effect of a normal *ac* electric field on the stability of the natural convection that occurs in a

dielectric fluid layer between two inclined plates that are maintained at different temperatures. The power series method is used to obtain the eigenvalue equation which is then solved numerically to obtain the stable and unstable solutions.

Orlik *et al*. (1998) investigated the electrochemical formation of luminescent electrohydrodynamic convective patterns in a thin layer of cells. Koulova-Nenova and Atten (1998) presented linear analysis of hydrodynamic instability of superposed layers of conducting and insulating liquids when injecting ions from the above a metallic electrode into an insulating liquid. Two instability mechanisms, convective and interfacial, are examined and the role of injection on decreasing the critical values is discussed.

Smorodin *et al*. (1999) have given results about the parametric excitation of thermoelectric instability in a fluid layer subject to a harmonically time varying heat flux normal to its top open surface. The boundaries of instability and characteristics of critical disturbances are found for the cases of coupled phenomena between thermoelectric effects and surface tension gradients and thermoelectric effects and buoyancy.

Ezzat and Othman (2000) investigated the effect of a vertical *ac* electric field on the onset of convective instability in a dielectric micropolar fluid heated from below under the simultaneous action of the rotation of the system and the vertical temperature gradient. The power series method is adopted to obtain the eigenvalue equation which is then computed numerically.

Smorodin and Velarde (2000) investigated the electrothermoconvective instability of a plane horizontal layer of a poorly conducting, Ohmic liquid subjected to a varying electric field in the *EHD* approximation. Floquet theory is applied for finding various instability thresholds in the linear approximation with and without the effect of buoyancy. It is shown that, depending on the amplitude and frequency of modulation, the electric field can stabilize an unstable basic state or destabilize the equilibrium of the liquid.

Smorodin (2001) analyzed the effect of an alternating arbitrary-frequency electric field on the stability of convective flow of a dielectric liquid occupying a vertical layer in the *EHD* approximation. The stability thresholds are determined in the linear approximation using Floquet theory.

Kosvintsev *et al.* (2002) investigated the effect of electrization of a poorly conducting liquid and the action of an electric field on the stability of the base flow in a vertical layer with unequal but constant temperatures at its vertical boundaries using a linear dependence of conductivity on temperature.

Siddheshwar (2002a) presented an analogy between Rayleigh-Bénard instability in Newtonian ferromagnetic/dielectric fluids by considering free-free, isothermal boundaries. The results obtained reveal that the problem of *RBC* in dielectric liquids could be extracted from an analogous problem in ferromagnetic liquids.

1.2.4 Rayleigh-Bénard/Marangoni Convection in Viscoelastic and Viscoelastic Ferromagnetic/Dielectric Fluids

Herbert (1963) and Green (1968) analyzed, for the first time, the problem of oscillatory convection in a viscoelastic fluid under the influence of infinitesimal disturbances. Herbert (1963) examined the stability of plane Couette flow heated from below and showed that finite elastic stress in the undisturbed state is necessary for the oscillatory motion. He also showed that the presence of elasticity has a destabilizing effect on the flow. Green (1968) through his investigations revealed that a large restoring force sets up an oscillating convective motion in a thin rectangular layer of the fluid heated from below.

Vest and Arpaci (1969) made significant contribution to the study of convection in a viscoelastic fluid layer heated from below. It is found that overstability would occur at the lowest value of possible adverse temperature gradient at which the rate of change of kinetic energy can balance in a synchronous manner.

Sokolov and Tanner (1972) studied Rayleigh-Bénard convection in a general viscoelastic fluid using an integral form of the constitutive equation. It has been shown that under certain conditions oscillatory motions are possible. Van der Borght *et al.* (1974) made theoretical investigations of finite amplitude thermal convection of non-Newtonian fluids in the steady state.

Nonlinear convection in viscoelastic fluids have been studied by Riahi (1976). The boundary layer method is used by assuming large Rayleigh number, Prandtl number and a small value of elasticity parameter. The study shows that elasticity effects do not affect the horizontal wavenumber significantly and also that the heat flux depends strongly on elasticity parameter and decreases with increasing elasticity parameter.

Eltayeb (1977) studied the linear and nonlinear Rayleigh-Bénard convection in a viscoelastic fluid using the Oldroyd model. He found that, in the study of nonlinear effects for slightly supercritical Rayleigh number, the plane disturbances for the case where the exchange of stabilities is valid and plane disturbances for the case of overstability are governed by equations similar to that for the plane Poiseuille flow. Eltayeb also studied the effect of linear and nonlinear convection and showed that the elasticity effect is to stabilize the layer in the linear theory and to destabilize it in the nonlinear theory provided the ratio of the mean temperature gradient of the layer to the actual temperature difference across the layer is large enough.

Stastna (1985) obtained sufficient conditions for the exchange of stabilities in the *RBC* problem of a viscoelastic fluid with different types of boundary conditions. A differential and a single integral model are applied and it is shown that the possibility of overstable motions can be ruled out.

Barbara *et al.* (1986) studied the multi-criticality in viscoelastic fluids heated from below. The phase diagrams near co-dimensions of two bifurcation points are analyzed for a Maxwell fluid. It is found that the phase diagram exhibit a novel

mixed phase in which both stationary and oscillatory modes are present. It is also shown that the system exhibit direct transition from the conductive to the mixed phase depending upon the coupling between the two modes.

Agrait and Castellanos (1986) investigated both stationary and oscillatory convection in a horizontal layer of a dielectric Oldroyd fluid under the simultaneous action of a vertical *ac* field and a vertical temperature gradient. The physical mechanisms relevant to the problem are expounded using a heuristic argument. They found that the observation of overstability could be possible in the case of low viscosity, high relaxation time fluid with the help of an *ac* potential.

Khayat (1994, 1995a) examined the onset of aperiodic or chaotic behaviour in viscoelastic fluids, *viz*., Oldroyd-B and upper-convected Maxwellian fluids in the context of the *RBC* problem. The truncated Fourier representation of the constitutive equations for the fluids considered leads to a generalized Lorenz model. He found that fluid elasticity tends to destabilize the convective cell structure, precipitating the onset of chaotic motion, at a Rayleigh number that may be well below that corresponding to Newtonian fluids. He also showed that, depending on the value of the Prandtl number, chaos is found to set in through the quasiperiodic route or periodic doubling.

Khayat (1995b) also analyzed the existence of overstability in the presence of non-negligible inertia for an Oldroyd-B fluid. Based on the generalization of the classical Lorenz system for a Newtonian fluid, he obtained conditions for the existence of the corresponding Hopf-bifurcation as a function of fluid elasticity, retardation and thermal conductivity.

Park and Lee (1995) investigated nonlinear hydrodynamic stability analysis for viscoelastic fluids heated from below by adopting a general constitutive model. The study confirms that the rigid boundaries cause smaller convective amplitudes and Nusselt number compared with free boundaries. The study also revealed that rigid boundaries have more tendency to cause subcritical bifurcation than the free

boundaries when compared at the same value of the elasticity parameter. Park and Lee (1996) also analyzed the Hopf-bifurcation of viscoelastic fluids heated from below with rigid-rigid and rigid-free boundary conditions for the range of viscoelastic parameters where the Hopf-bifurcation occurs. The nonlinear analysis based on power series methods reveals that various parameters have significant effects on hydrodynamic stability and suggest that Rayleigh-Bénard convective systems may be used at least in part, as a useful rheometric tool to assess the suitability of constitutive equations.

Khayat (1996) investigated the influence of weak shear thinning on the onset of chaos in thermal convection for a Carreau-Bird fluid. He found that the critical Rayleigh number at the onset of thermal convection remains the same as for a Newtonian fluid but the shear thinning dramatically alters the amplitude and nature of the convective cellular structure.

Otsubo and Edamura (1998) studied the viscoelasticity of a dielectric fluid in non-uniform electric field generated by electrodes with flocked fibers. They demonstrated that the electrorheological effect can be attributed to the electrohydrodynamic convection and external shear.

Siddheshwar (1998) studied the problem of oscillatory convection in an Oldroyd-Boussinesq-ferromagnetic fluid analytically. The linear stability analysis reveals the stabilizing nature of the strain retardation parameter, and the destabilizing effect of stress relaxation parameter and the magnetization parameters.

Kolodner (1999) presented experimental observations of the convective flow produced by heating a horizontal layer of viscoelastic fluid from below in a long narrow angular geometry. He observed that convective patterns take the form of spatially standing and travelling waves which exhibit small amplitudes and extremely long oscillation periods and that the threshold Rayleigh number for the onset of oscillation is lower than the value measured for steady convection in a

Newtonian fluid in the same apparatus. They also exhibit a decreasing trend with increasing elastic relaxation time. This behaviour agrees with the theoretical prediction of the linear instability of viscoelastic convection.

Siddheshwar (1999) studied the effect of second sound in the *RBC* problem of a second order ferromagnetic fluid. It is found that the results are noteworthy at short times and the critical eigenvalues are less than the classical ones. Odenbach (1999) examined the possibilities for combined terrestrial and microgravity investigations on viscoelastic behaviour in ferrofluids.

Martinez-Mardones *et al*. (1999) studied convective and absolute instabilities in viscoelastic fluids. Further, they analysed pattern selection and stability in deterministic and stochastic systems and discussed the possibility of noise induced phase transitions. The problem of the onset of convection in a horizontal layer of viscoelastic dielectric liquid (Walters' liquid B) in the presence of a vertical *ac* electric field is examined by Othman (2001).

Siddheshwar and Srikrishna (2002) made linear and nonlinear analyses of convection in a second-order fluid describable by the Rivlin-Ericksen constitutive equation. The linear theory, based on the normal mode technique, leads to a critical eigenvalue which is independent of viscoelastic parameters. The nonlinear analysis, based on the truncated representation of Fourier series, reveals that finite amplitudes have random behaviour. The onset of chaotic motion is also discussed.

Siddheshwar (2002b) studied oscillatory convection in viscoelastic ferromagnetic and dielectric liquids of the Rivlin-Ericksen, Maxwell and Oldroyd types. It is found that the Maxwell liquids are more unstable than the one subscribing to the Oldroyd description whereas the Rivlin-Ericksen liquid is comparatively more stable.

Ramadan *et al*. (2003) have considered a four-dimensional Lorenz model for an Oldroyd-B fluid to examine the viscoelastic flow in the context of *RBC* set-up. It is shown that fluid elasticity and fluid retardation alter the flow behaviour in

comparison to inertia-dominated Newtonian flow and the fluid elasticity tends to precipitate the onset of chaos. Numerical simulations are performed and the results are studied by means of time signature, phase portraits, power spectrum, Poincaré map, Lyapunov exponents and bifurcation diagrams.

Othman and Zaki (2003) analyzed the problem of the onset of instability in a horizontal layer of viscoelastic dielectric liquid under the simultaneous action of a vertical *ac* electric field and thermal relaxation time. Most recently, Siddheshwar (2005a) has studied the problems of *RBC* and *MC* in third-grade and Careau-Bird ferromagnetic/dielectric liquids.

1.3 PLAN OF WORK

Nowadays, the knowledge of the many processes involving ferromagnetic and dielectric fluids has reached a stage where a modelling may be done with a certain confidence. The problems of Rayleigh-Bénard and Marangoni convection in these fluids are extremely important from the viewpoint of technological and commercial applications. In many heat transfer problems involving these fluids as working media, it is now being increasingly realized that these may have enhanced/diminished heat transfer compared to classical Newtonian fluids. In the light of the above, the thesis is organized as follows:

The **Second Chapter** consists of basic equations, approximations, boundary conditions and a discussion on the dimensionless parameters. In **Chapter III**, linear and nonlinear analyses of ferroconvection are studied. The linear analysis is based on a normal mode technique. The nonlinear analysis exploits the representative Lorenz model of convection arrived at by considering a truncated Fourier series representation for the field variables. This chapter ends with a logical analogy between ferroconvection and electroconvection and the results are discussed in the last chapter.

The results of the study of the previous chapter are mathematically significant albeit physically restrictive. In **Chapter IV**, the effect of internal heat source/sink and the two non-Boussinesq effects, *viz.*, thermorheological and magnetorheological effects, are brought in. A higher order Rayleigh-Ritz technique is used for solving the resulting system of differential equations with space-varying coefficients. Both terrestrial and microgravity situations are considered and an analogy is presented between ferroconvection and electroconvection. The results obtained are discussed in the last chapter.

Chapter V deals with the effect of thermal radiation on Rayleigh-Bénard/Marangoni convection in a variable viscosity ferromagnetic fluid. The Milne-Eddington approximation is employed in obtaining the basic state and the limits of optically thin and thick fluid layer are considered. Higher order Rayliegh-Ritz method is used to handle the variable coefficient differential equations and an analogy between ferroconvection and electroconvection is presented. The important results of this chapter are given in the last chapter.

Chapter VI is devoted to the consideration of Rayleigh-Bénard/Marangoni convective instabilities in viscoelastic ferromagnetic/dielectric liquids with variable viscosity. Three constitutive equations of Jeffrey, Maxwell and Rivlin-Ericksen are considered and are shown to be limiting cases of the Jeffrey description. The results are discussed in the last chapter.

Finally, in **Chapter VII**, the main conclusions drawn from the investigation of the problems in chapters III – VI are stated. An exhaustive bibliography follows this last chapter.

CHAPTER II

BASIC EQUATIONS, APPROXIMATIONS, BOUNDARY CONDITIONS AND DIMENSIONLESS PARAMETERS

In this chapter, we discuss the mathematical modelling of convective phenomena involving a variable-viscosity ferromagnetic/dielectric liquid. Viscoelasticity of the fluid, the third mode of heat transport and volumetric internal heat generation have been considered in the modelling exercise. The relevant boundary conditions and dimensionless parameters arising in the problem are explained in a general manner.

It is now well-known that ferrofluids represent a class of magnetizable liquids with interesting properties capable of having a substantial impact on technology. In many commercial applications, ferrofluid is an essential component of the system or is an addition, which enhances the performance. Since the force exerted by a magnetic field gradient on the fluid is proportional to its susceptibility, even weak magnetic fields can exert reasonable forces to magnetic fluids.

It should be remarked that, upon application of a magnetic field, the entropy associated with the magnetic degree of freedom in magnetic fluids is changed due to the field-induced ordering. If performed adiabatically, this leads to a temperature change in the fluid (Resler and Rosensweig, 1964; Parekh *et al*., 2000). The magnitude of this effect depends on the physical and magnetic properties such as, size, temperature dependence of magnetization, heat capacity of the material and carrier liquid. We note, in view of this, that the energy conservation equation should account for heat sources (sinks) which have implications for magnetocaloric pumping. In the case of dielectric liquids, Joule heating by an alternating current provides the volumetric energy source (Kulacki and Goldstein, 1972) which proves to be beneficial in microfluidic ion-drag pumping.

On the other hand, if the magnetic force is to have any engineering application to the control of fluid motion, there must be an interface or temperature gradients. In what follows we elucidate briefly the development of some of the classical instabilities that arise in ferromagnetic fluids.

The interfacial phenomena provide an area where the fluid mechanics of a ferromagnetic liquid differs from that of a non-magnetic material. It is shown both theoretically and experimentally that when a vertical magnetic field is applied on a magnetic fluid having a flat surface with air above, the flat surface becomes unstable when the applied magnetic field exceeds the critical value of the magnetic field (Rosensweig and Cowley, 1967). This *normal field instability* (also known as *Rosensweig instability*) is a direct consequence of the interaction of nonlinear instabilities in magnetic fluids (Bajaj and Malik, 1996) and thanks to which a pattern of spikes appear on the fluid surface.

It is well-known that parametric stabilization can also be observed in fluid dynamics, the most impressive example being the inhibition of the *Rayleigh-Taylor instability*: a horizontal fluid layer placed above another one of smaller density could be stabilized by vertically vibrating their container (Racca and Annett, 1985). However, this requires a container with a rather small horizontal extension because modes with a large enough wavelength are not parametrically stabilized. It should be remarked that the parametric excitation of surface waves, the so-called *Faraday instability*, can also be achieved in magnetic fluids by temporal modulation of an external field (Mahr and Rehberg, 1998).

As has been discussed in Chapter I, dissipative instabilities, such as Rayleigh-Bénard instability arising due to density variation and Marangoni instability arising owing to surface-tension variation in ferromagnetic fluids in the presence of a temperature gradient, have been studied by many researchers (Finlayson, 1970; Siddheshwar, 1995; 2005; Zebib, 1996; Auernhammer and Brand, 2000; Abraham, 2003). It is worth noting that, in contrast to the dissipative Marangoni instability in magnetic fluids, the Rosensweig instability is static whose critical wavelength is

nearly independent of the layer thickness (Weilepp and Brand, 1996). It has been predicted recently that the Rosensweig instability could be inhibited by vertical vibrations with an appropriate choice of the fluid and vibration parameters (Muller, 1998; Petrelis *et al*., 2000).

It is interesting to note that the surface instability, similar to that proposed by Cowley and Rosensweig (1967), arises when an electric field is applied in the direction normal to the surface of a dielectric liquid (Melcher, 1963; Taylor and McEwan, 1965; Neron de Surgy *et al*., 1993). However, these studies reveal that the surface instability effect in magnetic fluids is much stronger than that of electric instability.

The foregoing remarks essentially suggest that a colloidal suspension of magnetic particles such as ferrofluid can be manipulated by field gradients. Most successful applications use permanent magnets to form field gradients and hold the liquid in position. Once in position, secondary properties of the liquid can be exploited such as its viscosity, its lubricity and the like.

Table 2.1: The ratio of some physical properties of various fluids at temperature of 20°C and 50°C (Shin and Cho, 1996).

Fluid	Density $\dfrac{\rho_{20^oC}}{\rho_{50^oC}}$	Specific heat capacity $\dfrac{c_{p\,20^oC}}{c_{p\,50^oC}}$	Thermal conductivity $\dfrac{k_{20^oC}}{k_{50^oC}}$	Dynamic viscosity $\dfrac{\mu_{20^oC}}{\mu_{50^oC}}$
Water	1.011	1.000	0.940	1.816
Ethylene glycol	1.021	0.945	0.961	3.263
Mineral oil (10 – NF)	1.022	0.933	1.027	3.592
FC – 77 (dielectric liquid)	1.041	0.954	1.041	1.616
Glycerin	1.015	0.923	0.997	8.809

The viscosity of any liquid system is related to the internal friction. Many works on convective instability took for granted the Boussinesq approximation which implies, in particular, that the fluid viscosity μ is temperature-independent. However, it is well-known that for a given class of fluids (Table 2.1), viscosity may vary significantly with temperature: the so-called *thermorheological effect*. Further, it is evident from Table 2.1 that viscosity decreases more drastically with temperature compared with other variations.

The viscosity of a magnetic fluid is greater than that of a carrier liquid due to the presence of suspended particles. In the absence of a magnetic field, the viscosity of dilute magnetic fluids can be described by the relations derived for non-magnetic colloidal solutions. Einstein gave the following relation for the effective viscosity μ of a dilute system, that is, for very small concentrations ($\varphi_S \simeq 0.05$) of spherical particles

$$\mu = \mu_c \left(1 + \frac{5}{2} \varphi_S \right), \tag{2.1}$$

where μ_c is the viscosity of the carrier liquid and φ_S is the solid volume fraction. Other fractions for the effective viscosity of magnetic fluids accounting for high concentration, fractional volume of surfactant and polydispersivity can be found elsewhere (Hall and Busenberg, 1969; Rosensweig, *et al.*, 1969; Fertman, 1990; Upadhyay, 2000).

In the classical experiment of McTague (1969), the viscosity of a suspension of cobalt nanoparticles was measured with an external magnetic field applied parallel and perpendicular to the flow direction (Figure 2.1). It was found that the external field hinders the free rotation of the magnetic particles and thus increases the viscosity of the fluid: the so-called *magnetorheological effect*. A theoretical treatment for the magnetically induced relative viscosity change $\mu_r = \Delta\mu / \mu$ was given by Shliomis (1972). The magnetorheological effect has subsequently been the subject of many works (Kamiyama, *et al.*, 1987; Kamiyama and Satoh, 1989;

44

Kobori and Yamaguchi, 1994; Chen *et al*., 2002; Bossis, *et al*., 2002 and references therein). These studies have assumed different laws of viscosity-magnetic field strength which include linear, quadratic and exponential proportionalities.
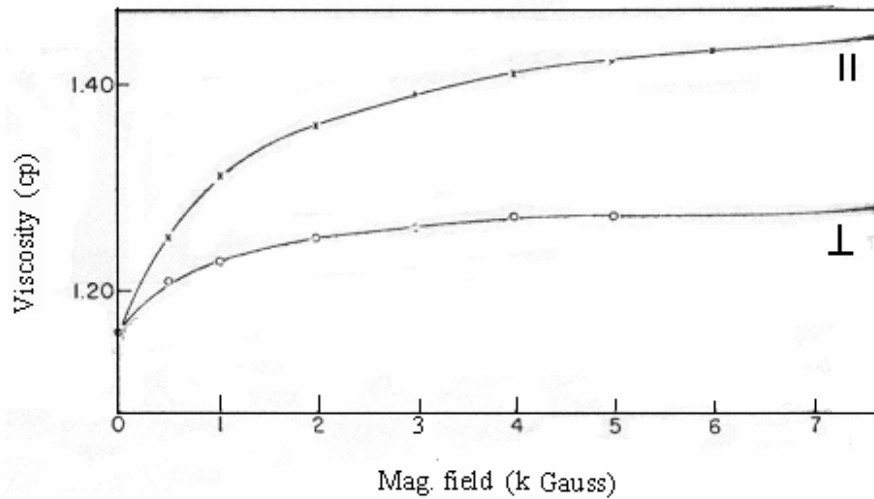
Figure 2.1 : Magnetic-field dependence of the viscosity of sample #12 for magnetic field parallel and perpendicular to the flow (from McTague, 1969).
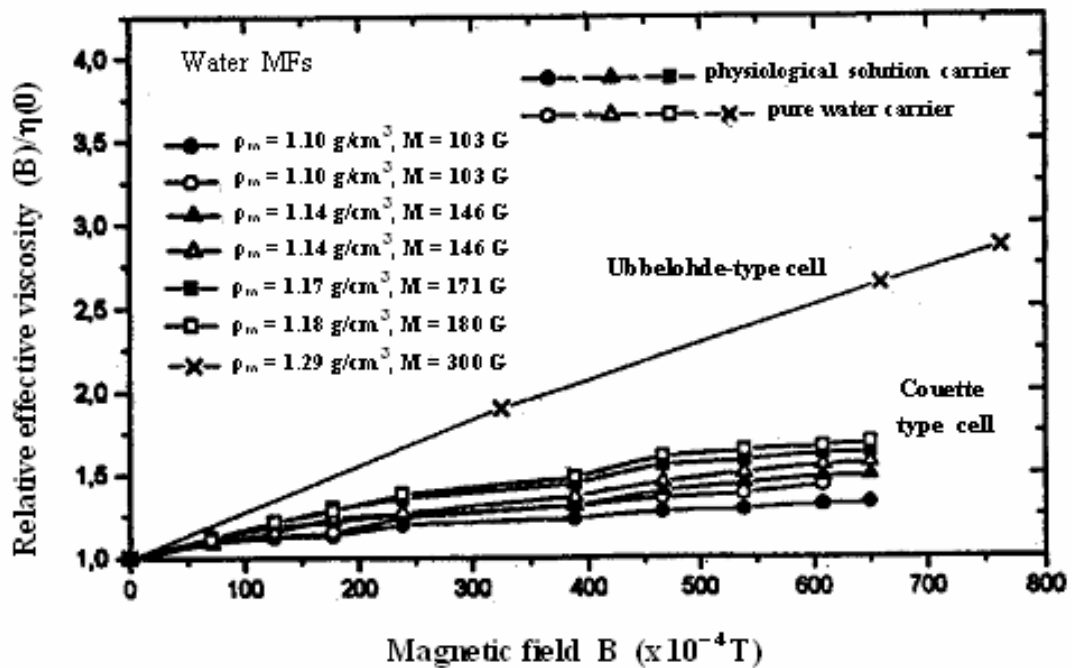
Figure 2.2 : Relative increase of the effective viscosity in magnetic field for pure water (from Balau *et al*., 2002).

More recently, Balau *et al.*, (2002) substantiated experimentally that magnetorheological effect is of significant importance in water-based and kerosene-based solutions, and in physiological-solution based magnetic liquids even for moderate strengths of applied magnetic field (Figure 2.2). It is pertinent to note here, although not considered in the present study, that Shliomis and Morozov (1994) postulated a *negative viscosity* contribution ($\Delta\mu < 0$) for a certain range of the field strength and frequency of the applied magnetic field. This negative viscosity effect can be understood as a transfer of energy from the magnetic field into rotational motion of the particles (Rosensweig, 1996).

The control of the effective viscosity of a dielectric suspension by an electric field has been intensively studied during the last two decades (Halsey, 1992; Sun and Rao, 1996; Klingenberg, 1998; Hao, 2002; Ma *et al.*, 2003) because of the numerous possible applications. Most of these studies have shown that the viscosity increases when an electric field is applied to a suspension of dielectric particles.
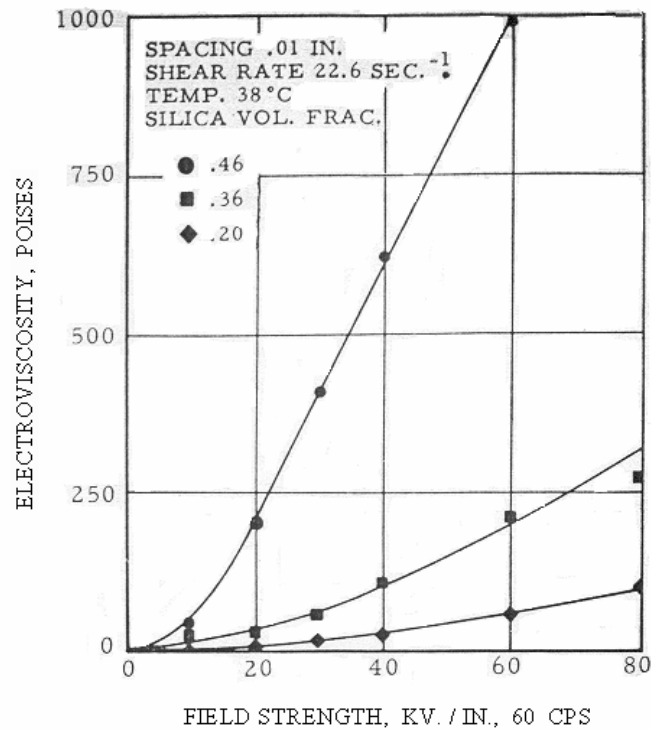
Figure 2.3 : Effect of field strength on electroviscosity of silica dispersions at different volume fractions (from Klass and Martinek, 1967).

In fact, it was shown experimentally (Klass and Martinek, 1967) that the electroviscosity continues to increase with the applied electric field (Figure 2.3). This effect, referred to as *electrorheological effect*, is due to a chain formation of the particles of the suspension induced by their polarization under an electric field. The concept of negative viscosity, that is a decrease in viscosity when an electric field is applied, has also been reported by a few authors (Wu and Conrad, 1997; Lobry and Lemaire, 1999). In this case the electric field is a *dc* field and the particles carry electric charges. In what follows we provide with several temperature-dependent viscosity correlations available in the literature.

a) **Linear law**

(Busse and Frick, 1985; Cloot and Lebon, 1985; Lam and Bayazitoglu, 1987; Selak and Lebon, 1993; Kozhoukharov and Roze, 1999; Straughan, 2002a)

$$\mu(T) = \mu_1 \left[1 - \gamma_1 \left(T - T_a \right) \right]. \tag{2.2}$$

b) **Palm-Jenssen law**

(Palm, 1960; Jenssen, 1963; Stengel *et al.*, 1982)

$$\mu(T) = \mu_1 \left[1 - \gamma_2 \cos \pi (1 - T) \right]. \tag{2.3}$$

c) **Exponential laws**

(Torrance and Turcotte, 1971; Stengel, *et al.*, 1982; Hirayama and Takaki, 1993; Selak and Lebon, 1993; 1997)

$$\text{(i)} \quad \mu(T) = \mu_1 \exp\left[-\gamma_3 \left(T - T_a \right) \right]. \tag{2.4a}$$

$$\text{(ii)} \quad \mu(T) = \mu_1 \exp\left[\gamma_e \left(\frac{1}{2} - T \right) \right]. \tag{2.4b}$$

d) **Inverse linear law**

(Lai and Kulacki, 1990; Kafoussias and Williams, 1995; Chakroborty and Borkakati, 2002)

$$\frac{1}{\mu(T)} = \frac{1}{\mu_1} \left[1 + \gamma_4 \left(T - T_a \right) \right]. \tag{2.5}$$

*e) **Linear-quadratic law***

(Straughan, 2004)

$$\mu(T) = \mu_1 \left[1 - \gamma_1 (T - T_a) - \gamma_5 (T - T_a)^2 \right]. \tag{2.6}$$

*f) **Quadratic law***

(Straughan, 2004; Siddheshwar, 2004; Siddheshwar and Chan, 2005)

$$\mu(T) = \mu_1 \left[1 - \gamma_5 (T - T_a)^2 \right]. \tag{2.7}$$

In the above equations (2.2) – (2.7), $\mu_1 = \mu(T_a)$, T_a is the average temperature, $\gamma_e = \log \dfrac{1 + \gamma_3}{1 - \gamma_3}$ and γ_i (i = 1, 2, 3, 4, 5) are small positive constants. It should be mentioned that the viscosity laws given by Eqs. (2.3) – (2.7) have been introduced thanks to the fact that the linear viscosity law given by Eq. (2.2) is found to be inadequate in delineating the essential attributes of temperature-dependent viscosity. Extensive investigation is required to be done on the temperature and electromagnetic field dependency of the viscosity and other physical properties of the complex liquids considered in the thesis. The theoretical investigation reported in the thesis is a first step in this direction. It is also on this reason that we take the specific heat capacity and thermal conductivity to be temperature-independent.

In the present study, the effective viscosity is taken to be a quadratic function of both temperature and strength of a uniform *dc* magnetic field in so far as the magnetic fluids are concerned. We have neglected the inertia of the suspended particles and their rotation so that the equations of motion could be tractable. Likewise, in the case of dielectric liquids, the effective viscosity is assumed to be a quadratic function of both temperature and magnitude of a uniform *ac* electric field.

In this thesis we are concerned with Rayleigh-Bénard/Marangoni convection in Newtonian ferromagnetic/dielectric liquids and also viscoelastic ferromagnetic/dielectric liquids. We now discuss about the basic equations

pertaining to the problems reported in the thesis after documenting the nomenclature of different quantities used in the governing equations and boundary conditions.

NOMENCLATURE

a	dimensionless wavenumber
$\vec{B}$	magnetic induction
Bi	Biot number
C_{VE}	effective heat capacity at constant volume and electric field
C_{VH}	effective heat capacity at constant volume and magnetic field
d	depth of the fluid layer
$\vec{D}$	electric displacement
$\vec{E}$	electric field
E_{o}	root mean square value of the electric field at the lower surface
$\vec{g}$	gravitational acceleration $(0, 0, -g)$
G	rate of radiative heating per unit volume
$\vec{H}$	magnetic field
H_C	convective heat transfer coefficient
$\vec{H}_{o}$	applied uniform vertical magnetic field
H_T	rate of heat transfer per unit area
I^{+}	intensity of radiation in the upward direction
I^{-}	intensity of radiation in the downward direction
$(\hat{i}, \hat{j}, \hat{k})$	unit vectors in the x, y and z directions respectively
k	dimensional wave number
k_1	thermal conductivity
K_1	pyromagnetic coefficient $(= -(\partial M / \partial T)_{H_{o}, T_a})$
K_a	absorption coefficient of the fluid

49

$k_x,\ k_y$	wavenumber in the x and y directions
$\vec{M}$	Magnetization
M_1	buoyancy-magnetization parameter
M_3	non-buoyancy-magnetization parameter
Ma	thermal Marangoni number
Ma_E	electric Marangoni number
Ma_H	magnetic Marangoni number
M_o	mean value of magnetization at $H = H_\mathrm{o}$, $T = T_a$
N_S	heat source (sink) parameter
Nu	Nusselt number
p	effective pressure $\left(= p^* + p_m + p_s \right)$
p^*	hydrostatic pressure
p_m	fluid magnetic pressure $\left(= \mu_\mathrm{o} \int_0^H M\ dH \right)$
p_s	magneto-strictive pressure $\left(= \mu_\mathrm{o} \int_0^H V \left(\dfrac{\partial M}{\partial V} \right)_{H,\,T} dH \right)$
$\vec{P}$	dielectric polarization
P_B	Planck black-body intensity $\left(= S_c\, T_a^4 / \pi \right)$
Pr	Prandtl number
$\vec{q}$	velocity vector $\ (= (u,\, v,\, w)\,)$
R	thermal Rayleigh number
R_E	electric Rayleigh number
R_M	magnetic Rayleigh number
S	strength of volumetric heat source per unit volume and per unit time
S_c	Stefan-Boltzmann constant
s_r	heat content per unit volume
t	time
T	temperature

T_1	constant temperature of the lower boundary $(=T_o + \Delta T)$
T_a	arithmetic mean of boundary temperatures $(=(T_o + T_1)/2)$
T_o	constant temperature of the upper boundary
Tr	transpose
(x, y, z)	Cartesian coordinates with z-axis vertically upwards
ΔT	temperature difference between the lower and upper surfaces $(=T_1 - T_o)$
∇	vector differential operator $\left(= (\partial/\partial x)\hat{i} + (\partial/\partial y)\hat{j} + (\partial/\partial z)\hat{k} \right)$
D/Dt	material or substantial derivative $(= \partial/\partial t + (\vec{q}.\nabla))$
∇^2	three dimensional Laplacian operator $\left(=(\partial^2/\partial x^2)+(\partial^2/\partial y^2) + (\partial^2/\partial z^2)\right)$
∇_1^2	two dimensional Laplacian operator $\left(= (\partial^2/\partial x^2) + (\partial^2/\partial y^2)\right)$

Subscripts

b	basic state
c	critical value

Greek symbols

α	thermal expansion coefficient $\left(= (-1/\rho_o)(d\rho/dT)_{T=T_a}\right)$
β	adverse basic temperature gradient $(= \Delta T/d)$
Φ	magnetic (electric) scalar potential
κ	thermal diffusivity
τ	absorptivity parameter
μ	temperature and electric/magnetic field strength dependent effective viscosity
μ_1	reference viscosity at $H = H_o$, $T = T_a$
μ_2	fluid elasticity coefficient
μ_o	magnetic permeability of vacuum
λ_1	stress-relaxation coefficient

51

λ_2	strain-retardation coefficient
Γ	effective viscosity parameter
Γ_V	stress-relaxation parameter
η	viscoelastic parameter
ε_o	electric permittivity of free space
ε_r	relative permittivity or dielectric constant
θ	non-dimensional temperature $(=(T_b - T_a)/\Delta T)$
ρ	fluid density
ρ_o	reference density at $T = T_a$
ω	frequency of oscillations
σ_o	reference surface tension at $H = H_\mathrm{o}$, $T = T_a$
σ_r	growth rate
σ_S	temperature and electric/magnetic field strength dependent surface tension
χ	conduction-radiation parameter
χ_e	electric susceptibility
χ_m	magnetic susceptibility $(=(\partial M /\partial H)_{H_\mathrm{o}, T_a})$

2.1 BASIC EQUATIONS

2.1.1 Basic Equations for a Newtonian Ferromagnetic Liquid

To derive the basic equations, we make the following approximations:

a) The ferromagnetic fluid is a homogeneous, incompressible medium and the total magnetic moment of the particles is equally distributed throughout any elementary fluid volume. Since the carrier fluids are good insulators, forces due to interaction of magnetic fields with currents of free charge, such as found in magnetohydrodynamics, are negligible (Cowley and Rosensweig, 1967). The particles are prevented from agglomerating in the presence of a magnetic field as they are surrounded by a surfactant such as oleic acid. The combination of

the short-range repulsion due to the surfactant and the thermal agitation yields a material which behaves as a continuum (Papell and Faber, 1966).

b) Since we are considering small particle concentrations dipole-dipole interactions are negligible and hence the applied magnetic field is not distorted by the presence of the ferromagnetic fluid (Bean, 1955). Hysteresis is unlikely in ferromagnetic fluids since the applied magnetic field is not rapidly changing (Cowley and Rosensweig, 1967).

c) When the fluid is at equilibrium, the flow field is isothermal, *i.e.*, the temperature of the fluid is everywhere below the boiling point leading to an equation of state where the density of the fluid is a linear function of temperature according to $\rho = \rho_{\mathrm{o}}\left[1 - \alpha\left(T - T_a\right)\right]$.

d) The Boussinesq approximation is assumed to be valid, *i.e.*, $(1/\rho)(D\rho/Dt) \ll \nabla \cdot \vec{q}$. As a result, the equation of continuity, *viz.*, $(D\rho/Dt) + \rho\,(\nabla \cdot \vec{q}) = 0$, reduces to $\nabla \cdot \vec{q} = 0$. In other words, Boussinesq fluids behave as incompressible fluids. This assumption also allows the fluid density to vary only in the buoyancy force term in the momentum equation and elsewhere it is treated as a constant. This is valid provided the velocity of the fluid is much less than that of sound, *i.e.*, Mach number $\ll 1$. The basic idea of this approximation is to filter out high frequency phenomena such as sound waves since they do not play an important role in transport processes (Spiegel and Veronis, 1960).

e) Maxwell's equations are considered for non-conducting liquids with no displacement currents.

f) The effective viscosity is assumed to be a function of both temperature (thermorheological effect) and magnitude of the magnetic field (magnetorheological effect).

g) Other fluid properties such as thermal conductivity and heat capacity are assumed to be constants.

h) The heating due to magnetocaloric effect of the magnetic substance in the presence of a magnetic field is assumed negligible.

i) The effect of internal heat generation in the liquid is considered in Chapter IV and the volumetric heat source is assumed to be uniform.

j) The viscous dissipation effect is neglected. The influence of radiative heat transfer on the onset of convection is considered in Chapter V. The radiating fluid is assumed to be gray and the effect of scattering is neglected.

k) The temperature range of operation is below the Curie point.

l) Magnetization induced by temperature variations is small compared to that induced by the external magnetic field, *i.e.* $K_1 \Delta T << \left(1 + \chi_m\right) H_0$.

m) The magnetization is assumed to get aligned with the magnetic field. Experiments indicate that there is only a small dependence of viscosity and surface tension on magnetization. Thus, the magnetization is taken as a function of both magnetic field and temperature.

The governing equations for ferrofluids (Neuringer and Rosensweig, 1964; Finlayson, 1970) are the following:

Conservation of Mass (Continuity Equation)

The general form of the continuity equation is

$$\frac{D\rho}{Dt} + \rho \left(\nabla . \vec{q} \right) = 0 . \tag{2.1.1}$$

Eq. (2.1.1), for a fluid with Boussinesq approximation, reduces to

$$\nabla \cdot \vec{q} = 0.$$ (2.1.2)

Conservation of Momentum (Momentum Equation)

The momentum equation for a ferromagnetic fluid under the Boussinesq approximation with variable viscosity is

$$\rho_0 \frac{D\vec{q}}{Dt} = -\nabla p - \rho g \,\hat{k} + \mu_0 (\vec{M} \cdot \nabla) \vec{H} + \nabla \cdot \left[\mu(H,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right].$$ (2.1.3)

The left side of Eq. (2.1.3) represents the rate of change of momentum per unit volume. The first, second, third and fourth terms on the right side represent respectively the pressure force due to normal stress, body force due to gravity, pondermotive force arising due to the magnetization of the fluid, called the *Maxwell's stress*, and the viscous force arising due to shear. For a constant viscosity ferromagnetic fluid, the last term on the right side simplifies to $\mu \nabla^2 \vec{q}$.

Conservation of Energy

The heat transport equation for the considered ferromagnetic fluid which obeys modified Fourier law is

$$\rho_0 C_{VH} \frac{DT}{Dt} = k_1 \nabla^2 T + S + \rho_0 C_{VH} \frac{G}{s_r}.$$ (2.1.4)

The second and third terms on the right side of Eq. (2.1.4) account for the uniform heat source and radiation respectively.

Equation of State

The equation of state for a single component fluid is

$$\rho(T) = \rho_0 \left[1 - \alpha (T - T_a) \right].$$ (2.1.5)

Eq. (2.1.5) is derived by expanding the density $\rho(T)$ using a Taylor's series at $T = T_a$ and neglecting the second and higher terms. The expression for temperature and magnetic field strength dependent surface tension $\sigma_S(H, T)$ for a single component fluid can be, in a similar fashion, written as

$$\sigma_S(H, T) = \sigma_o + \sigma_H(H - H_o) - \sigma_T(T - T_a),$$ (2.1.6)

where $\sigma_H = (\partial \sigma_S / \partial H)_{H_o, T_a}$ and $\sigma_T = -(\partial \sigma_S / \partial T)_{H_o, T_a}$.

Maxwell's Equations

Maxwell's equations, simplified for a non-conducting ferromagnetic fluid with no displacement currents, become

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = \vec{0}.$$ (2.1.7a, b)

The magnetic induction $\vec{B}$, in terms of the magnetization $\vec{M}$ and magnetic field $\vec{H}$, is expressed as

$$\vec{B} = \mu_o \left(\vec{M} + \vec{H} \right).$$ (2.1.8)

Since the magnetization is aligned with the magnetic field and is a function of temperature and magnetic field, we have

$$\vec{M} = \frac{\vec{H}}{H} M(H, T).$$ (2.1.9)

The magnetic equation of state is linearized about the magnetic field H_o and the average temperature T_a to become

$$M = M_o + \chi_m(H - H_o) - K_1(T - T_a).$$ (2.1.10)

2.1.2 Basic Equations for a Newtonian Dielectric Liquid

In deriving the basic equations we make the following assumptions:

a) The dielectric liquid is a non-conducting, homogeneous, incompressible continuum and the total dipole moment of the particles is equally distributed throughout any elementary fluid volume.

b) When the fluid is at equilibrium, the flow field is isothermal, *i.e.*, the temperature of the fluid is everywhere below the boiling point leading to an equation of state where the density of the fluid is a linear function of temperature according to $\rho = \rho_{\mathrm{o}}\left[1 - \alpha\left(T - T_a\right)\right]$.

c) The Boussinesq approximation is assumed to be valid.

d) Maxwell's equations are considered for non-conducting fluids with no displacement currents.

e) The effective viscosity is assumed to be a function of both temperature (thermorheological effect) and magnitude of the electric field (electrorheological effect).

f) Other fluid properties such as thermal conductivity and heat capacity are assumed to be constants.

g) If the frequency of the electric field becomes too high, there can be appreciable heating associated with dielectric loss. We assume this form of dielectric heating is negligible for the frequencies discussed in our problem.

h) The effect of internal heat generation in the liquid is considered in Chapter IV and the volumetric heat source is assumed to be uniform.

i) The viscous dissipation effect is neglected. The influence of radiative heat transfer on the onset of convection is considered in Chapter V. The radiating fluid is assumed to be gray and the effect of scattering is neglected.

j) The applied electric field $\vec{E}$ is assumed to be an *ac* field and the root mean square of the magnitude of the electric field is assumed as the effective value.

k) Polarization induced by temperature variations is small compared to that induced by the external electric field, *i.e.*, $e\,\Delta T << \left(1+\chi_e\right)$ where the quantity e is defined below following Eq. (2.1.18).

l) The dielectric constant ε_r is assumed to be a linear function of temperature.

The governing equations for a variable-viscosity dielectric liquid (Stiles *et al.*, 1993) under the Boussinesq approximation are

$$\nabla . \vec{q} = 0, \tag{2.1.11}$$

$$\rho_{\rm o} \frac{D\vec{q}}{Dt} = -\nabla p - \rho\,g\,\hat{k} + (\vec{P}.\nabla)\,\vec{E} + \nabla .\left[\mu(E,T)\left(\nabla\vec{q} + \nabla\vec{q}^{Tr}\right)\right], \tag{2.1.12}$$

$$\rho_{\rm o} C_{VE} \frac{DT}{Dt} = k_1\,\nabla^2 T + S + \rho_{\rm o} C_{VE}\,\frac{G}{s_r}, \tag{2.1.13}$$

$$\rho(T) = \rho_{\rm o}\left[1 - \alpha(T - T_a)\right], \tag{2.1.14}$$

where the third term on the right side of Eq. (2.1.12) represents a polarization force called the *dielectrophoretic force*. The expression for temperature and electric field strength dependent surface tension $\sigma_s(E,T)$ takes the form

$$\sigma_s\left(E,T\right) = \sigma_{\rm o} + \sigma_E\left(E - E_{\rm o}\right) - \sigma_T\left(T - T_a\right), \tag{2.1.15}$$

where $\sigma_E = (\partial\sigma_s/\partial E)_{E_{\rm o},\,T_a}$ and $\sigma_T = -(\partial\sigma_s/\partial T)_{E_{\rm o},\,T_a}$.

The electrical equations are

$$\nabla \cdot \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0}, \tag{2.1.16a, b}$$

where

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_o \left(\varepsilon_r - 1 \right) \vec{E}. \tag{2.1.17a, b}$$

The equation of state for the dielectric constant ε_r is

$$\varepsilon_r = \varepsilon_r^o - e\left(T - T_a \right), \tag{2.1.18}$$

where $\varepsilon_r^o = \varepsilon_r(T_a)$ and $e = -(\partial \varepsilon_r / \partial T)_{T=T_a}$. It is expedient to write $\varepsilon_r^o = (1 + \chi_e)$ for it facilitates us to arrive at the conventional definition $\vec{P} = \varepsilon_o \chi_e \vec{E}$ in the absence of the temperature dependence of ε_r, *i.e.*, when $e = 0$. In writing Eq. (2.1.18), we have assumed that ε_r varies with the electric field strength quite insignificantly (Stiles *et al.*, 1993).

2.1.3 Basic Equations for a Viscoelastic Liquid

Viscoelastic fluids are non-Newtonian fluids which are viscous and have an elastic nature. The viscoelasticity in a fluid is indicated by the existence of normal stress differences. Non-Newtonian fluids demonstrate a nonlinear dependence of shearing stress on velocity gradient. Fluids belonging to this category are any number of thick gooey substances such as paste, printer's ink and slurries. As we noted earlier, the most famous influence of magnetic fields on magnetic fluids is the change of viscosity provided by the hindrance of free rotation of the particles in a shear flow due to the action of magnetic field. As extension of this effect the appearance of viscoelasticity in magnetic fluids has been discussed for many years (Odenbach, 1999).

Viscoelastic fluids do have the property of partially recovering their original state after the stress is removed. In other words, fluids possessing a certain degree of elasticity in addition to viscosity are called viscoelastic. The elastic property of such fluids leads to several unusual behaviour, such as, the *Weissenberg effect* or the *rod-climbing phenomenon*. If a rotating rod is immersed in a Newtonian liquid, the liquid surface is depressed near the rod because of the centrifugal force, whereas in a viscoelastic liquid, the liquid climbs up the rod because of normal stress generated by elastic properties, which can also be observed during polymerization reactions. It is found that the rise of a free fluid surface at a rotating axis is too small in viscoelastic ferromagnetic fluids to be observed under normal terrestrial conditions and that the microgravity situations can help to amplify the effect of the normal stress differences (Odenbach, 1999).

Another phenomenon is the marked swelling in a viscoelastic liquid issuing from a die. As a result, extrusion dies must be designed with care to produce the desired product cross section. Markovitz and Coleman (1964) demonstrated these phenomena. When a viscoelastic fluid is in motion, certain amount of energy is stored up in the fluid as the strain energy and some is dissipated in order to overcome viscous forces. In viscoelastic fluids, therefore, we have to consider the strain however small it may be. The strain is responsible for the partial recovery of the fluid to the original state and that ensures the reverse flow when the stress is removed. When flow takes place, the natural state of the fluid constantly changes and it tries to attain the instantaneous position of the deformed state but does not succeed completely. This lag is referred to as the *memory* of the fluid which is a measure of elasticity of the fluid. Thus for viscoelastic fluids the flow behaviour cannot be represented as a relation between stress and shear rate alone, but it depends on the recent history of these quantities as well as their current values. The constitutive equations for such fluids must therefore involve shear stress, shear rate and their local time derivatives and can be written only by a nonlinear functional relation. However, linear viscoelasticity has been studied in great detail, as it represents a property of the material in well-defined limits.

Oldroyd (1950, 1958) considered rheological equations of state for idealized, incompressible, viscoelastic fluids whose behaviour at small variable shear stress is characterized by μ_1, the coefficient of viscosity, λ_1 the relaxation time and λ_2 $(<\lambda_1)$, the retardation time. Frohlich and Sack (1946) gave the following rheological equation for constant viscosity viscoelastic liquids

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_o\frac{D\vec{q}}{Dt}+\rho g\,\hat{k}+\nabla p\right]=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu_1\nabla^2\vec{q}. \qquad (2.1.19)$$

This equation is based on a structural model for a colloidal suspension in which Hookean elastic spherical particles are supposed to be distributed in a Newtonian viscous liquid. The material is essentially a liquid and the physical model is such that $\lambda_1>\lambda_2$. When λ_1 tends to λ_2, the material reduces to a Newtonian fluid of viscosity μ_1.

Eq. (2.1.19) for a viscoelastic ferromagnetic fluid with temperature- and magnetic field strength-dependent viscosity becomes

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_o\frac{D\vec{q}}{Dt}+\rho g\,\hat{k}+\nabla p-\mu_o(\vec{M}.\nabla)\vec{H}\right]$$
$$=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla.\left[\mu(H,T)\left(\nabla\vec{q}+\nabla\vec{q}^{Tr}\right)\right] \qquad (2.1.20)$$

Similarly, Eq. (2.1.19) for a viscoelastic dielectric liquid with temperature- and electric field strength-dependent viscosity becomes

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_o\frac{D\vec{q}}{Dt}+\rho g\,\hat{k}+\nabla p-(\vec{P}.\nabla)\vec{E}\right]$$
$$=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla.\left[\mu(E,T)\left(\nabla\vec{q}+\nabla\vec{q}^{Tr}\right)\right] \qquad (2.1.21)$$

Several limiting cases of Eqs. (2.1.19) – (2.1.21) including both Newtonian and viscoelastic descriptions are documented in Table 2.2.

Table 2.2: Limiting cases of Eqs. (2.1.19) – (2.1.21) for a constant viscosity liquid.

Nature of liquid $\rightarrow$ / Type of liquid $\downarrow$	Ordinary viscous liquid	Ferromagnetic liquid	Dielectric liquid
Newtonian	$\lambda_1 = \lambda_2$ in Eq. (2.1.19) (Chandrasekhar, 1961)	$\lambda_1 = \lambda_2$ in Eq. (2.1.20) (Finlayson, 1970)	$\lambda_1 = \lambda_2$ in Eq. (2.1.21) (Stiles *et al.*,1993)
Maxwell	$\lambda_2 \rightarrow 0$ in Eq. (2.1.19) (Vest and Arpaci, 1969)	$\lambda_2 \rightarrow 0$ in Eq. (2.1.20) (Siddheshwar, 1998; 2002b)	$\lambda_2 \rightarrow 0$ in Eq. (2.1.21) (Siddheshwar, 2002b)
Jeffrey	Eq. (2.1.19) as it is (Sokolov and Tanner, 1972)	Eq. (2.1.20) as it is (Siddheshwar, 2002b)	Eq. (2.1.21) as it is (Takashima and Ghosh, 1979)
Rivlin - Ericksen	$\lambda_1 \rightarrow 0$ and $\lambda_2 \rightarrow \mu_2/\mu_1$ in Eq. (2.1.19) (Siddheshwar and Srikrishna, 2002)	$\lambda_1 \rightarrow 0$ and $\lambda_2 \rightarrow \mu_2/\mu_1$ in Eq. (2.1.20) (Siddheshwar, 1999; 2002b)	$\lambda_1 \rightarrow 0$ and $\lambda_2 \rightarrow \mu_2/\mu_1$ in Eq. (2.1.21) (Siddheshwar, 2002b)

In what follows we discuss about various boundary conditions arising in the convective instability problems of ferromagnetic/dielectric liquids.

2.2 BOUNDARY CONDITIONS

2.2.1 Velocity Boundary Conditions

The boundary conditions on velocity are obtained from conservation of mass, the no-slip condition and the Cauchy's stress principle depending on the nature of the bounding surfaces of the fluid. The following combinations of boundary surfaces are considered in the convective instability problems:

(i) Both lower and upper boundary surfaces are rigid.

(ii) Both lower and upper boundary surfaces are free.

(iii) Lower surface is rigid and upper surface is free.

a) *Rigid surfaces*

If the fluid layer is bounded above and below by rigid surfaces, then the viscous fluid adheres to its bounding surface; hence the velocity of the fluid at a rigid boundary surface is that of the boundary. This is known as the *no-slip* condition and it indicates that the tangential components of velocity in the x and y directions are zero, *i.e.* $u = 0$, $v = 0$. If the boundary surface is fixed or stationary, then in addition to $u = 0$, $v = 0$, the normal component of velocity $\vec{q} \cdot \hat{n}$ is also zero, *i.e.*, $w = 0$. Hence at the rigid boundary we have

$$u = v = w = 0. \tag{2.2.1}$$

Since $u = v = 0$ for all values of x and y at the boundary, we have $\dfrac{\partial u}{\partial x} = 0$ and $\dfrac{\partial v}{\partial x} = 0$, and hence from the continuity equation subject to the Boussinesq approximation, it follows that

$$\frac{\partial w}{\partial z} = 0$$

at the boundaries. Thus, in the case of rigid boundaries, the boundary conditions for the z-component of velocity are

$$w = \frac{\partial w}{\partial z} = 0. \tag{2.2.2}$$

b) *Free surfaces*

In the case of a free surface the boundary conditions for velocity depend on whether we consider the surface-tension or not. If there is no surface-tension at the boundary, *i.e.*, the free surface does not deform in the direction normal to itself, we must require that

$$w = 0. \tag{2.2.3}$$

We have taken the *z*-axis perpendicular to the *xy* plane, therefore *w* does not vary with respect to *x* and *y*, *i.e.*

$$\frac{\partial w}{\partial x} = 0 \quad \text{and} \quad \frac{\partial w}{\partial y} = 0. \tag{2.2.4}$$

In the absence of surface tension, the non-deformable free surface (assumed flat) is free from shear stresses so that

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0. \tag{2.2.5}$$

From the equation of continuity subject to the Boussinesq approximation, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{2.2.6}$$

Differentiating this equation with respect to '*z*' and using Eq. (2.2.5) yields

$$\frac{\partial^2 w}{\partial z^2} = 0. \tag{2.2.7}$$

Thus, in the absence of surface-tension, the conditions for the *z*-component of velocity at the free surfaces are

$$w = \frac{\partial^2 w}{\partial z^2} = 0. \tag{2.2.8}$$

This condition is the *stress-free* condition.

In the presence of surface-tension, the boundary conditions for a 'constant viscosity fluid' can be obtained by equating the shear stresses at the surface to the variations of surface-tension, *i.e.*,

$$\tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{\partial \sigma_s}{\partial x} \tag{2.2.9}$$

and

$$\tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{\partial \sigma_s}{\partial y}, \tag{2.2.10}$$

where μ is assumed constant for a moment. The expression for the surface-tension σ_s, appearing in the above equation, is given by Eqs. (2.1.6) and (2.1.15). Using Eq. (2.2.3), differentiating Eq. (2.2.9) with respect to '*x*' and Eq. (2.2.10) with respect to '*y*' and adding the two, we obtain

$$\mu\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \nabla_1^2 \sigma_s. \tag{2.2.11}$$

In arriving at Eq. (2.2.11) we have used the fact that x and y variations of w cease to exist at the boundaries.

Eq. (2.2.11), using the continuity equation (2.2.6), reduces to

$$-\mu\frac{\partial^2 w}{\partial z^2} = \nabla_1^2 \sigma_s. \tag{2.2.12}$$

Equation (2.2.12), on using Eq. (2.1.6), becomes

$$\mu\frac{\partial^2 w}{\partial z^2} = \sigma_T \nabla_1^2 T - \sigma_H \nabla_1^2 H. \tag{2.2.13}$$

Thus the boundary conditions for a free surface of magnetic fluids in the presence of surface-tension and magnetic field are

$$w = 0 \quad \text{and} \quad \mu \frac{\partial^2 w}{\partial z^2} = \sigma_T \nabla_1^2 T - \sigma_H \nabla_1^2 H \,. \tag{2.2.14a}$$

Similarly, using Eq. (2.1.15), the boundary conditions for a free surface of dielectric liquids in the presence of surface-tension and an *ac* electric field become

$$w = 0 \quad \text{and} \quad \mu \frac{\partial^2 w}{\partial z^2} = \sigma_1 \nabla_1^2 T - \sigma_E \nabla_1^2 E \,. \tag{2.2.14b}$$

We stress once more that the boundary conditions given by Eqs. (2.2.14a) and (2.2.14b) are applicable to constant viscosity ferromagnetic and dielectric liquids respectively. The modified boundary conditions which include the variable viscosity effect are discussed in Appendix B. These new boundary conditions involving surface-tension have not been reported in the literature so far.

2.2.2 Thermal Boundary Conditions

The thermal boundary conditions depend on the nature of the boundaries (Sparrow *et al*., 1964). Four different types of thermal boundary conditions are discussed below.

(i) *Fixed surface temperature*

If the bounding wall of the fluid layer has high heat conductivity and large heat capacity, the temperature in this case would be spatially uniform and independent of time, *i.e.* the boundary temperature would be unperturbed by any flow or temperature perturbation in the fluid. Thus

$$T = 0 \tag{2.2.15}$$

at the boundaries. The effect is to maintain the temperature and this boundary condition is known as *isothermal* or *boundary condition of the first kind* which is the *Dirichlet type* boundary condition.

(ii) *Fixed surface heat flux*

Heat exchange between the free surface and the environment takes place in the case of free surfaces. According to Fourier's law, the heat flux Q_T passing through the boundary per unit time and area is

$$Q_T = -k_1 \frac{\partial T}{\partial z} \tag{2.2.16}$$

where $\dfrac{\partial T}{\partial z}$ is the temperature gradient of the fluid at the boundary. If Q_T is unperturbed by thermal or flow perturbations in the fluid, it follows that

$$\frac{\partial T}{\partial z} = 0 \tag{2.2.17}$$

at the boundaries. This thermal boundary condition is known as *adiabatic* boundary condition or *insulating* boundary condition or *boundary condition of the second kind* which is the *Neumann* type boundary condition.

(iii) *Boundary condition of the third kind*

This is a general type of boundary condition on temperature which is given by

$$\frac{\partial T}{\partial z} = -Bi\, T \,. \tag{2.2.18}$$

When $Bi \to \infty$, we are led to the isothermal boundary condition $T = 0$ and when $Bi \to 0$, we obtain the adiabatic boundary condition $\dfrac{\partial T}{\partial z} = 0$.

The boundary conditions that we are discussing in what follows concern magnetic and electric potential. These new boundary conditions are the first of their kind in literature and suggest that most theoretical investigations in the area need to be reworked and analyzed.

2.2.3 Magnetic Potential Boundary Conditions

The general boundary conditions for the perturbed magnetic potential Φ are

$$
\left.\begin{aligned}
D\Phi + \frac{a\Phi}{1+\chi_m} - T = 0 \qquad \text{at} \quad z = \frac{1}{2}, \\
D\Phi - \frac{a\Phi}{1+\chi_m} - T = 0 \qquad \text{at} \quad z = -\frac{1}{2},
\end{aligned}\right\}
\tag{2.2.19}
$$

where $D = \dfrac{d}{dz}$. If we take $a \to \infty$ in Eq. (2.2.19), we obtain the boundary condition of the first kind, *i.e.*, $\Phi = 0$ at both the boundaries. This type of boundary condition has been used by Gotoh and Yamada (1982) for a liquid layer confined between two ferromagnetic boundaries. In this case the magnetic permeability of the boundary is much higher than that of the fluid. If we consider isothermal boundary conditions for temperature and take $\chi_m \to \infty$ in (2.2.19) at both boundaries, we obtain the boundary condition of the second kind, *i.e.*, $D\Phi = 0$. Finlayson (1970) used this type of boundary condition in order to obtain exact solution to the convective instability problem of ferromagnetic fluids for free-free, isothermal boundaries. The derivation of the new, general boundary conditions in Eq. (2.2.19) is given in detail in Appendix B.

2.2.4 Electric Potential Boundary Conditions

We take the general boundary conditions for the perturbed electric potential Φ as

$$
\left.\begin{aligned}
D\Phi + \frac{a\Phi}{1+\chi_e} - T = 0 \qquad \text{at} \quad z = \frac{1}{2}, \\
D\Phi - \frac{a\Phi}{1+\chi_e} - T = 0 \qquad \text{at} \quad z = -\frac{1}{2},
\end{aligned}\right\}
\tag{2.2.20}
$$

If we take $a \to \infty$ in Eq. (2.2.20), we obtain the boundary condition of the first kind, *i.e.*, $\Phi = 0$ at both the boundaries. This type of boundary condition signifies

perfect electrically conducting surfaces and has been used by Stiles (1991). If we consider isothermal boundary conditions for temperature and take $\chi_m \to \infty$ in (2.2.20) at both boundaries, we obtain the boundary condition of the second kind, *i.e.* $D\Phi = 0$. Takashima and Ghosh (1979) used this type of boundary condition in order to obtain exact solutions to the problem of electrohydrodynamic instability in a viscoelastic fluid layer for free-free isothermal boundaries. The derivation of the new, general boundary conditions in Eq. (2.2.20) is given in detail in Appendix B.

2.3 DIMENSIONAL ANALYSIS AND SCALING

Exact solutions are rare in many branches of fluid mechanics because of nonlinearities and general boundary conditions. Hence to determine approximate solutions of the problem, numerical techniques or analytical techniques or a combination of both are used. The key to tackle modern problems is mathematical modelling. This process involves keeping certain elements, neglecting some, and approximating yet others. To accomplish this important step one needs to decide the order of magnitude, *i.e.*, smallness or largeness of the different elements of the system by comparing them with one another as well as with the basic elements of the system. This process is called non-dimensionalization or making the variables dimensionless. Expressing the equations in dimensionless form brings out the important dimensionless parameters that govern the behaviour of the system. The first method used to make the equations dimensionless is by introducing the characteristic quantities and the other is by comparing similar terms. We use the former method of introducing characteristic quantities. The scales used in the thesis for non-dimensionalization are as given below:

Quantity	Characteristic quantity used for scaling
Time	d^2/κ
Length	d
Velocity	κ/d
Temperature	ΔT
Magnetic potential	$K_1 \Delta T d \,/1+\chi_m$
Electric potential	$e E_0 \Delta T d \,/1+\chi_e$
Wavenumber	$1/d$

2.4 DIMENSIONLESS PARAMETERS

We have seen from the previous sections that the flows considered are governed by a system of nonlinear partial differential equations. Using dimensional analysis, these complicated differential equations can be reduced to simpler dimensionless form. The two general methods of obtaining dimensionless parameters are (i) inspectional analysis and (ii) dimensional analysis (Hughes and Young, 1966) of which the latter method is used in the present study in obtaining the dimensionless parameters. Study of the effect of dimensionless parameters appearing in the equations helps us understand the qualitative and quantitative nature of the flows.

The dimensionless parameters appearing in the thesis are the following:

(i) Thermal Rayleigh number – R

The thermal Rayleigh number is defined as

$$R = \frac{\alpha \rho_0 g \Delta T d^3}{\mu_1 \kappa}.$$

70

The thermal Rayleigh number plays a significant role in fluid layers where the buoyancy forces are predominant. Physically it represents the balance of energy released by the buoyancy force and the energy dissipation by viscous and thermal effects. We observe from the expression of R that the terms in the numerator drive the motion and the terms in the denominator oppose the motion. Mathematically, this number denotes the eigenvalue in the study of stability of thermal convection in the absence of surface-tension. The critical thermal Rayleigh number is the value of the eigenvalue at which the conduction state breaks down and convection sets in. Thermal Rayleigh number can also be interpreted as the non-dimensional form of the temperature by writing R as $R = \Delta T \bigg/ \left(\dfrac{\mu_1 \kappa}{\alpha \rho_o g d^3} \right)$.

(ii) Thermal Marangoni number – *Ma*

The thermal Marangoni number is defined as

$$Ma = \frac{\sigma_T \Delta T \, d}{\mu_1 \kappa},$$

where σ_T represents the rate of decrease of surface tension with increase in temperature. The thermal Marangoni number plays a significant role in fluid layers where the surface tension force is predominant. It represents the relative importance of surface tension forces to viscous forces. At a critical value of the thermal Marangoni number, Ma_{Tc}, convection sets in. The thermal Marangoni number Ma_T is an eigenvalue in the study of problems of thermal convection driven by surface tension (Pearson, 1958).

(iii) Prandtl number – *Pr*

The Prandtl number, which is a property of a particular fluid, is defined as

$$Pr = \frac{\mu_1}{\rho_o \kappa}.$$

Pr is the ratio between diffusivity of momentum and vorticity to diffusivity of heat. High *Pr* liquids are very viscous ones and low *Pr* ones have high thermal diffusivities. When *Pr* is large the velocity boundary layer is thick compared with the temperature boundary layer. The Prandtl number is very high for non-Newtonian fluids.

(iv) Nusselt number – *Nu*

The Nusselt number is defined as

$$Nu = \frac{H_T}{\left(\frac{\kappa \Delta T}{d}\right)}.$$

Nusselt number measures the ratio of the total heat transport across a horizontal plane to the heat transport by conduction alone. In general, the Nusselt number is a function of z but should be constant in a steady state. For Rayleigh numbers below the critical value, the heat transport is purely by conduction and in that case the Nusselt number turns out to be unity.

(v) Biot number – *Bi*

The Biot number is defined as

$$Bi = \frac{H_C d}{\kappa}.$$

Biot number represents the ratio of conductive resistance within the fluid layer to convective resistance at the free surface of the fluid layer. The Biot number plays a significant role in the heat transfer problems where the fluid layer has a free surface. As has been mentioned earlier, small enough values of *Bi* describe the insulating boundary condition on temperature, while large values signify an isothermal boundary condition.

(vi) Heat source (sink) parameter – N_S

$$N_S = \frac{Sd^2}{k_1 \Delta T}.$$

The parameter N_S measures the ratio of strength of the internal heat source to external heating. Positive values of N_S characterize either the effect of heat source $(S > 0)$ on the stability of a fluid which is heated from below $(\Delta T > 0)$ or the effect of heat sink $(S < 0)$ on a fluid layer which is heated from above $(\Delta T < 0)$. On the other hand, negative values of N_S describe either the effect of heat source on the stability of a fluid which is heated from above or the effect of heat sink on a fluid layer which is heated from below. In the thesis, as we are only interested in the case in which the fluid layer is heated from below, it is clear that positive values of N_S signify a heat source and the negative values a heat sink.

(vii) Buoyancy-magnetization parameter – M_1

The parameter M_1 is defined as

$$M_1 = \frac{\mu_0 K_1^2 \Delta T}{\alpha g \rho_0 (1 + \chi_m) d}.$$

M_1 is a ratio of the magnetic to gravitational forces. Large values of M_1 imply that the magnetic mechanism is very large when compared to the buoyancy effect. When both magnetic and buoyancy forces cause convection, the Rayleigh number depends on M_1 and both are coupled. When the buoyancy force has a negligible influence (that is, for very large M_1), we define another parameter, referred to as the *magnetic Rayleigh number*, $R_M = R M_1 = \mu_0 (K_1 \Delta T d)^2 / (1 + \chi_m) \mu_1 \kappa$. The latter definition is also applicable to a thin layer of ferromagnetic fluid where the surface tension force is important. When $M_1 = 0$, we obtain the nonmagnetic classical Rayleigh-Bénard problem for buoyancy induced convection (Finlayson, 1970).

(viii) Non-buoyancy-magnetization parameter – M_3

We define M_3 as

$$M_3 = \frac{\left(1 + \dfrac{M_o}{H_o}\right)}{(1 + \chi_m)}.$$

The parameter M_3 measures the departure of linearity in the magnetic equation of state. $M_3 = 1$ corresponds to linear magnetization. As the equation of state becomes more nonlinear (*i.e.* M_3 large), the fluid layer is destabilized slightly. When $M_3 \rightarrow \infty$, which means very strong nonlinearity of magnetization of the fluid, the entire problem reduces to the classical Rayleigh-Bénard problem (Finlayson, 1970).

(ix) Electric Rayleigh number – R_E

The electric Rayleigh number R_E is defined as

$$R_E = \frac{\varepsilon_o (E_o e \beta d^2)^2}{\mu_1 \kappa (1 + \chi_e)}.$$

The electric Rayleigh number R_E (also called *Roberts number*) is the ratio of electric forces to viscous forces. The electric Rayleigh number plays a significant role in not-so-thin fluid layers where the buoyancy and electric forces are equally predominant. It has been shown (Roberts, 1969) that in the absence of gravity, electrically induced convection commences when R_E reaches a critical value. Since R_E depends on the square of the temperature gradient, this result is independent of whether the upper or lower plate is warmer (Stiles *et al.*, 1993).

(x) Effective viscosity parameter – Γ

The effective viscosity parameter Γ is defined as

$$\Gamma = \delta (\Delta T)^2.$$

The quantity δ appearing in the above expression is defined in the context of Chapter IV. The dominance of magnetic/electric field strength dependency over temperature dependency of the effective viscosity is signified by $\Gamma < 0$, while $\Gamma > 0$ characterizes dominance of temperature dependency.

(xi) Magnetic Marangoni number – Ma_H

The dimensionless number Ma_H is defined as

$$Ma_H = \frac{K_1 \Delta T \sigma_H d}{\mu_1 \kappa (1 + \chi_m)}.$$

The magnetic Marangoni number, Ma_H, is the ratio of magnetorheological factors affecting motion to forces opposing motion. In general, Ma_H has a negligible influence on the stability or instability of a ferromagnetic fluid layer and hence we neglect the same in the present study.

(xii) Electric Marangoni number – Ma_E

We define the electric Marangoni number Ma_E as

$$Ma_E = \frac{e E_0 \Delta T \sigma_E d}{\mu_1 \kappa (1 + \chi_e)}.$$

The dimensionless number Ma_E is the ratio of electrorheological factors affecting motion to forces opposing motion. In general, Ma_E has a negligible influence on the stability or instability of a dielectric fluid layer and hence we neglect the same in the present study.

(xiii) Conduction-radiation parameter – χ

We define χ as

$$\chi = \frac{4\pi Q}{3\kappa K_a s_r},$$

where $Q = 4 S_c T_a^3 / \pi$. The conduction-radiation parameter χ is indicative of the temperature in the equilibrium state. The results pertaining to non-radiating fluids can be obtained in the limit of $\chi \to 0$ (Khosla and Murgai, 1963).

(xiv) Absorptivity parameter – τ

The absorptivity parameter τ is defined as

$$\tau = K_a d \sqrt{3(1+\chi)}.$$

The dimensionless parameter τ is the characteristic of absorption coefficient and distance between the horizontal planes. The results relating to non-radiating fluids can be obtained in the limit of $\tau \to 0$ (Khosla and Murgai, 1963).

(xv) Stress-relaxation parameter – Γ_V

The stress-relaxation parameter Γ_V is defined as

$$\Gamma_V = \frac{\lambda_1 \kappa}{d^2}.$$

The parameter Γ_V indicates the non-dimensional relaxation time. It is an elastic parameter which may be interpreted as a Fourier number in terms of λ_1 (Vest and Arpaci, 1969).

(xvi) Viscoelastic parameter – η

We define η as

$$\eta = \frac{\lambda_2}{\lambda_1},$$

which is the ratio of the retardation time to the relaxation time (Sokolov and Tanner, 1972). The results of the Newtonian viscous fluid can be obtained by taking both Γ_V and η to be zero. Alternatively, we could take $\eta = 1$.

In Table 2.3, we have documented the actual values of the important physical quantities pertaining to four different ferromagnetic liquids.

Table 2.3. Physical data of the ferro liquids EMG 905, EMG 901, 90 G and W-40. The value of magnetic permeability of vacuum, μ_o, is $4\pi \times 10^{-7}$ W / A .m.

Properties	Ferro liquid type			
	Hydrocarbon-based			**Water-based**
	EMG 905 (Stiles and Kagan, 1990)	**EMG 901** (Weilepp and Brand, 1996)	**90 G** (Tangthieng et al., 1999)	**W-40** (Snyder et al., 2003)
Density ρ_o (kg / m^3)	1.22×10^3	1.53×10^3	0.972×10^3	1.4×10^3
Thermal expansion coefficient α (K^{-1})	8.6×10^{-4}	6×10^{-4}	9×10^{-4}	0.026×10^{-4}
Heat capacity C_{VH} (J / kg . K)	2.09×10^3	1.45×10^3	1.764×10^3	3×10^3
Thermal conductivity k_1 (W / m . K)	2.2×10^{-1}	1.85×10^{-1}	1.5×10^{-1}	27×10^{-1}
Thermal diffusivity κ (m^2/ s)	8.6×10^{-8}	8.2×10^{-8}	8.7×10^{-8}	64.3×10^{-8}
Dynamic viscosity μ_1 (kg . m^{-1}. s^{-1})	8.25×10^{-3}	9.95×10^{-3}	15×10^{-3}	3.99×10^{-2}
Pyromagnetic coefficient K_1 (A / m . K)	27.3	~ 30	30	240
Prandtl number Pr	78.4	79.3	176.4	44.3
Rayleigh number R ($g = 9.8$ m/ s^2)	14.492 (1.479×10^{-6}) K^{-1} mm^{-3} $\Delta T\, d^3$	11.026 (1.126×10^{-6}) K^{-1} mm^{-3} $\Delta T\, d^3$	6.569 (6.666×10^{-7}) K^{-1} mm^{-3} $\Delta T\, d^3$	0.001 (1.419×10^{-10}) K^{-1} mm^{-3} $\Delta T\, d^3$
Buoyancy - magnetization parameter M_1	0.046 K^{-1} mm $\Delta T\, d^{-1}$	0.063 K^{-1} mm $\Delta T\, d^{-1}$	0.066 K^{-1} mm $\Delta T\, d^{-1}$	1014.04 K^{-1} mm $\Delta T\, d^{-1}$
Magnetic Rayleigh number R_M	0.659 K^{-2}mm$^{-2}\Delta T^2 d^2$	0.693 K^{-2}mm$^{-2}\Delta T^2 d^2$	0.431 K^{-2}mm$^{-2}\Delta T^2 d^2$	1.409 K^{-2}mm$^{-2}\Delta T^2 d^2$

Note: 1. Values of R in the parentheses are calculated for $g = 10^{-6}$ m/ s^2.

2. At the present time data is not available for these liquids on their viscosity variation with temperature and magnetic field.

CHAPTER III

LINEAR AND NONLINEAR FERRO- AND ELECTRO-CONVECTION

3.1 INTRODUCTION

The occurrence of cellular convection in nonmagnetic liquid layers heated from below is generally ascribed to two different mechanisms: the buoyancy and surface tension mechanisms. The buoyancy driven convection is popularly known as "Rayleigh-Bénard Convection (*RBC*)" while the surface-tension driven convection is referred to as "Marangoni Convection (*MC*)". In the case of magnetic liquids, thermally and magnetically induced gradients of magnetization also contribute to the convective motion besides the two aforesaid candidates pertaining to nonmagnetic liquids. The problem of convection in ferromagnetic fluids is of relevance in many fields of applications as discussed in Chapter I. The Rayleigh-Bénard convection in ferromagnetic fluids, using the classical linear stability theory, has been exhaustively studied (Finlayson, 1970; Sekhar, 1990; Siddheshwar, 1993; 1995; 1999; 2002a; Siddheshwar and Abraham, 1998; 2003; Abraham, 2002a; 2002b and references therein).

The study of finite amplitude convection (Veronis, 1966), using a truncated Fourier representation, has gained momentum in recent years owing to its simplicity and nonlinear complexity of the solution. It is found handy by the researchers at least for four reasons: It can be used (i) to determine the plan-forms of cellular motion that can occur in the fluid, (ii) to explicate the convective processes of many non-isothermal situations of practical interest, (iii) to quantify the heat transfer and (iv) to advance a bit closer to the challenging problem of the onset of chaotic motion. More recently, the use of a minimal representation of Fourier series in finite amplitude analysis finds its mention in the study of chaotic thermal convection in viscoelastic fluids of different genre (Khayat, 1994; 1995a;

1995b; Siddheshwar and Srikrishna, 2002; Ramadan *et al*., 2003 and references therein). Khayat (1996) examined, using a truncated Fourier representation of the flow and temperature fields, the influence of weak shear thinning on the onset of chaos in thermal convection for a Carreau-Bird fluid. He found that the critical Rayleigh number at the onset of thermal convection remains the same as for a Newtonian fluid but the shear thinning dramatically alters the amplitude and nature of the convective cellular structure.

The reported works on nonlinear convection in ferromagnetic fluids are very scant owing to the involvedness of both the governing equations and the solution procedure. Blennerhassett *et al*. (1991) examined steady convective heat transfer in strongly magnetized ferro fluids heated from above and predicted that the Nusselt numbers for a given supercritical temperature gradient are significantly higher than when the ferrofluid is heated from below. Stiles *et al*. (1992) presented a weakly nonlinear, steady thermoconvective stability in a thin layer of ferrofluid heated from above and confined between two rigid horizontal plates. They found that the magnitude of the critical horizontal wavenumber is substantially higher than that when the ferro fluid is heated from below.

Qin and Kaloni (1994) developed a nonlinear energy stability theory to study the buoyancy-surface tension effects in a ferromagnetic fluid layer heated from below. Assuming the free surface to be flat and non-deformable, they pointed out the existence of sub-critical instabilities. Russell *et al*. (1995) studied the long wavenumber steady convection in a strongly magnetized ferrofluid which is heated from above and deduced that the heat transfer depends nonlinearly on the temperature difference. Elhefnawy (1997) considered the heat and mass transfer in ferrofluids using a nonlinear approach. Kaloni and Lou (2004) carried out a weakly nonlinear analysis to investigate the thermal instability of a rotating ferromagnetic fluid. Making use of the multi-scale perturbation method, they found that the ratio of heat transfer by convection to that by conduction decreases as magnetic field increases. Most recently, Siddheshwar (2005b) has made a spectral study of

nonlinear ferroconvection wherein cross interaction of different modes and effect of magnetization parameters on nonlinear ferroconvection is discussed.

We note that the study of finite amplitude Rayleigh-Bénard convection in a ferromagnetic fluid by means of a minimal Fourier series representation does not seem to have been undertaken. Accordingly, we study in this chapter a weakly nonlinear analysis of thermal convection in a ferromagnetic fluid permeated by a vertical uniform magnetic field. An analogy for nonlinear *RBC* problem between ferromagnetic and dielectric liquids is also presented.

3.2 MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of a Boussinesquian, electrically non-conducting ferromagnetic fluid of depth '*d*' that supports a temperature gradient ΔT and a *dc* magnetic field $\vec{H}_{\text{o}}$ in the vertical direction. The upper and lower boundaries are maintained at constant temperatures T_{o} and $T_{\text{o}} + \Delta T$ $(\Delta T > 0)$ respectively. The schematic of the same is shown in Figure 3.1. For mathematical tractability we confine ourselves to two-dimensional rolls so that all physical quantities are independent of y, a horizontal co-ordinate. Further, the boundaries are assumed to be free and perfect conductors of heat. In this chapter we assume the effective viscosity μ to be constant and the reference viscosity μ_1 will be used to denote the constant viscosity.

The governing equations describing the Rayleigh-Bénard instability situation in a constant viscosity ferromagnetic fluid in the notation of Chapter II are

$$\nabla . \vec{q} = 0, \tag{3.2.1}$$

$$\rho_{\text{o}} \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} . \nabla \right) \vec{q} \right] = -\nabla p - \rho \, g \, \hat{k} + \mu_{\text{o}} \, (\vec{M} . \nabla) \vec{H} + \mu_1 \nabla^2 \vec{q}, \tag{3.2.2}$$

$$\rho_o C_{VH} \left[\frac{\partial T}{\partial t} + \left(\overrightarrow{q} \cdot \nabla \right) T \right] = k_1 \nabla^2 T . \tag{3.2.3}$$

The effects of heat source and radiation are assumed to be negligible in writing the energy equation (3.2.3).

The density equation of state is

$$\rho = \rho_o \left[1 - \alpha \left(T - T_o \right) \right] . \tag{3.2.4}$$

It should be observed that the nonlinear terms $(\overrightarrow{q} \cdot \nabla) \overrightarrow{q}$ and $\overrightarrow{q} \cdot \nabla T$ plus the nonlinear terms arising in the term $(\overrightarrow{M} \cdot \nabla) \overrightarrow{H}$ are to be retained in this analysis and are the overriding objects of interest in so far as the finite amplitude theory is concerned.

Maxwell's equations, simplified for a non-conducting ferromagnetic fluid with no displacement current, become

$$\nabla \cdot \overrightarrow{B} = 0, \quad \nabla \times \overrightarrow{H} = \overrightarrow{0}, \tag{3.2.5}$$

$$\overrightarrow{B} = \mu_o \left(\overrightarrow{M} + \overrightarrow{H} \right) . \tag{3.2.6}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as on the temperature. Thus the magnetic equation of state is

$$M = M_o + \chi_m \left(H - H_o \right) - K_1 \left(T - T_o \right) . \tag{3.2.7}$$

The magnetic boundary conditions specify that the normal component of the magnetic induction and tangential component of the magnetic field are continuous across the boundary. Taking the components of temperature, density, magnetization

and magnetic field in the basic state to be $T_b(z)$, $\rho_b(z)$, $[0, M_b(z)]$ and $[0, H_b(z)]$, we obtain the quiescent state solution

$$
\left.
\begin{aligned}
&\vec{q}_b = (0, 0), \quad T_b = T_o - \frac{\Delta T}{d} z, \quad \rho_b = \rho_o \left[1 + \alpha \frac{\Delta T}{d} z \right], \\
&\vec{H}_b = \left[H_o - \frac{K_1 \Delta T}{(1+\chi_m)d} z \right] \hat{k}, \quad \vec{M}_b = \left[M_o + \frac{K_1 \Delta T}{(1+\chi_m)d} z \right] \hat{k} .
\end{aligned}
\right\}
\tag{3.2.8}
$$

On this basic state we superpose finite amplitude perturbations of the form

$$
\left.
\begin{aligned}
&\vec{q} = \vec{q}_b + (u', w'), \quad T = T_b + T', \quad p = p_b + p', \quad \rho = \rho_b + \rho', \\
&\vec{M} = \vec{M}_b + (M_1', M_3'), \quad \vec{H} = \vec{H}_b + (H_1', H_3'),
\end{aligned}
\right\}
\tag{3.2.9}
$$

where the prime indicates perturbation. Using Eq. (3.2.9) in Eq. (3.2.7), we obtain

$$
\left.
\begin{aligned}
&M_1' + H_1' = \left[1 + \frac{M_o}{H_o} - \frac{K_1 T'}{H_o} \right] H_1', \\
&M_3' + H_3' = (1+\chi_m) H_3' - K_1 T',
\end{aligned}
\right\}
\tag{3.2.10}
$$

where we have assumed $K_1 \Delta T << (1+\chi_m) H_o$. The second of Eq. (3.2.5) implies

one can write $\vec{H}' = \nabla \Phi'$, where Φ' is the perturbed magnetic scalar potential.

Since we consider only two-dimensional disturbances, we introduce the stream function ψ'

$$
u' = \frac{\partial \psi'}{\partial z}, \quad w' = -\frac{\partial \psi'}{\partial x},
\tag{3.2.11}
$$

which satisfy the continuity equation (3.2.1) in the perturbed state. Introducing the magnetic potential Φ', eliminating the pressure p in Eq. (3.2.2), incorporating the

83

quiescent state solution and non-dimensionalizing the resulting equation as well as Eq. (3.2.3) using the following definition

$$\left(x^*, z^*\right) = \left(\frac{x}{d}, \frac{z}{d}\right), \quad t^* = \frac{\kappa}{d^2}t, \quad \psi^* = \frac{\psi'}{\kappa}, \quad T^* = \frac{T'}{\Delta T}, \quad \Phi^* = \frac{(1+\chi_m)}{K_1 \Delta T d}\Phi',$$

(3.2.12)

we obtain the dimensionless form of the vorticity and heat transport equations as

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2 \psi\right) = -R(1+M_1)\frac{\partial T}{\partial x} + RM_1\frac{\partial^2 \Phi}{\partial x \partial z} + \nabla^4 \psi$$

$$+ RM_1 J\left(T, \frac{\partial \Phi}{\partial z}\right) + \frac{1}{Pr} J\left(\psi, \nabla^2 \psi\right),$$

(3.2.13)

$$\frac{\partial T}{\partial t} = -\frac{\partial \psi}{\partial x} + \nabla^2 T + J\left(\psi, T\right),$$

(3.2.14)

where the Jacobian, J, is defined as

$$J(f, h) = \frac{\partial f}{\partial x}\frac{\partial h}{\partial z} - \frac{\partial f}{\partial z}\frac{\partial h}{\partial x}$$

and the Laplacian operator for the two-dimensional case takes the form $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial z^2)$. In the above equations, the asterisks have been dropped for simplicity and we continue doing so in the remaining part of the chapter. Using Eq. (3.2.10) into the first of Eq. (3.2.5) and non-dimensionalizing the resulting equation, we obtain

$$M_3\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial T}{\partial z} = 0.$$

(3.2.15)

The non-dimensional parameters appearing in Eqs. (3.2.13) – (3.2.15) are as defined in Chapter II. Equations (3.2.13) – (3.2.15) are solved using the boundary conditions

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = \frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = 0, 1.$$

(3.2.16)

84

From Eq. (3.2.16) it is clear that the boundaries are taken to be flat, stress-free and perfect conductors of heat. We also note that the boundary condition for the magnetic potential Φ, which allows periodic solutions in the vertical direction, is tantamount to assuming that the magnetic susceptibility χ_m with respect to the perturbed field is large at both the boundaries (Finlayson, 1970). In the next section, we discuss the linear stability analysis, which is of great utility in the local nonlinear stability analysis.

3.3 LINEAR STABILITY THEORY

In order to study the linear theory we consider the linear version of Eqs. (3.2.13) – (3.2.15) and assume the solutions to be periodic waves of the form (Chandrasekhar, 1961)

$$
\begin{bmatrix} \psi \\ T \\ \Phi \end{bmatrix} = e^{\sigma t} \begin{bmatrix} \psi_o & \sin \pi a x & \sin \pi z \\ \theta_o & \cos \pi a x & \sin \pi z \\ \left(\dfrac{\Phi_o}{\pi} \right) & \cos \pi a x & \cos \pi z \end{bmatrix},
\tag{3.3.1}
$$

which satisfy the boundary conditions in Eq. (3.2.16). In Eq. (3.3.1), $\sigma = \sigma_r + i\omega$, in which σ_r is the growth rate and ω is the frequency of oscillations, πa is the horizontal wavenumber and π is the vertical wavenumber. ψ_o, θ_o and Φ_o are, respectively, amplitudes of the stream function, temperature and the magnetic potential. Substituting Eq. (3.3.1) into the linear version of Eqs. (3.2.13) – (3.2.15), we obtain

$$
\left(\frac{\sigma}{Pr} + \eta_1^2 \right) \eta_1^2 \psi_o + R \left(1 + M_1 \right) \pi a \, \theta_o + R M_1 \pi a \, \Phi_o = 0,
\tag{3.3.2}
$$

$$
\pi a \, \psi_o + \left(\frac{\sigma}{Pr} + \eta_1^2 \right) \theta_o = 0,
\tag{3.3.3}
$$

$$
\theta_o + \left(1 + M_3 a^2 \right) \Phi_o = 0,
\tag{3.3.4}
$$

85

where $\eta_1^2 = \pi^2 (1 + a^2)$. For a non-trivial solution for ψ_o, θ_o and Φ_o, we require

$$R = \frac{\left(\sigma + \eta_1^2\right)\left(\frac{\sigma}{Pr} + \eta_1^2\right)\eta_1^2\left(1 + M_3 a^2\right)}{\pi^2 a^2 \left[1 + M_3 a^2 + M_1 M_3 a^2\right]}. \tag{3.3.5}$$

The onset of convection in ferromagnetic liquids can occur in one of the following ways:

    (i) stationary convection (steady convection),

    (ii) oscillatory convection (unsteady convection).

The thermal Rayleigh number R is the eigenvalue of the problem that throws light on the stability or otherwise of the system. The critical value of R, *i.e.*, R_c signifies the onset of convection via one of the above modes. R_c of stationary is different from R_c of oscillatory. If R_c of stationary convection is less than that of oscillatory convection, then we say the "*Principle of Exchange of Stabilities (PES)*" is valid. We now move over to the discussion on the stationary instability followed by that on the validity or otherwise of the *PES*.

3.3.1 Marginal State

If σ is real, then the marginal instability occurs when $\sigma = 0$. This gives the stationary thermal Rayleigh number (Finlayson, 1970)

$$R^S = \frac{\left(1 + M_3 a^2\right)\eta_1^6}{\pi^2 a^2 \left(1 + M_3 a^2 + M_1 M_3 a^2\right)}. \tag{3.3.6}$$

The critical wavenumber a_c satisfies the equation

$$\left[(1 + M_1) M_3^2 \left(a_c^2\right)^2 + 2 M_3 a_c^2 + 1\right]\left[2a_c^2 - 1\right]$$

$$+ M_1 M_3 a_c^2 \left[a_c^2 - 2\right] = 0. \tag{3.3.7}$$

Equation (3.3.7) clearly shows that a_c depends on the magnetic parameters M_1 and M_3. When $M_1 = 0$, we obtain the results of the classical Rayleigh-Bénard instability with $a_c^2 = 0.5$ and $R_c^s = 657.5$ (Chandrasekhar, 1961).

3.3.2 Oscillatory Motions

Taking $\sigma = i\omega$ (ω being the frequency of oscillations) in Eq. (3.3.5) and separating the real and imaginary parts, we obtain the oscillatory thermal Rayleigh number

$$R^O = \frac{\left[\eta_1^6\left(1+M_3\,a^2\right) - \dfrac{\omega^2\eta_1^2}{Pr}\right]}{\pi^2\,a^2\left(1+M_3\,a^2 + M_1\,M_3\,a^2\right)} + i\,\omega N, \qquad (3.3.8)$$

where $N = \dfrac{\eta_1^4\left(1+\dfrac{1}{Pr}\right)\left(1+M_3\,a^2\right)}{\pi^2\,a^2\left(1+M_3\,a^2 + M_1\,M_3\,a^2\right)}.$

Since R^O is a real quantity, the imaginary part of Eq. (3.3.8) has to vanish. This gives us two possibilities:

   (i) $\omega \neq 0,\ N=0$ (oscillatory instability),

   (ii) $\omega = 0,\ N \neq 0$ (stationary instability).

Taking $N=0$, we get $\eta_1^4\left(1+\dfrac{1}{Pr}\right)\left(1+M_3\,a^2\right) = 0$, which is independent of ω. In problems wherein oscillatory convection is preferred to stationary, the condition $N=0$ leads to an expression for ω^2 that is in turn substituted in the real part of the expression for R^O, thereby yielding the oscillatory thermal Rayleigh number. In view of the fact that N is independent of ω, we infer that oscillatory convection is not possible in the present problem. In support of the impossibility of the occurrence of oscillatory instability in Rayleigh-Bénard ferroconvection, we can

also seek the assistance of the Einstein formula for the effective viscosity, *viz*.,
$\mu = \mu_c\left(1 + (5/2)\,\alpha_s\,\varphi_s\right)$, where μ_c is the viscosity of the carrier liquid, φ_s is the
solid volume fraction and α_s is the shape factor. For a sphere α_s takes a value of 1.
From the above formula it is obvious that the viscosity of the ferromagnetic liquid
is much higher than the viscosity of the carrier liquid and thus the former has to
have a large *Pr* value in the presence of a magnetic field. Thus oscillatory
instability can justifiably be discounted. This essentially means that the *PES* holds
good for the problem at hand.

3.3.3 Parametric Perturbation Method

Our main objective in this section is to decipher analytically the effect of
magnetic parameters M_1 and M_3 on the monotonicity of the thermal Rayleigh
number R using the concept of self-adjoint operator. In view of the fact that the
PES is valid, we consider steady motions.

We assume the steady solution to the linear version of Eqs. (3.2.13) – (3.2.15)
in the form

$$\begin{bmatrix} \psi \\ T \\ \Phi \end{bmatrix} = \begin{bmatrix} \psi(z) & \sin \pi a x \\ \theta(z) & \cos \pi a x \\ \Phi(z) & \cos \pi a x \end{bmatrix}. \tag{3.3.9}$$

Substituting Eq. (3.3.9) into the linear form of Eqs. (3.2.13) – (3.2.15), we obtain

$$\left(D^2 - \pi^2 a^2\right)^2 \psi + \pi a\, R\left(1 + M_1\right)T - R M_1\, \pi a\, D\Phi = 0, \tag{3.3.10}$$

$$\pi a \psi - \left(D^2 - \pi^2 a^2\right)T = 0, \tag{3.3.11}$$

$$DT - \left(D^2 - M_3\, \pi^2 a^2\right)\Phi = 0, \tag{3.3.12}$$

where $D = \dfrac{d}{dz}$. Eliminating Φ between Eqs. (3.3.10) and (3.3.12), we obtain

$$\left(D^2 - \pi^2 a^2\right)^2 \left(D^2 - M_3 \pi^2 a^2\right)\psi + \pi a R\left\{D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right\} T = 0.$$

(3.3.13)

Eq. (3.3.11) can be rewritten as

$$\pi a R\left[D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right]\psi$$
$$- R\left(D^2 - \pi^2 a^2\right)\left[D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right] T = 0.$$

(3.3.14)

We now define a symmetric operator L as follows:

$$L = \begin{bmatrix} \left(D^2 - \pi^2 a^2\right)^2 \left[D^2 - M_3 \pi^2 a^2\right] & \pi a R\left\{D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right\} \\ \pi a R\left\{D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right\} & -R\left(D^2 - \pi^2 a^2\right)\left\{D^2 - \left(1+M_1\right)M_3 \pi^2 a^2\right\} \end{bmatrix}.$$

(3.3.15)

We next define a vector $\vec{V}$ such that $\vec{V} = \begin{bmatrix} \psi \\ T \end{bmatrix}$. Eqs. (3.3.13) and (3.3.14) can now be written as

$$L\vec{V} = \vec{0}.$$

(3.3.16)

We define the inner product between two vectors $\vec{a}$ and $\vec{b}$ such that

$$< \vec{a}, \vec{b} > = \int_V \vec{a}^{*Tr} \cdot \vec{b} \, dV,$$

(3.3.17)

where V represents the domain of the integral operator in which $\vec{a}$ and $\vec{b}$ are defined, the asterisk represents the complex conjugate and Tr represents the transpose. As the operator L and the boundary conditions in Eq. (3.2.16) are symmetric, one may easily prove that L is self-adjoint and so are the boundary conditions in Eq. (3.2.16).

To seek information on the variation of R with respect to M_1, we differentiate Eq. (3.3.16) with respect to M_1 and obtain

$$L \vec{V}_{d1} = \vec{h}_{d1}, \tag{3.3.18}$$

where

$$\vec{h}_{d1} = \begin{bmatrix} \left\{ \pi^3 a^3 R M_3 - \pi a R_{d1} \left(D^2 - (1+M_1) M_3 \pi^2 a^2 \right) \right\} T \\ \\ 0 \end{bmatrix}$$

and the subscript '$d1$' represents the derivative with respect to M_1. Applying a Fredholm alternative condition to Eq. (3.3.18), we obtain

$$\pi a R_{d1} \left\{ \int_V (D\psi^* DT) \, dV + (1+M_1) M_3 \pi^2 a^2 \int_V (\psi^* T) \, dV \right\}$$
$$= -\pi^3 a^3 R M_3 \int_V (\psi^* T) \, dV . \tag{3.3.19}$$

From the above equation it is clear that $R_{d1} < 0$ if R is positive. This means that R is a decreasing function of M_1 and hence the effect of M_1 is to destabilize the system. In order to extract information on the dependence of R on the parameter M_3, we differentiate Eq. (3.3.16) with respect to M_3 and obtain

$$L \vec{V}_{d3} = \vec{h}_{d3}, \tag{3.3.20}$$

where

$$\vec{h}_{d3} = \begin{bmatrix} \left\{ \pi^2 a^2 \left(D^2 - \pi^2 a^2 \right)^2 \right\} \psi \\ + \left\{ \pi^3 a^3 R_{d3} (1+M_1) M_3 + \pi^3 a^3 R (1+M_1) - \pi \alpha R_{d3} D^2 \right\} T \\ \\ 0 \end{bmatrix}$$

and the subscript '$d3$' represents differentiation with respect to M_3. On applying the solvability condition, we obtain

$$\pi a R_{d3} \left\{ (1+M_1) M_3 \pi^2 a^2 \int_V (\psi^* T)\, dV + \int_V (D\psi^* DT)\, dV \right\}$$

$$= -\pi^3 a^3 R(1+M_1) \int_V (\psi^* T)\, dV - \pi^2 a^2 \int_V \left| D^2 \psi \right|^2 dV \qquad (3.3.21)$$

$$- \pi^6 a^6 \int_V |\psi|^2\, dV - 2\pi^4 a^4 \int_V |D\psi|^2\, dV < 0 .$$

This yields the condition that R decreases if M_3 increases.

3.4 NONLINEAR THEORY

The linear theory discussed in a previous section reveals that the stationary mode of instability is preferred to the oscillatory one. In deed, the linear theory predicts only the condition for the onset of convection and is silent about the heat transfer. We now embark on a weakly nonlinear analysis by means of a truncated representation of Fourier series for velocity, temperature and magnetic fields to find the effect of various parameters on finite amplitude convection and to know the amount of heat transfer. We note that the results obtained from such an analysis can serve as starting values while solving a more general nonlinear convection problem.

The first effect of nonlinearity is to distort the temperature field through the interaction of ψ and T, and Φ and T. The distortion of temperature field will correspond to a change in the horizontal mean, *i.e.*, a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal double Fourier series which describes the finite amplitude convection in a ferromagnetic fluid is

$$\begin{bmatrix} \psi \\ T \\ \Phi \end{bmatrix} = \begin{bmatrix} A(t) & \sin \pi a x & \sin \pi z \\ B(t) & \cos \pi a x & \sin \pi z \\ \dfrac{1}{\pi} E(t) & \cos \pi a x & \cos \pi z \end{bmatrix} + \begin{bmatrix} 0 \\ C(t) \\ 0 \end{bmatrix} \sin(2\pi z), \qquad (3.4.1)$$

where the amplitudes *A, B, C* and *E* are to be determined from the dynamics of the system. Substituting Eq. (3.4.1) into Eqs. (3.2.13) – (3.2.15), equating the coefficients of like terms, we obtain the following nonlinear autonomous system (generalized Lorenz model, Sparrow, 1981) of differential equations

$$\dot{A} = -Pr\,\eta_1^2 A - \frac{R\pi a\,Pr\,(1+M_1)}{\eta_1^2}B - \frac{R\pi a\,PrM_1}{\eta_1^2}E - \frac{RM_1\pi^2 a\,Pr}{\eta_1^2}CE$$

(3.4.2)

$$\dot{B} = -\pi a\,A - \eta_1^2 B - \pi^2 a\,AC,$$

(3.4.3)

$$\dot{C} = \frac{\pi^2 a}{2}AB - 4\pi^2 C,$$

(3.4.4)

$$0 = B + (1+M_3 a^2)E,$$

(3.4.5)

where the over dot denotes time derivative. It is important to observe that the nonlinearities in the equations (3.4.2) – (3.4.5) stem from the convective terms in the energy equation (3.2.3) as in the Lorenz system (Lorenz, 1963) and from the Maxwell's stress term in Eq. (3.2.2). This is in contrast to the case of a viscoelastic fluid, where the type of nonlinearity is the same as in the Lorenz system (Siddheshwar and Srikrishna, 2002; Ramadan *et al.*, 2003).

It is advantageous to eliminate the variable *E* between Eqs. (3.4.2) and (3.4.5) noting that Eq. (3.4.5) does not have a time derivative term on the left side. This process reduces the system of equations (3.4.2) – (3.4.5) to

$$\dot{A} = -Pr\,\eta_1^2 A - \frac{R\pi a\,Pr\left(1+M_3 a^2 + M_1 M_3 a^2\right)}{\left(1+M_3 a^2\right)\eta_1^2}B + \frac{RM_1\pi^2 a\,Pr}{\left(1+M_3 a^2\right)\eta_1^2}BC,$$

(3.4.6)

$$\dot{B} = -\pi a\,A - \eta_1^2 B - \pi^2 a\,AC,$$

(3.4.7)

$$\dot{C} = \frac{\pi^2 a}{2}AB - 4\pi^2 C.$$

(3.4.8)

The third order Lorenz system described by Eqs. (3.4.6) – (3.4.8) is uniformly bounded in time and possesses many properties of the full problem. Moreover, the phase-space volume contracts at a uniform rate

$$\frac{\partial \dot{A}}{\partial A} + \frac{\partial \dot{B}}{\partial B} + \frac{\partial \dot{C}}{\partial C} = -\left[(\mathrm{Pr}+1)\eta_1^2 + 4\pi^2 \right], \tag{3.4.9}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase-space; in particular, they may be attracted to a fixed point, a limit cycle or perhaps, a strange attractor. Before solving the nonlinear system of equations, we consider the linear autonomous system and analyze the critical points. The nature of the critical points obtained from the linear system discloses information about the trajectories in the phase plane. The nature of these trajectories is retained by the nonlinear system but with distortions dictated by the nonlinear terms.

3.4.1 Linear Autonomous System

The linearized autonomous system is

$$\dot{A} = -Pr\,\eta_1^2 A - \frac{R\,\pi a\,Pr\left(1 + M_3\,a^2 + M_1 M_3 a^2\right)}{\left(1 + M_3\,a^2\right)\eta_1^2}\,B, \tag{3.4.10}$$

$$\dot{B} = -\pi a\,A - \eta_1^2 B, \tag{3.4.11}$$

$$\dot{C} = -4\pi^2 C. \tag{3.4.12}$$

To explore the critical points of the above linear autonomous system of equations, we follow Simmons (1974) and write the auxiliary equation

$$\begin{vmatrix} -Pr\,\eta_1^2 - \xi & \dfrac{-R\,\pi a Pr\left(1+M_3\,a^2+M_1 M_3 a^2\right)}{\left(1+M_3\,a^2\right)\eta_1^2} & 0 \\[3ex] -\pi a & -\eta_1^2 - \xi & 0 \\[2ex] 0 & 0 & -4\pi^2 - \xi \end{vmatrix} = 0.$$

On expansion, we obtain

$$\xi^2 + (Pr+1)\eta_1^2\,\xi + \left[Pr\,\eta_1^4 - \frac{R\,\pi^2 a^2 Pr\left(1+M_3 a^2+M_1 M_3 a^2\right)}{\left(1+M_3\,a^2\right)\eta_1^2} \right] = 0. \tag{3.4.13}$$

Let ξ_1 and ξ_2 be the roots of Eq. (3.4.13). We now discuss three cases according to the nature of these roots.

Case (i) ξ_1 and ξ_2 are real and equal. In this case, we have

$$(Pr+1)^2\,\eta_1^4 = 4\left[Pr\,\eta_1^4 - \frac{R\,\pi^2 a^2 Pr\left(1+M_3\,a^2+M_1 M_3 a^2\right)}{\left(1+M_3\,a^2\right)\eta_1^2} \right].$$

On simplification, the above yields an expression for R

$$R = \frac{\left[4\,Pr - (Pr+1)^2\right]\left(1+M_3 a^2\right)\eta_1^6}{4\,\pi^2 a^2\,Pr\left(1+M_3 a^2+M_1 M_3 a^2\right)}. \tag{3.4.14}$$

For the above value of R, the critical point is a *node* and the system becomes stable as the paths approach and enter the critical point.

Case (ii) ξ_1 and ξ_2 are real and distinct. In this case, we have the condition

$$R > \frac{\left[4\,Pr - (Pr+1)^2\right]\left(1+M_3\,a^2\right)\eta_1^6}{4\pi^2 a^2\,Pr\left(1+M_3\,a^2+M_1 M_3\,a^2\right)}. \tag{3.4.15}$$

For this range of values of R, the critical point is a *saddle point* and the system is unstable as paths never approach the critical points.

Case (iii) ξ_1 and ξ_2 are imaginary. The requirement in this case takes the form

$$R < \frac{\left[4\,Pr - (Pr+1)^2\right]\left(1+M_3\,a^2\right)\eta_1^6}{4\pi^2 a^2\,Pr\left(1+M_3\,a^2 + M_1 M_3 a^2\right)}. \tag{3.4.16}$$

For this range of values of R, the critical point is a *spiral* and the system is asymptotically stable if paths approach the critical point as $t \to -\infty$ and the system becomes unstable as $t \to \infty$ if the paths spiral out.

Having made a qualitative analysis of the linear autonomous system, we note that the nonlinear system of autonomous differential equations (3.4.6) – (3.4.8) is not amenable to analytical treatment for the general time-dependent variables and we need to solve it by means of a numerical method. However, in the case of steady motions, these equations can be solved in closed form. Such solutions prove very useful because they may show that a finite amplitude steady solution to the system is possible for sub-critical values of the thermal Rayleigh number and that the minimum value of R for which finite amplitude steady solution is possible lies below the critical values corresponding to a steady infinitesimal disturbance or an overstable infinitesimal disturbance. Thus in the case of steady motions, Eqs. (3.4.6) – (3.4.8) take the form

$$\left(1+M_3\,a^2\right)\eta_1^4 A + R\pi a\left(1+M_3\,a^2 + M_1 M_3 a^2\right)B - R M_1 \pi^2 a\,BC = 0,$$
$$\tag{3.4.17}$$

$$\pi a\,A + \eta_1^2 B + \pi^2 a\,AC = 0, \tag{3.4.18}$$

$$8C - a\,AB = 0. \tag{3.4.19}$$

Writing B and C in terms of A using Eqs. (3.4.18) and (3.4.19) and substituting the resulting expressions into Eq. (3.4.17), we obtain

$$A \left\{ \begin{vmatrix} a^4 \pi^4 \eta_1^4 \left(\dfrac{A^2}{8} \right)^2 + \left[a^2 \pi^2 \left\{ 2\eta_1^6 - R(M_1+1)a^2\pi^2 \right\} \right] \left(\dfrac{A^2}{8} \right) \\ + \eta_1^2 \left[\eta_1^6 - \dfrac{Ra^2\pi^2 \left(1 + M_3 a^2 + M_1 M_3 a^2 \right)}{\left(1 + M_3 a^2 \right)} \right] \end{vmatrix} \right\} = 0. \quad (3.4.20)$$

The solution $A = 0$ corresponds to pure conduction and the rest of the solutions are given by

$$\frac{A^2}{8} = \left[\frac{1}{2\pi^2 a^2 \left(1 + M_3 a^2 \right) \eta_1^4} \right]$$

$$\times \left[\frac{\left\{ R\pi^2 a^2 (1 + M_1) - 2\eta_1^6 \right\} \left(1 + M_3 a^2 \right)}{\pm \pi a \sqrt{R \left\{ R\pi^2 a^2 (1 + M_1)^2 \left(1 + M_3 a^2 \right) - 4 M_1 \eta_1^6 \right\} \left(1 + M_3 a^2 \right)}} \right].$$

$$(3.4.21)$$

We take the positive sign in front of the radical in Eq. (3.4.21) on the ground that the amplitude of the stream function is real.

3.5 HEAT TRANSPORT

In the study of convection in ferromagnetic fluids the quantification of heat transport across the layer plays a crucial role. This is because the onset of convection, as the thermal Rayleigh number R is increased, is more readily detected by its effect on the heat transfer. In the basic state, heat transport is due to conduction alone. Hence if H_T is the rate of heat transfer per unit area, then we have

$$H_T = -\kappa \left\langle \frac{\partial T_{\text{total}}}{\partial z} \right\rangle_{z=0}, \quad (3.5.1)$$

where the angular bracket denotes a horizontal average and

$$T_{\text{total}} = T_{\text{o}} - \frac{\Delta T}{d} z + T(x, z, t). \quad (3.5.2)$$

Substituting the second of Eq. (3.4.1) into Eq. (3.5.2) and using the resultant in Eq. (3.5.1), we obtain

$$H_T = \frac{\kappa \Delta T}{d} - \frac{\kappa \Delta T}{d} 2\pi C .$$
(3.5.3)

The Nusselt number Nu is defined as

$$Nu = \frac{H_T}{\left(\dfrac{\kappa \Delta T}{d} \right)} = 1 - 2\pi C .$$
(3.5.4)

Writing C in terms of A using Eqs. (3.4.17) – (3.4.19) and substituting the resulting expression into Eq. (3.5.4), we obtain

$$Nu = 1 + \frac{2\pi^2 a^2 \left(\dfrac{A^2}{8} \right)}{\eta_1^2 + a^2 \pi^2 \left(\dfrac{A^2}{8} \right)} .$$
(3.5.5)

The second term on the right side of Eq. (3.5.5) characterizes the convective contribution to the heat transport.

Numerous works have appeared in parallel on ferroconvection with a *dc* magnetic field and electroconvection with an *ac* electric field. In a recent study Siddheshwar (2002a, b) has proved an analogy between the Rayleigh-Bénard instability in ferromagnetic and dielectric liquids. In what follows we discuss an analogy for finite amplitude convection between ferromagnetic and dielectric liquids.

3.6 ANALOGY FOR NONLINEAR CONVECTION BETWEEN FERROMAGNETIC AND DIELECTRIC LIQUIDS

The system of equations of electrohydrodynamics describing the Rayleigh-Bénard instability situation in a constant viscosity dielectric liquid, using the notation of Chapter II, becomes

$$\nabla \cdot \vec{q} = 0, \tag{3.6.1}$$

$$\rho_o \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{q} \right] = -\nabla p - \rho g \, \hat{k} + \left(\vec{P} \cdot \nabla \right) \vec{E} + \mu_1 \nabla^2 \vec{q}, \tag{3.6.2}$$

$$\rho_o C_{VE} \left[\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla \right) T \right] = k_1 \nabla^2 T, \tag{3.6.3}$$

$$\rho = \rho_o \left[1 - \alpha (T - T_o) \right], \tag{3.6.4}$$

$$\nabla \cdot \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0}, \tag{3.6.5}$$

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_o \left(\varepsilon_r - 1 \right) \vec{E}, \tag{3.6.6}$$

$$\varepsilon_r = \varepsilon_r^o - e (T - T_o), \tag{3.6.7}$$

where $\vec{E}$ is an *ac* electric field, which is assumed to oscillate sufficiently rapidly so as to make the body force on any free charges in the liquid inconsequential, Φ is the scalar electric potential and the rest of the quantities have their usual meaning. We note here that the assumed strength of $\vec{E}$ is such that it does not induce any non-Newtonian characteristics in the dielectric liquid.

It is expedient to write $\varepsilon_r^o = \left(1 + \chi_e \right)$, where χ_e is the electric susceptibility, for it enables us to arrive at the conventional definition $\vec{P} = \varepsilon_o \chi_e \vec{E}$ in the absence of the temperature dependence of ε_r, that is, when $e = 0$. We continue using Eq. (3.6.7) with ε_r^o replaced by $\left(1 + \chi_e \right)$ in the remaining chapters. In writing Eq. (3.6.7) we have assumed that ε_r varies with the electric field strength quite insignificantly (Stiles *et al.*, 1993).

We restrict ourselves to the two-dimensional analysis, as in the case of ferroconvection, so that all physical quantities are independent of y, a horizontal

co-ordinate. The electric boundary conditions are that the normal component of the electric displacement $\overrightarrow{D}$ and tangential component of the electric field $\overrightarrow{E}$ are continuous across the boundaries.

Taking the components of polarization and electric field in the basic state to be $\left[0,\,P_b(z)\right]$ and $\left[0,\,E_b(z)\right]$, we obtain the quiescent state solution

$$
\left.
\begin{array}{l}
\overrightarrow{q}_b = (0,\,0),\quad T_b = T_o - \dfrac{\Delta T}{d}z,\quad \rho_b = \rho_o\left[1+\alpha\dfrac{\Delta T}{d}z\right], \\[4mm]
\overrightarrow{E}_b = \left[\dfrac{(1+\chi_e)E_o}{(1+\chi_e)+\dfrac{e\Delta T}{d}z}\right]\hat{k},\quad P_b = \varepsilon_o E_o(1+\chi_e)\left[1-\dfrac{1}{(1+\chi_e)+\dfrac{e\Delta T}{d}z}\right]\hat{k},
\end{array}
\right\}
$$

$$(3.6.8)$$

where E_o is the root mean square value of the electric field at the lower surface. On this basic state we superpose finite amplitude perturbations of the form

$$
\left.
\begin{array}{l}
\overrightarrow{q} = \overrightarrow{q}_b + (u',\,w'),\quad T = T_b + T^{'},\quad p = p_b + p',\quad \rho = \rho_b + \rho', \\[3mm]
\overrightarrow{P} = \overrightarrow{P}_b + (P_1',\,P_3'),\quad \overrightarrow{E} = \overrightarrow{E}_b + (E_1',\,E_3'),
\end{array}
\right\}
$$

$$(3.6.9)$$

where the prime denotes perturbation. The second of Eq. (3.6.6) now leads to

$$
\left.
\begin{array}{l}
P_1' = \varepsilon_o\chi_e E_1' - e\varepsilon_o T' E_1', \\[3mm]
P_3' = \varepsilon_o\chi_e E_3' - e\varepsilon_o E_o T' - e\varepsilon_o T' E_3',
\end{array}
\right\}
$$

$$(3.6.10)$$

where it has been assumed that $e\Delta T << (1+\chi_e)$. Introducing the perturbed electric potential $\Phi^{'}$ through the relation $\overrightarrow{E}^{'} = \nabla\Phi'$, eliminating the pressure p in Eq. (3.6.2), incorporating the quiescent state solution and non-dimensionalizing the resulting equation as well as Eq. (3.6.3) using the definitions given in Chapter II, we obtain the dimensionless form of the vorticity and heat transport equations

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right) = -\left(R+R_E\right)\frac{\partial T}{\partial x} + R_E\frac{\partial^2\Phi}{\partial x\partial z} + \nabla^4\psi$$
$$+R_E\ J\left(T,\frac{\partial\Phi}{\partial z}\right) + \frac{1}{Pr}\ J\left(\psi,\nabla^2\psi\right)^{\text{,}}$$
(3.6.11)

$$\frac{\partial T}{\partial t} = -\frac{\partial\psi}{\partial x} + \nabla^2 T + J\left(\psi,T\right).$$
(3.6.12)

Using Eq. (3.6.10) in Eq. (3.6.5) and non-dimensionalizing the resulting equation, we obtain

$$\nabla^2\Phi - \frac{\partial T}{\partial z} = 0.$$
(3.6.13)

Equations (3.6.11) – (3.6.13) are solved subject to the boundary conditions

$$\psi = \frac{\partial^2\psi}{\partial z^2} = T = \frac{\partial\Phi}{\partial z} = 0 \quad \text{at } z = 0,\ 1.$$
(3.6.14)

Comparing the set of Eqs. (3.2.13) – (3.2.15) and that of Eqs. (3.6.11) – (3.6.13), the analogy between 'Rayleigh-Bénard ferroconvection' and 'Rayleigh-Bénard electroconvection' can easily be recognized. Indeed, we can recover the equations of the latter from those of the former when we replace RM_1 by R_E and M_3 by 1. This reiterates the fact that the problem of nonlinear electroconvection of the type reported here can be construed from that of nonlinear ferroconvection. In view of the above analogy, there is no need to study the nonlinear electroconvection problem in isolation. This analogy can, in fact, be proved for the more general boundary conditions discussed in Chapter IV.
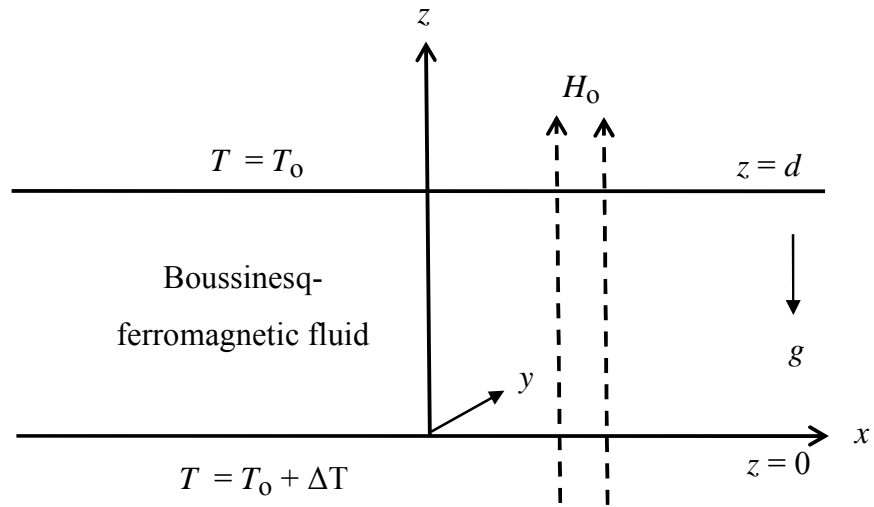
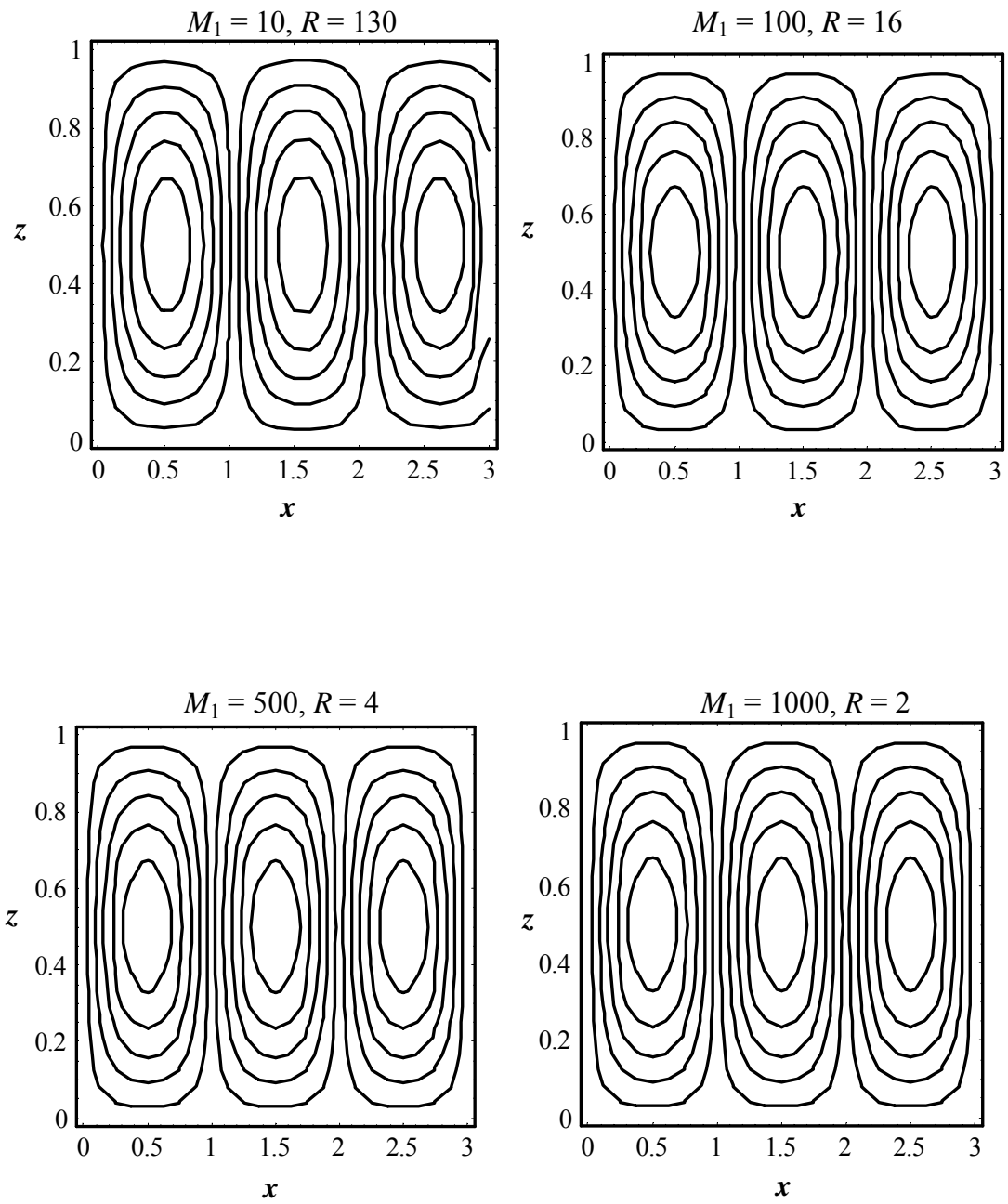**Figure 3.1:** Configuration of the problem.

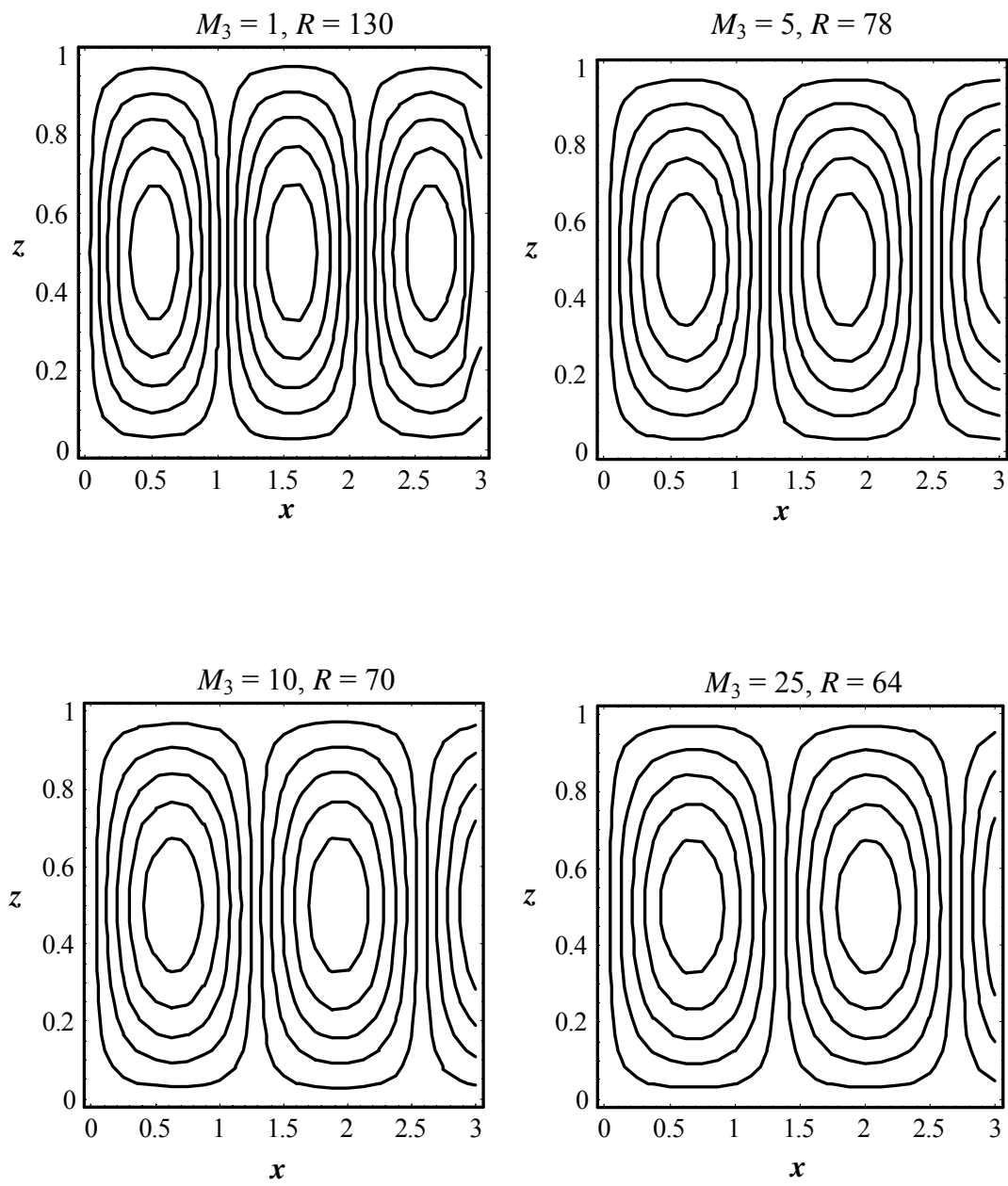**Figure 3.2:** Streamlines for different values of M_1 and for $M_3 = 1$.

Figure 3.3: Streamlines for different values of M_3 and for $M_1 = 10$.
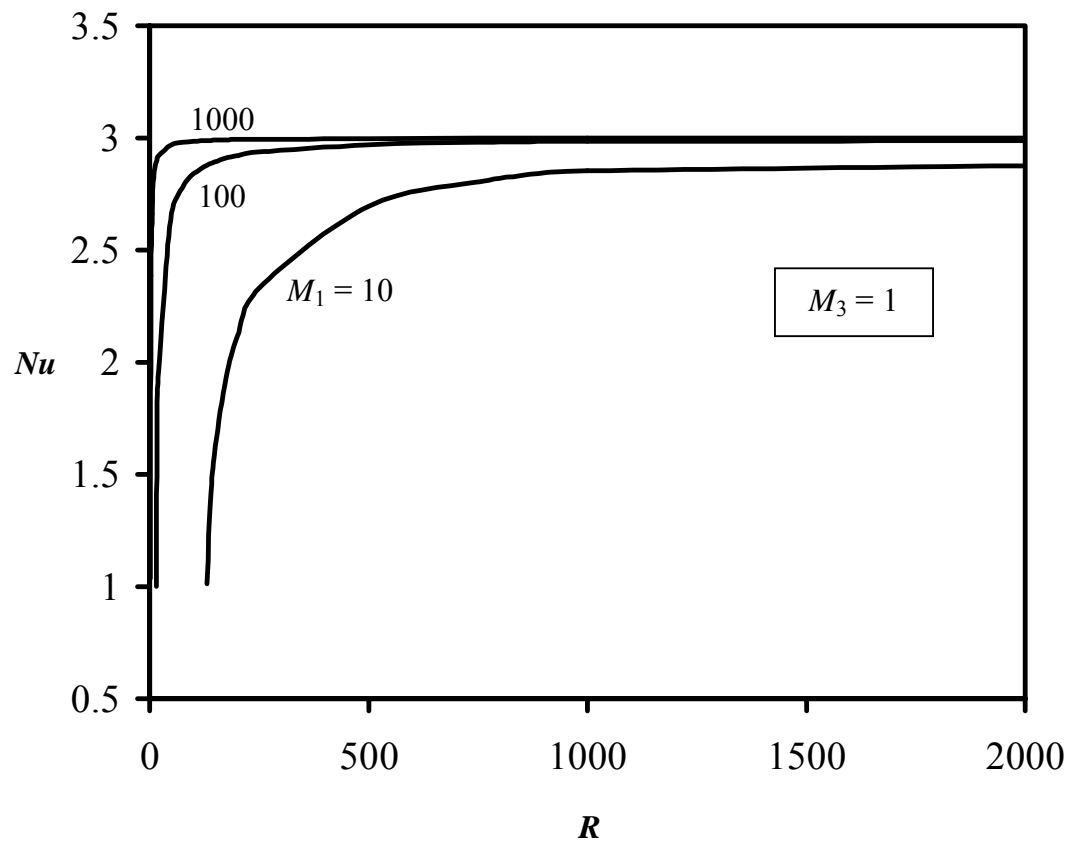
Figure 3.4: Plot of the Nusselt number *Nu* versus the thermal Rayleigh number *R* for different values of the buoyancy-magnetization parameter M_1.
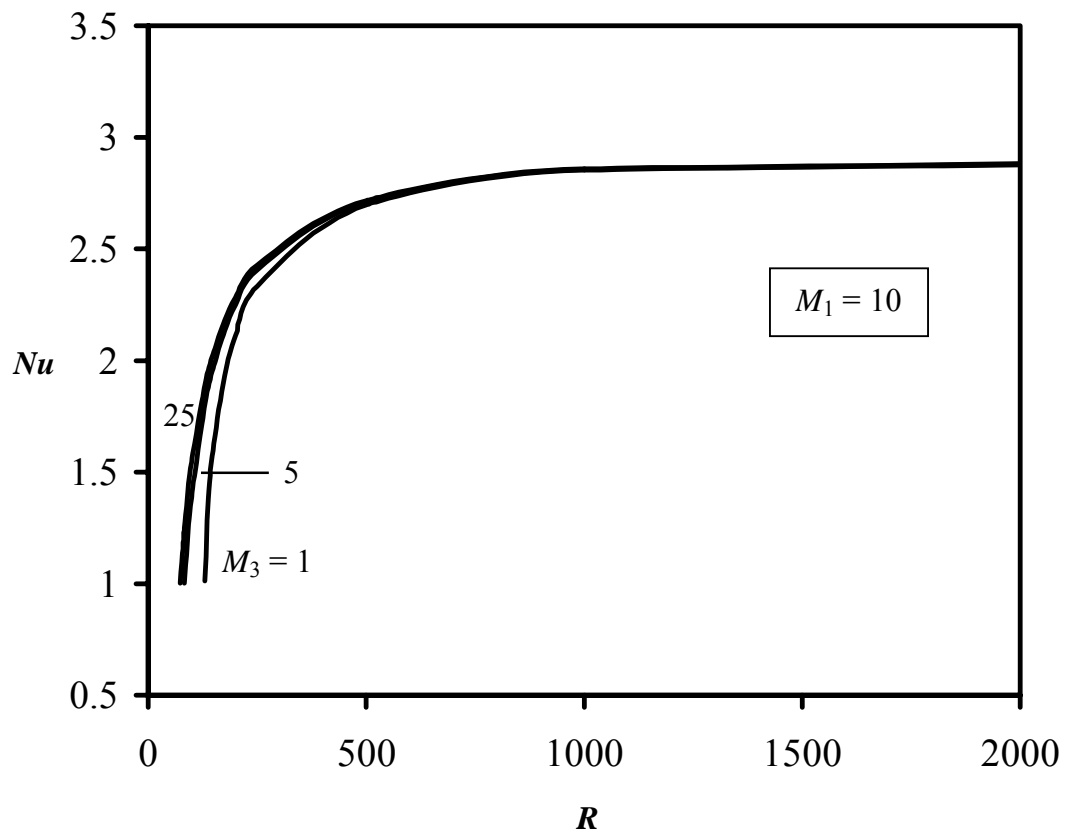
Figure 3.5: Plot of Nu versus R for different values of the non-buoyancy magnetization parameter M_3.
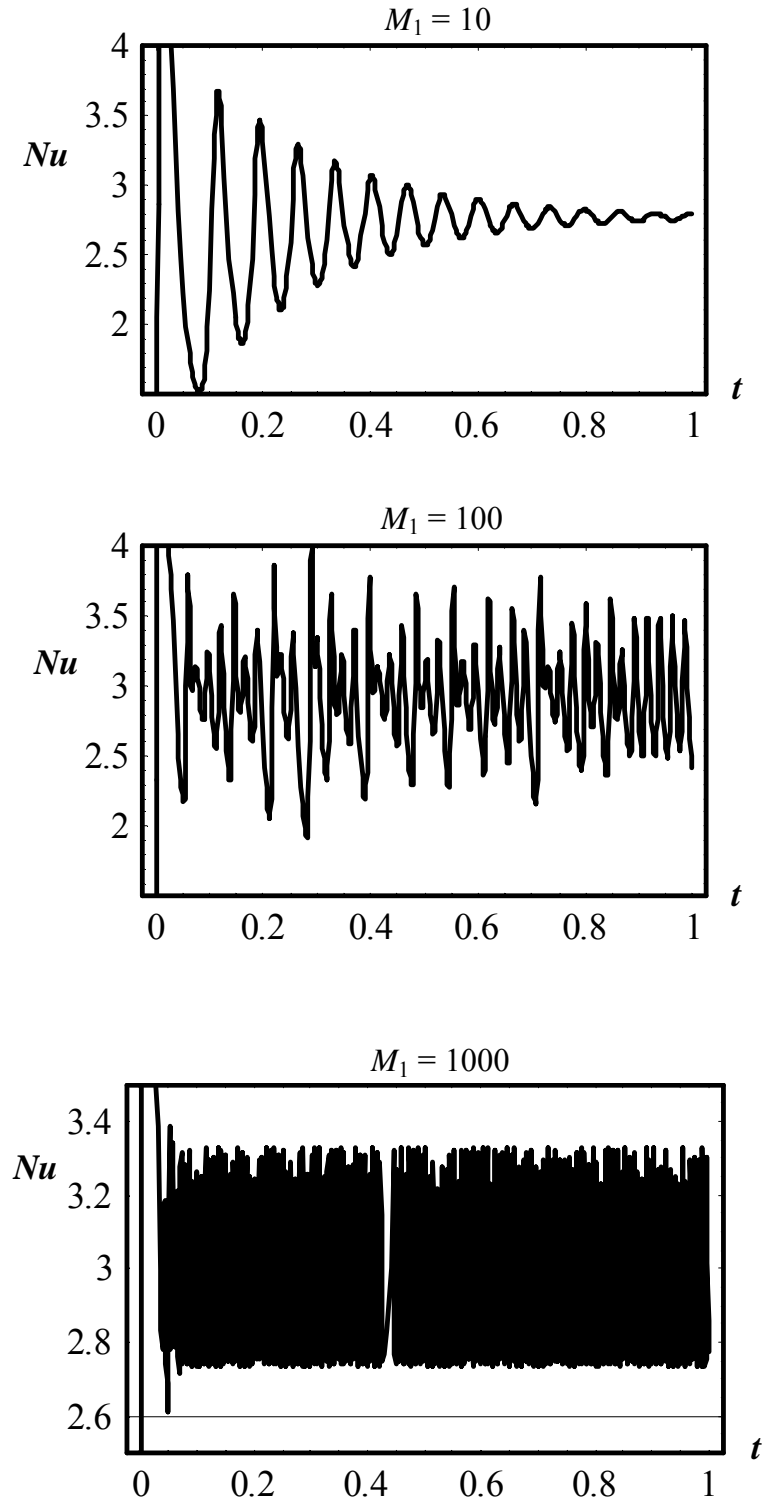
Figure 3.6: Variations in the Nusselt number, *Nu*, with time for different values of M_1 and for $M_3 = 1$, $R = 658$ and $Pr = 10$.
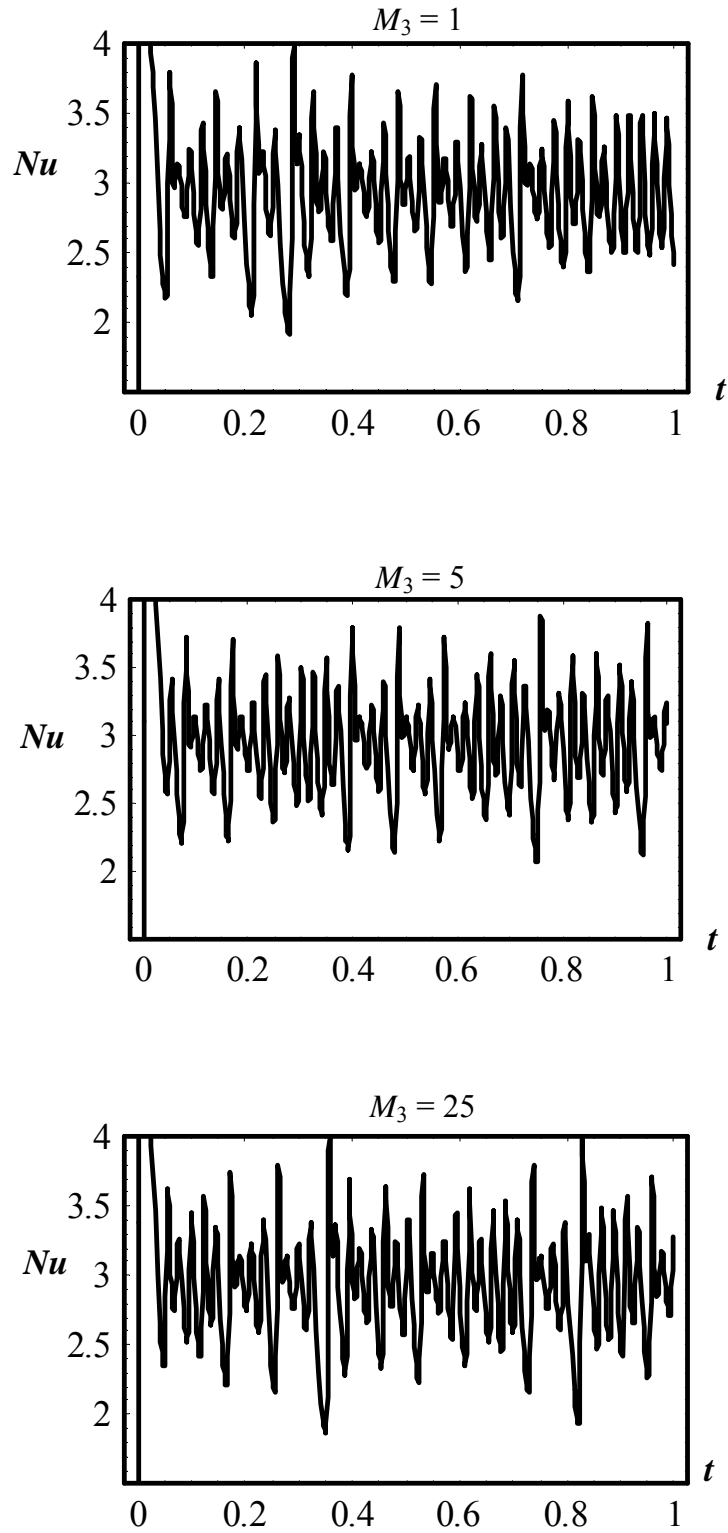
Figure 3.7: Variations in the Nusselt number, Nu, with time for different values of M_3 and for $M_1 = 100$, $R = 658$ and $Pr = 10$.
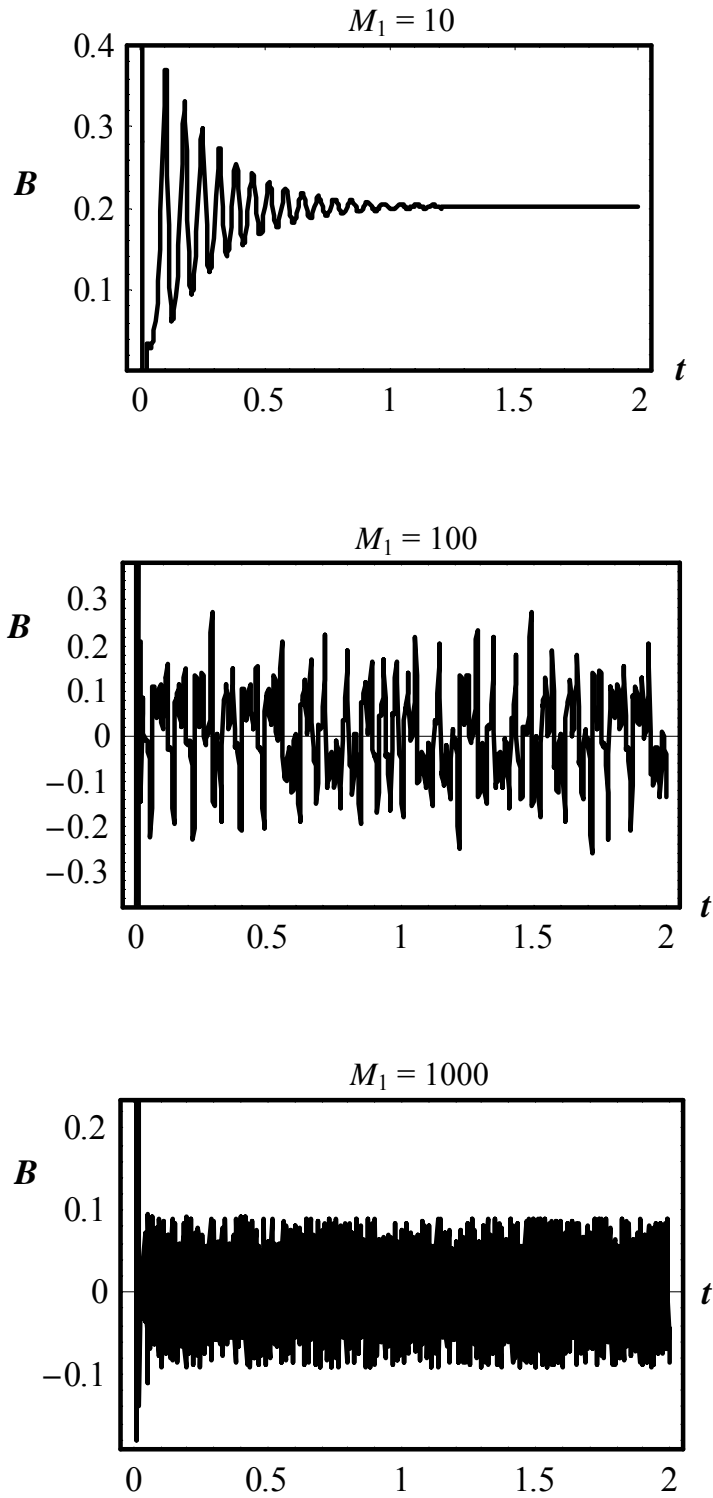
Figure 3.8: Variations in amplitude, $B(t)$ versus time for different values of M_1 and for $M_3 = 1$, $R = 658$ and $Pr = 10$.
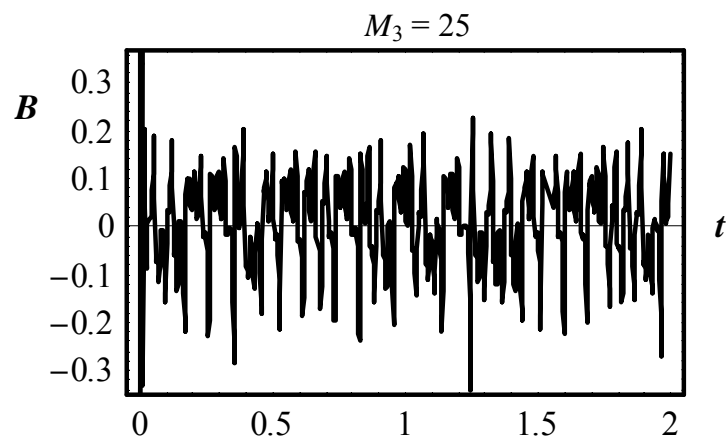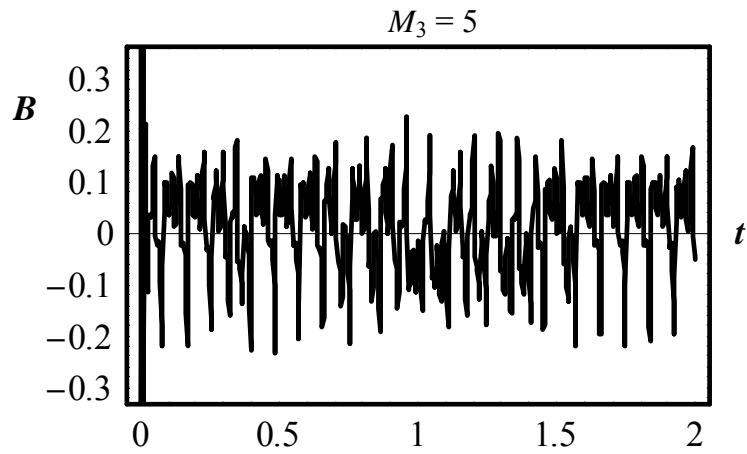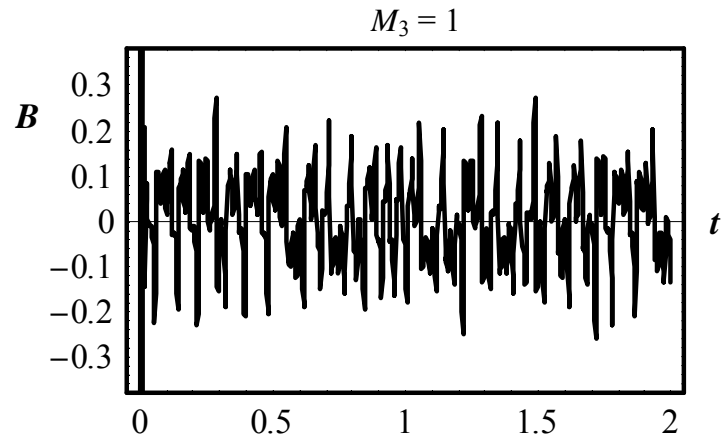
Figure 3.9: Variations in amplitude, $B(t)$ versus time for different values of M_3 and for $M_1 = 100$, $R = 658$ and $Pr = 10$.
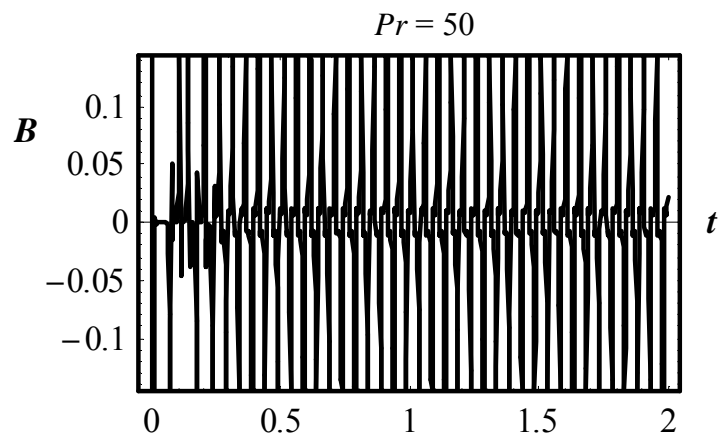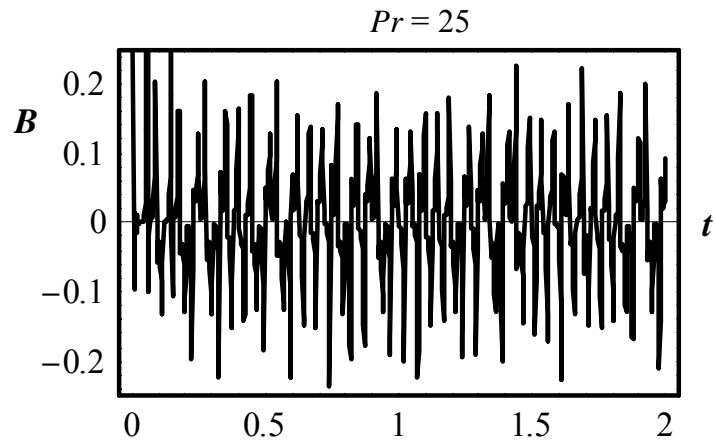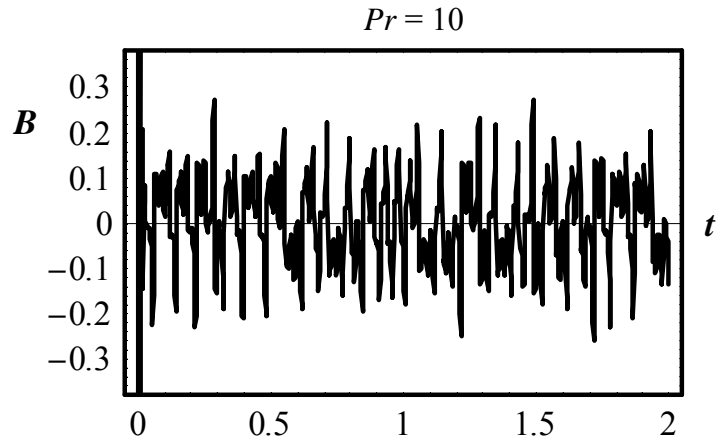
Figure 3.10: Variations in amplitude, $B(t)$ versus time for different values of Pr and for $M_1 = 100$, $R = 658$ and $M_3 = 1$.
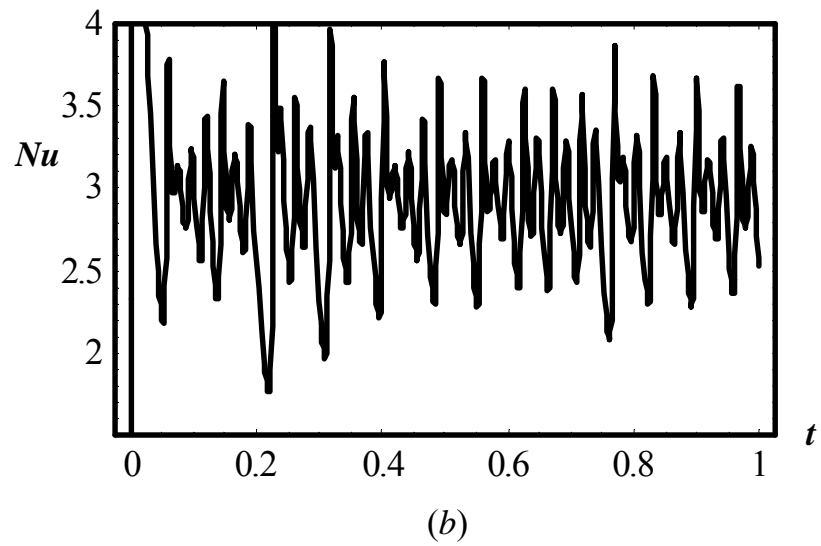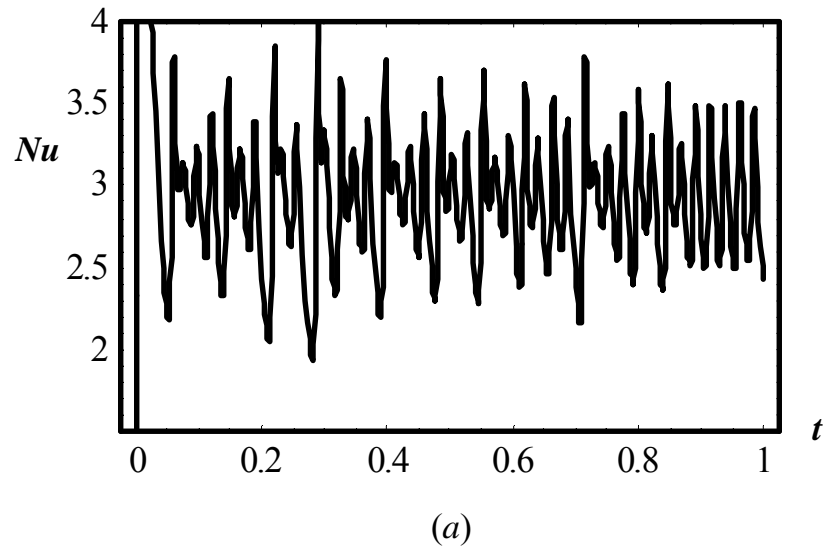
Figure 3.11: Variations in the Nusselt number, Nu, with time for different initial conditions and for $M_1 = 100$, $M_3 = 1$, $R = 658$ and $Pr = 10$. (a) $(A, B, C) = (0, 1, 0)$ and (b) $(A, B, C) = (0, 1.0001, 0)$.
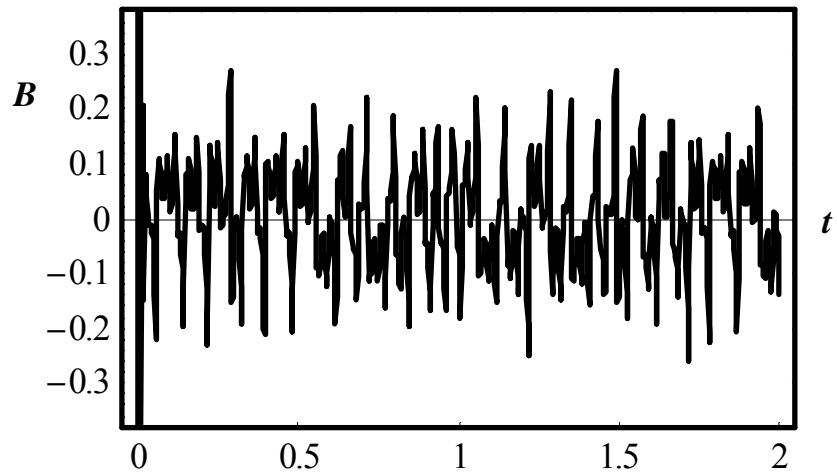
(a)
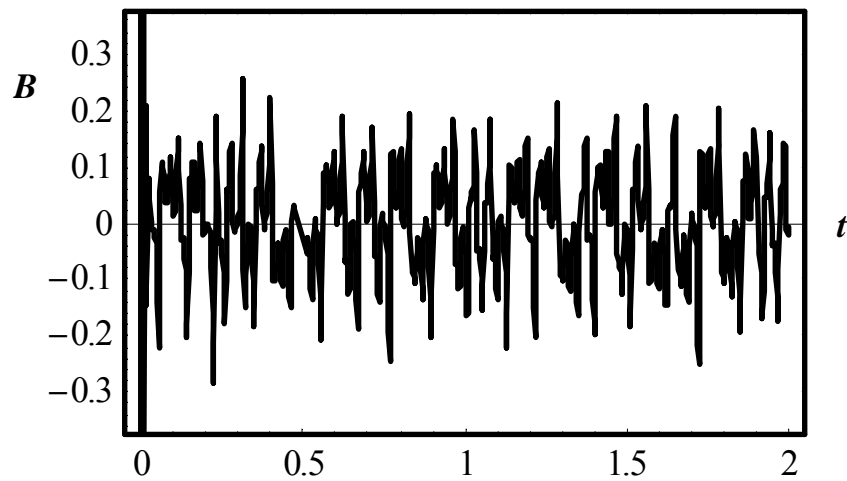
(b)

Figure 3.12: Variations in amplitude, $B(t)$ versus time for different initial conditions and for $M_1 = 100$, $R = 658$, $M_3 = 1$ and Pr $= 10$.

(a) $(A, B, C) = (0, 1, 0)$ and (b) $(A, B, C) = (0, 1.0001, 0)$.

Table 3.1:   Critical thermal Rayleigh number (R_c^s), critical wavenumber (α_c) and critical wavelength (λ_c) pertaining to the stationary instability for a constant viscosity ferromagnetic fluid.

M_1	M_3	R_c^s	α_c	$\lambda_c = \dfrac{2\pi}{\pi\alpha_c}$
10	1	129.390	0.956	2.092
	5	77.821	0.818	2.445
	10	69.475	0.776	2.577
	25	63.894	0.739	2.706
100	1	15.279	0.995	2.010
	5	8.677	0.829	2.413
	10	7.667	0.782	2.558
	25	6.999	0.743	2.692
500	1	3.105	0.999	2.002
	5	1.753	0.830	2.410
	10	1.548	0.783	2.554
	25	1.412	0.743	2.692
1000	1	1.555	0.999	2.002
	5	0.878	0.830	2.410
	10	0.775	0.783	2.554
	25	0.707	0.743	2.692

CHAPTER IV

THERMORHEOLOGICAL AND MAGNETORHEOLOGICAL EFFECTS ON FERROCONVECTION WITH INTERNAL HEAT SOURCE

4.1 INTRODUCTION

The investigation of convective instability due to the buoyancy force or surface-tension force or both in the presence of a vertical temperature gradient has been the subject of interest since the carrying out of an experiment by Bénard (1901). In the case of usual gravity situation *RBC* predominates over *MC* in not-so-thin layers and the opposite is true for microgravity ($g = o(10^{-6})$) situation when the liquid layer with a free upper surface is thin (Pearson, 1958). The main disparity between *RBC* and *MC* is that the surface is elevated in regions of rising hot fluid in the former case, whereas in the latter the surface is depressed. The idea of tuning the properties of magnetic liquids with a magnetic field has led to numerous innovative and fascinating applications (Popplewell, 1984; Rosensweig, 1986; Berkovskii *et al.*, 1993; Horng *et al.*, 2001). *RBC* in constant viscosity ferromagnetic liquids, Newtonian as well as non-Newtonian, is fairly well studied (Finlayson, 1970; Sekhar and Rudraiah, 1991; Siddheshwar, 1993, 1995, 1998, 1999; 2002a; Siddheshwar and Abraham, 1998; 2003; Abraham, 2002a; 2002b; Yamaguchi *et al.*, 2002).

It has been corroborated by Finlayson (1970) that in very thin layers of magnetic liquids only magnetic forces contribute to convection and that the effect of buoyancy forces could be ignored in such layers. Motivated by the microgravity research in magnetic fluids, Odenbach (1993, 1995b, 1999) investigated the thermomagnetic convection in magnetic liquids using sounding rocket and drop tower experiments. Schwab (1990), Qin and Kaloni (1994), Weilepp and Brand (1996) and Shivakumara *et al.* (2002) have studied the effect of surface tension on

thermomagnetic convection in a constant viscosity ferromagnetic liquid. The study of both *RBC* and *MC* in a constant viscosity nonmagnetic liquid has been refined by several investigators taking into account the effects of gravity, rotation, magnetic field, internal heat generation and micro rotation of suspended particles (Nield, 1964; Sparrow *et al.*, 1964; Watson, 1968; Riahi, 1986; Lam and Bayazitoglu, 1987; Benguria and Depassier, 1989; Char and Chiang, 1994; Wilson, 1997; Rudraiah and Siddheshwar, 2000; Siddheshwar and Pranesh, 2001, 2002; Kim *et al.*, 2002).

Technological and biomedical applications of magnetic liquids indicate that these liquids depend greatly on their rheological properties. Several studies such as those of McTague (1969), Hall and Busenberg (1969), Shliomis (1972), Kamiyama *et al.* (1987), Kobori and Yamaguchi (1994) and Chen *et al.* (2002) specify that the effective viscosity of a ferromagnetic liquid is enhanced by the application of a magnetic field. The contemporary applications of this effect, known as magnetorheological effect, include dampers, brakes, pumps, clutches, valves, robotic control systems and the like (Carlson *et al.*, 1996). Recently, Balau *et al.* (2002) have pointed out through their experiments that magnetorheological effect is of significant importance in water-based and kerosene-based solutions, and in physiological-solution-based magnetic liquids even for moderate strengths of applied magnetic field. This is more so in the extraterrestrial context.

Another fact about the viscosity of any carrier liquid decreasing with temperature is also well known (Stengel *et al.*, 1982; Platten and Legros, 1984; Gebhart *et al.*, 1988; Severin and Herwig, 1999; Siddheshwar, 2004; Siddheshwar and Chan, 2005) and is referred to as thermorheological effect. It is imperative therefore to envisage the importance of the both *RBC* and *MC* problems in ferromagnetic liquids involving both temperature and magnetic field strength dependent effective viscosity. Apart from the aforementioned rheological effects, the effect of volumetric internal heat source is also important in ferromagnetic liquids from the viewpoint of magnetocaloric pumping. In this chapter we aim at studying the effect of uniform internal heat generation on the threshold of both

RBC and *MC* in a variable viscosity ferromagnetic liquid subjected to a vertical temperature gradient and a vertical *dc* magnetic field. The assumed strength of the magnetic field is such that the liquid does not exhibit any non-Newtonian characteristics. We study both stationary and oscillatory modes of instability and show that the former one is the preferred mode. Two important explorations are made in this chapter:

   (i)   the effect of heat source and heat sink on the onset of both *RBC* and *MC* in ferromagnetic liquids and
   (ii) an analogy between ferroconvection and electroconvection.

   We also introduce new set of boundary conditions on the potential for both ferroconvection and electroconvection.

4.2 MATHEMATICAL FORMULATION

   Consider an infinite horizontal layer of a ferromagnetic liquid that supports a temperature gradient and a *dc* magnetic field $\vec{H}_o$ in the vertical direction (Figure 4.1). The gradient in temperature is due to a prescribed temperature difference ΔT (> 0 for fluid heated from below) across the layer and a uniform distribution of heat source/sink of intensity S in the liquid. The liquid is assumed to have an effective variable viscosity μ that depends on the temperature as well as on the magnitude of the magnetic field.

   The system of equations describing the Rayleigh-Bénard instability situation in a variable viscosity ferromagnetic liquid with uniform heat source in the notation of Chapter II is

$$\nabla \cdot \vec{q} = 0, \tag{4.2.1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} . \nabla \right) \vec{q} \right] = -\nabla p - \rho g \, \hat{k} + \mu_0 \, (\vec{M} . \nabla) \vec{H}$$

$$+ \nabla . \left[\mu(H,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right] , \tag{4.2.2}$$

$$\rho_0 C_{VH} \left[\frac{\partial T}{\partial t} + \left(\vec{q} . \nabla \right) T \right] = k_1 \nabla^2 T + S , \tag{4.2.3}$$

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_a \right) \right] , \tag{4.2.4}$$

$$\nabla . \vec{B} = 0, \quad \nabla \times \vec{H} = \vec{0} , \tag{4.2.5}$$

$$\vec{B} = \mu_0 \left(\vec{M} + \vec{H} \right) , \tag{4.2.6}$$

$$M = M_0 + \chi_m \left(H - H_0 \right) - K_1 \left(T - T_a \right) , \tag{4.2.7}$$

where the superscript *Tr* in Eq. (4.2.2) denotes the transpose. The effects of viscous dissipation and radiation are neglected in writing the energy equation (4.2.3).

The magnetic boundary conditions specify that the normal component of the magnetic induction and tangential components of the magnetic field are continuous across the boundary. The velocity boundary condition is $\vec{q} = \vec{0}$ on a rigid wall and different temperatures are maintained on each boundary. We assume the effective viscosity $\mu(H,T)$, based on the information given in Chapter II, to be a quadratic function of H and T in the form

$$\mu \left(H,T \right) = \mu_1 \left[1 + \delta_1 \left(H - H_0 \right)^2 - \delta_2 \left(T - T_a \right)^2 \right] , \tag{4.2.8}$$

where δ_1 and δ_2 are small positive quantities. Taking the components of magnetization and magnetic field in the basic state to be $[0, 0, M_b(z)]$ and $[0, 0, H_b(z)]$, we obtain the quiescent state solution in the form

$$\vec{q}_b = \vec{0}, \qquad T_b(z) = T_a - f(z), \qquad p = p_b(z),$$

$$\mu_b(z) = \mu_1 \left[1 - \delta \{ f(z) \}^2 \right], \qquad \rho_b(z) = \rho_o [1 + \alpha f(z)],$$

$$\vec{H}_b = \left[H_o - \frac{K_1}{1 + \chi_m} f(z) \right] \hat{k}, \qquad \vec{M}_b = \left[M_o + \frac{K_1}{1 + \chi_m} f(z) \right] \hat{k},$$

$$\tag{4.2.9}$$

where

$$\delta = \delta_2 - \delta_1 \left[\frac{K_1}{1 + \chi_m} \right]^2 \tag{4.2.10}$$

and $f(z) = (S z^2 / 2 k_1) + (\Delta T z / d) - (S d^2 / 8 k_1)$. In arriving at the above basic state solution we have assumed

$$T = T_o \qquad \qquad \text{at } z = d/2$$
$$T = T_1 \, (= T_o + \Delta T) \quad \text{at } z = -d/2, \tag{4.2.11}$$

where d is the thickness of the layer. The dominance of magnetic dependency over temperature dependency of viscosity is implied by $\delta < 0$ while $\delta > 0$ signifies dominance of temperature dependency. It should be noted that the quiescent state temperature profile is quadratic over the cross section, and in the absence of internal heat generation, $i.e.$, when $S = 0$, it is characterized by a liquid temperature that decreases linearly with height. We also note that, unlike the Rayleigh-Bénard ferroconvection problem of a constant viscosity liquid with no internal heating, the expression for $H_b(z)$ and $M_b(z)$ in the present problem become nonlinear due to the presence of the heat source term in the energy equation (4.2.3). In the succeeding section we study the stability of the quiescent state within the framework of the linear theory.

4.2.1 Linear Stability Analysis

Let the components of the perturbed magnetization and the magnetic field be $\left(M_1', M_2', M_b(z) + M_3' \right)$ and $\left(H_1', H_2', H_b(z) + H_3' \right)$ respectively. The

temperature T is taken to be $T_b(z)+T'$ with T' being the perturbation from the quiescent state. The effective viscosity μ, likewise, is taken to be $\mu_b(z)+\mu'$. Using these in Eq. (4.2.7) and linearizing the resulting equation, we obtain

$$
\left.
\begin{aligned}
H_i' + M_i' &= \left(1+\frac{M_o}{H_o}\right)H_i', \quad (i=1,2)\\
H_3' + M_3' &= (1+\chi_m)H_3' - K_1 T',
\end{aligned}
\right\}
\tag{4.2.12}
$$

where it has been assumed that $K_1 \Delta T \ll (1+\chi_m)H_o$ and $K_1 S d^2 \ll (1+\chi_m)H_o$. The second of Eq. (4.2.5) implies one can write $\vec{H}' = \nabla \Phi'$, where Φ' is the perturbed magnetic scalar potential.

Introducing the magnetic potential Φ', eliminating the pressure p in Eq. (4.2.2) and incorporating the quiescent state solution, we obtain the perturbed state vorticity transport equation in the form

$$
\begin{aligned}
&\rho_o \frac{\partial}{\partial t}\left(\nabla^2 w'\right) - 2\delta\mu_1\left[f(z)D^2 f(z) + \{Df(z)\}^2\right]\left(\nabla_1^2 - D^2\right)w'\\
&\quad + 4\delta\mu_1 f(z)Df(z)\nabla^2(Dw') - \mu_1\left[1 - \delta\{f(z)\}^2\right]\nabla^4 w'\\
&\quad - \alpha\rho_o g\nabla_1^2 T' - \frac{\mu_o K_1^2}{1+\chi_m}Df(z)\nabla_1^2 T' + \mu_o K_1 Df(z)\nabla_1^2(D\Phi') = 0,
\end{aligned}
\tag{4.2.13}
$$

where $\nabla^2 = \nabla_1^2 + D^2$, $\nabla_1^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ and $D = \partial/\partial z$. We note that the expression for μ_b in Eq. (4.2.9) is quadratic in z when the internal heat source is absent, *i.e.*, when $S = 0$, because of the quadratic viscosity variation as assumed in Eq. (4.2.8). It should be mentioned that the second term on the left side of Eq. (4.2.13) results from the term $\left(D^2\mu_b/Dz^2\right)\left(\nabla_1^2 - D^2\right)w'$ and it is obvious that it would vanish, in the absence of internal heat source, if μ_b is a linear function of z, *i.e.*, if the effective viscosity $\mu(H,T)$ is a linear function of temperature or the

strength of the magnetic field or both. Thus it is evident that the contribution of this term is quite important in problems involving variable-viscosity liquids.

The linear form of equation (4.2.3) in the perturbed state, on incorporation of the quiescent state solution, becomes

$$\frac{\partial T'}{\partial t} - Df(z)\, w' = \kappa \nabla^2 T' . \tag{4.2.14}$$

Using Eq. (4.2.12) in the first of Eq. (4.2.5), we obtain

$$(1 + \chi_m)\, D^2 \Phi' + \left(1 + \frac{M_o}{H_o}\right) \nabla_1^2 \Phi' - K_1 D T' = 0 . \tag{4.2.15}$$

The infinitesimal perturbations w', T' and Φ' are supposed to be periodic waves that lead to a separable solution to Eqs. (4.2.13) – (4.2.15) in the form

$$\begin{bmatrix} w' \\ T' \\ \Phi' \end{bmatrix} = \begin{bmatrix} w(z,t) \\ T(z,t) \\ \Phi(z,t) \end{bmatrix} \exp \left[i \left(k_x x + k_y y \right) \right] . \tag{4.2.16}$$

Substituting Eq. (4.2.16) into Eqs. (4.2.13) – (4.2.15), we obtain

$$\rho_o \left(D^2 - k^2 \right) \frac{\partial w}{\partial t} + 2 \delta \mu_1 \left[f(z)\, D^2 f(z) + \{ Df(z) \}^2 \right] \left(D^2 + k^2 \right) w$$

$$+ 4 \delta \mu_1 f(z) Df(z) \left(D^2 - k^2 \right) Dw - \mu_1 \left[1 - \delta \{ f(z) \}^2 \right] \left(D^2 - k^2 \right)^2 w$$

$$+ \alpha \rho_o g\, k^2 T - \frac{\mu_o K_1}{1 + \chi_m} k^2 Df(z) \left[(1 + \chi_m) D\Phi - K_1 T \right] = 0 , \tag{4.2.17}$$

$$\frac{\partial T}{\partial t} - Df(z) w - \kappa \left(D^2 - k^2 \right) T = 0 , \tag{4.2.18}$$

$$(1 + \chi_m)\, D^2 \Phi - \left(1 + \frac{M_o}{H_o} \right) k^2 \Phi - K_1 DT = 0 , \tag{4.2.19}$$

107

where $k = \sqrt{k_x{}^2 + k_y{}^2}$ is the wave number in the horizontal direction. Non-dimensionalizing Eqs. (4.2.17) – (4.2.19), using the scaling given in Chapter II, we obtain

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left[g(z)\,D^2 g(z) + \{Dg(z)\}^2\right]\left(D^2 + a^2\right)w$$
$$+ 4\Gamma g(z)\,Dg(z)\left(D^2 - a^2\right)Dw - \left[1 - \Gamma\{g(z)\}^2\right]\left(D^2 - a^2\right)^2 w \quad (4.2.20)$$
$$+ R\,a^2 T - R M_1\,a^2 Dg(z)\left[D\Phi - T\right] = 0\,,$$

$$\frac{\partial T}{\partial t} = \left(D^2 - a^2\right)T + Dg(z)\,w\,, \qquad\qquad (4.2.21)$$

$$(D^2 - M_3\,a^2)\,\Phi - DT \;=\; 0\,, \qquad\qquad (4.2.22)$$

where $g(z) = (N_S\,z^2/2) + z - (N_S/8)$ and the asterisks have been dropped for simplicity. We note that, as to the variable viscosity parameter Γ, the condition $\Gamma < 0$ characterizes the dominance of magnetic field strength dependence of effective viscosity over temperature dependence while $\Gamma > 0$ signifies the dominance of temperature-dependent effective viscosity. The above becomes apparent on examining the expression for δ given in Eq. (4.2.10) and noting that δ_1 and δ_2 appearing in Eq. (4.2.10) are small positive quantities.

Equations (4.2.20) – (4.2.22) are solved subject to the following boundary combinations

    (a)   Free – free, isothermal
$$w = D^2 w = T = 0 \quad \text{at}\ \ z = \pm\ 1/2$$
    (b)   Free – rigid, isothermal
$$w = D^2 w = T = 0 \quad \text{at}\ \ z = +\ 1/2$$
$$w = Dw = T = 0 \quad\ \ \text{at}\ \ z = -\ 1/2$$
    (c)   Rigid – rigid, isothermal
$$w = Dw = T = 0 \quad\ \ \text{at}\ \ z = \pm\ 1/2$$

In each of the boundary combinations mentioned above, the boundary conditions for the magnetic potential given in Eq. (2.2.19) become

$$
\left.
\begin{aligned}
D\Phi + \frac{a\Phi}{1+\chi_m} &= 0 \qquad \text{at} \quad z = +1/2, \\
D\Phi - \frac{a\Phi}{1+\chi_m} &= 0 \qquad \text{at} \quad z = -1/2 .
\end{aligned}
\right\}
$$

Since the system of equations (4.2.20) – (4.2.22) consists of space varying coefficients, it is no longer possible to obtain a closed form solution to the problem. We therefore use the Rayleigh-Ritz method to solve the eigenvalue problem described by Eqs. (4.2.20) – (4.2.22) using simple polynomial trial functions. We choose suitable trial functions for the z-component of velocity, temperature perturbation and magnetic potential that satisfy some of the given boundary conditions, but may not exactly satisfy the differential equations. This results in residuals when the trial functions are substituted into the differential equations. The Rayleigh-Ritz method warrants the residuals be orthogonal to each trial function. It should be made explicit that the Rayleigh-Ritz method is equivalent to the Galerkin method (Finlayson, 1972) for the problem at hand.

We now move on to discuss the validity or otherwise of the principle of exchange of stabilities (*PES*). The *PES* has already been shown to be valid for the Rayleigh-Bénard problem of constant viscosity fluids (Pellew and Southwell, 1940; Chandrasekhar, 1961; Finlayson, 1970; Sekhar and Rudraiah, 1991). On the other hand, as to variable viscosity fluids, the *PES* has been either assumed to be valid or verified to be valid by means of a numerical search (Torrance and Turcotte, 1971; Stengel *et al.*, 1982; Kozhoukharova and Roze, 1999). Nevertheless, it is by no means an arduous task to show analytically that the *PES* holds good for fluid layers with variable-viscosity. This task is expounded in what follows.

4.2.2 Oscillatory Instability

We examine the possibility of the oscillatory instability by means of the single-term Rayleigh-Ritz technique. Multiplying Eqs. (4.2.20) – (4.2.22) by w, T and Φ respectively, integrating with respect to z between the limits $z = -1/2$ and $z = 1/2$, taking $w(z, t) = A_1(t) w_1(z)$, $T(z, t) = B_1(t) T_1(z)$, $\Phi(z, t) = C_1(t) \Phi_1(z)$ (in which $w_1(z)$, $T_1(z)$ and $\Phi_1(z)$ are trial functions), and using the boundary conditions mentioned in Section 4.2.1 give rise to the following system of ordinary differential equations

$$\frac{Q_1}{Pr} \frac{dA_1}{dt} = -Q_2 A_1 + R[Q_3 + M_1 Q_4] B_1 - R M_1 Q_5 C_1, \tag{4.2.23}$$

$$Q_6 \frac{dB_1}{dt} = Q_7 A_1 - Q_8 B_1, \tag{4.2.24}$$

$$0 = Q_9 B_1 + Q_{10} C_1, \tag{4.2.25}$$

where

$$Q_1 = \left\langle (Dw_1)^2 \right\rangle + a^2 \left\langle w_1^2 \right\rangle,$$

$$\begin{aligned}
Q_2 &= \left\langle w_1 \left(1 - \Gamma \{g(z)\}^2 \right) D^4 w_1 \right\rangle - 2 a^2 \left\langle w_1 \left(1 - \Gamma \{g(z)\}^2 \right) D^2 w_1 \right\rangle \\
&\quad + a^4 \left\langle w_1 \left(1 - \Gamma \{g(z)\}^2 \right) w_1 \right\rangle - 2 \Gamma \left\langle w_1 \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] D^2 w_1 \right\rangle \\
&\quad - 2 \Gamma a^2 \left\langle w_1 \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] w_1 \right\rangle - 4 \Gamma \left\langle w_1 g(z) Dg(z) D^3 w_1 \right\rangle \\
&\quad + 4 \Gamma a^2 \left\langle w_1 g(z) Dg(z) Dw_1 \right\rangle,
\end{aligned}$$

$$Q_3 = a^2 \left\langle w_1 T_1 \right\rangle, \quad Q_4 = a^2 \left\langle w_1 Dg(z) T_1 \right\rangle,$$

$$Q_5 = a^2 \left\langle w_1 Dg(z) D\Phi_1 \right\rangle, \quad Q_6 = \left\langle T_1^2 \right\rangle, \quad Q_7 = \left\langle T_1 Dg(z) w_1 \right\rangle,$$

$$Q_8 = \left\langle (DT_1)^2 \right\rangle + a^2 \left\langle T_1^2 \right\rangle, \quad Q_9 = \left\langle \Phi_1 DT_1 \right\rangle,$$

$$Q_{10} = \left\langle (D\Phi_1)^2 \right\rangle + M_3\, a^2 \left\langle \Phi_1^2 \right\rangle + \frac{a}{1+\chi_m}\left[\left\{\Phi_1\!\left(\frac{1}{2}\right)\right\}^2 + \left\{\Phi_1\!\left(\frac{-1}{2}\right)\right\}^2\right]$$

and $\left\langle uv \right\rangle = \displaystyle\int_{-1/2}^{1/2} uv\, dz$. Eliminating C_1 between Eqs. (4.2.23) and (4.2.25), the

resulting system of equations can be rearranged into the matrix form

$$\frac{dA}{dt} = F A, \tag{4.2.26}$$

where

$$A = \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \quad \text{and} \quad F = \begin{pmatrix} -\dfrac{Pr\, Q_2}{Q_1} & \dfrac{R\, Pr}{Q_1}\left[Q_3 + M_1\left\{Q_4 + \dfrac{Q_5 Q_9}{Q_{10}}\right\}\right] \\[3ex] \dfrac{Q_7}{Q_6} & -\dfrac{Q_8}{Q_6} \end{pmatrix}.$$

Following classical works, we study the linear stability of the system (both stationary and oscillatory) by letting $A = \exp[i\omega t]\,A_o$ in Eq. (4.2.26), where ω is the frequency of oscillations, $A_o = \begin{pmatrix} A_{1o} \\ B_{1o} \end{pmatrix}$, and A_{1o} and B_{1o} are constants. This yields

$$\left(F - i\omega I\right) A_o = 0, \tag{4.2.27}$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. For a non-trivial solution of the above homogeneous system, we require

$$\left| F - i\omega I \right| = 0. \tag{4.2.28}$$

After minor simplification, this equation can be rearranged to yield an expression for R

$$R = \frac{1}{Q_7 N_1} \left\{ Q_2 Q_8 - \frac{Q_1 Q_6}{Pr} \omega^2 \right\} + i\omega N_2, \tag{4.2.29}$$

where

$$N_1 = Q_3 + \left\{ Q_4 + \frac{Q_5 Q_9}{Q_{10}} \right\} M_1 \quad \text{and} \quad N_2 = \frac{Pr\, Q_2 Q_6 + Q_1 Q_8}{Pr\, Q_7 N_1}.$$

Since R is a real quantity, the imaginary part of Eq. (4.2.29) has to vanish. This gives us two possibilities:

(i) $\omega \neq 0$, $N_2 = 0$ (oscillatory instability),

(ii) $\omega = 0$, $N_2 \neq 0$ (stationary instability).

Taking $N_2 = 0$, we get $Pr\, Q_2 Q_6 + Q_1 Q_8 = 0$, which is independent of ω. We therefore conclude, based on the reasoning given in Chapter III, that oscillatory convection is not possible in the present problem. So we take $\omega = 0$ in the subsequent analysis. In view of the above observation, we now proceed to discuss the preferred mode of instability, *viz.*, stationary convection.

4.2.3 Stationary Instability

The system of equations associated with the stationary instability reads as

$$\left[1 - \Gamma \{g(z)\}^2 \right] (D^2 - a^2)^2 w - 2\,\Gamma \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] (D^2 + a^2) w$$
$$- 4\,\Gamma g(z) Dg(z)(D^2 - a^2)Dw - a^2 \left[R + RM_1 Dg(z) \right] T$$
$$+ RM_1 a^2 Dg(z) D\Phi = 0, \tag{4.2.30}$$

$$(D^2 - a^2)\, T + Dg(z) w = 0, \tag{4.2.31}$$

$$\left(D^2 - M_3 a^2 \right) \Phi - DT = 0. \tag{4.2.32}$$

The system of Eqs. (4.2.30) – (4.2.32) together with the boundary conditions specified Section 4.2.2 poses an eigenvalue problem for R with Γ, N_S, M_1, M_3

and χ_m as parameters. Despite the fact that stationary instability could be studied with the help of the single-term Rayleigh-Ritz technique, it is however, advantageous to use the more accurate 'Higher Order Rayleigh-Ritz Technique (*HORT*)' (Finlayson, 1972) to obtain the eigenvalue and the associated wavenumber. To this end, we expand $w(z)$, $T(z)$ and $\Phi(z)$ in a series of trial functions

$$w(z) = \sum_{i=1}^{n} \alpha_i w_i(z), \quad T(z) = \sum_{i=1}^{n} \beta_i T_i(z) \quad \text{and} \quad \Phi(z) = \sum_{i=1}^{n} \gamma_i \Phi_i(z), \quad (4.2.33)$$

where α_i, β_i and γ_i are constants, and $w_i(z)$, $T_i(z)$ and $\Phi_i(z)$ are trial functions. Applying *HORT* to Eqs. (2.28) – (2.30), one obtains the following system of homogeneous equations:

$$\left.\begin{aligned} A_{ji}\alpha_i + B_{ji}\beta_i + C_{ji}\gamma_i &= 0, \\ D_{ji}\alpha_i + E_{ji}\beta_i &= 0, \\ F_{ji}\beta_i + G_{ji}\gamma_i &= 0, \end{aligned}\right\} \qquad (4.2.34)$$

where

$$\begin{aligned} A_{ji} &= \left\langle w_j \left(1 - \Gamma\{g(z)\}^2\right) D^4 w_i \right\rangle - 2a^2 \left\langle w_j \left(1 - \Gamma\{g(z)\}^2\right) D^2 w_i \right\rangle \\ &\quad + a^4 \left\langle w_j \left(1 - \Gamma\{g(z)\}^2\right) w_i \right\rangle - 2\Gamma \left\langle w_j \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] D^2 w_i \right\rangle \\ &\quad - 2\Gamma a^2 \left\langle w_j \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] w_i \right\rangle - 4\Gamma \left\langle w_j g(z) Dg(z) D^3 w_i \right\rangle \\ &\quad + 4\Gamma a^2 \left\langle w_j g(z) Dg(z) Dw_i \right\rangle, \end{aligned}$$

$$B_{ji} = -R\, a^2 \left\langle w_j\, T_i \right\rangle - R M_1 a^2 \left\{ \left\langle w_j\, Dg(z) T_i \right\rangle \right\},$$

$$C_{ji} = R M_1 a^2 \left\{ \left\langle w_j\, Dg(z) D\Phi_i \right\rangle \right\}, \quad D_{ji} = \left\{ \left\langle T_j\, Dg(z) w_i \right\rangle \right\},$$

$$E_{ji} = -\left\{ \left\langle DT_j DT_i \right\rangle + a^2 \left\langle T_j T_i \right\rangle \right\}, \quad F_{ji} = \left\langle \Phi_j DT_i \right\rangle,$$

113

$$G_{ji} = \left\langle D\Phi_j \, D\Phi_i \right\rangle + M_3 \, a^2 \left\langle \Phi_j \Phi_i \right\rangle$$

$$+ \frac{a}{1+\chi_m} \left\{ \Phi_j\!\left(\frac{1}{2}\right)\Phi_i\!\left(\frac{1}{2}\right) + \Phi_j\!\left(\frac{-1}{2}\right)\Phi_i\!\left(\frac{-1}{2}\right) \right\}.$$

In the above use has been made of the inner product $\left\langle uv \right\rangle = \int\limits_{-1/2}^{1/2} uv \, dz$. We

now choose the following trial functions guided by the boundary conditions given in Section 4.2.1 and variational considerations.

(a) Free – free, isothermal

$$w_i = \left[\left(z^2 - \frac{1}{4}\right)^2 - \left(z^2 - \frac{1}{4}\right) \right] z^{i-1}, \quad T_i = \left(z^2 - \frac{1}{4}\right) z^{i-1}, \quad \Phi_i = z^{2i-1}$$

(b) Free – rigid, isothermal

$$w_i = \left(2z^2 - z - 1\right)\left(z^2 - \frac{1}{4}\right) z^{i-1}, \quad T_i = \left(z^2 - \frac{1}{4}\right) z^{i-1}, \quad \Phi_i = z^{2i-1}$$

(c) Rigid – rigid, isothermal

$$w_i = \left(z^2 - \frac{1}{4}\right)^2 z^{i-1}, \quad T_i = \left(z^2 - \frac{1}{4}\right) z^{i-1}, \quad \Phi_i = z^{2i-1}$$

From the experience gathered in the context of *RBC*, we understand that heat sink and heat source have identical effect on convection. This was, in fact, proved by Watson (1968) for free-free and rigid-rigid boundaries in the case of Newtonian, nonmagnetic liquids. In what follows, we investigate the applicability or otherwise of this observation to the *RBC* problem in ferromagnetic liquids.

4.3 HEAT SINK WITHIN THE FERROMAGNETIC LIQUID

In this section, we analyze in detail the influence of heat sink on the Rayleigh-Bénard instability of a variable-viscosity ferromagnetic liquid. It is worthwhile making a mention of the finding of Watson (1968), who studied the effects of both uniform internal heat source and sink on the onset of *RBC* in a constant viscosity nonmagnetic fluid. Considering two sets of boundary conditions corresponding to

two free and two rigid boundaries, he demonstrated that the two problems relating to heat source and sink are identical and that they lead to the same eigenvalue.

In order to perceive the influence of uniform heat sink associated with the problem at hand, we apply the transformations $N_S \to -N_S$ and $z \to -z$, as discussed by Watson (1968), to Eqs. (4.2.30) – (4.2.32) and to the boundary conditions given in Section 4.2.1. It can be seen that Eqs. (4.2.30) – (4.2.32) and the boundary conditions on velocity, temperature and magnetic potential are invariant under the aforementioned transformation except for the boundary conditions on velocity pertaining to the free-rigid case. Thus the effects of heat source and heat sink on the onset of Rayleigh-Bénard ferroconvection with variable viscosity are no different for the free-free and rigid-rigid boundaries as reported by Watson (1968). One might expect that, for free-rigid boundaries, the effect of heat sink would not be identical with the effect of heat source. However, as will be demonstrated later (see Chapter VII), the dissimilarity between the effect of heat source and heat sink vanishes for large values of $|N_S|$. The results on the effect of heat source (sink) are also true for the case of RBC in constant viscosity magnetic liquids, as can be seen from Eqs. (4.2.30) – (4.2.32) by taking $\Gamma = 0$. It is in place to note here that Sekhar (1990) incorrectly reported in the context of RBC for a constant viscosity ferromagnetic liquid that the effects of heat source and heat sink were not identical for all the three boundary combinations. We have verified that it was due to a computational error.

In the succeeding section we explore an interesting possibility of the existence of an analogy for the RBC problems between ferro liquids and dielectric liquids in the presence of volumetric heating.

4.4 ANALOGY FOR RAYLEIGH-BÉNARD CONVECTION BETWEEN FERROMAGNETIC AND DIELECTRIC LIQUIDS WITH INTERNAL HEAT SOURCE

The system of equations of electrohydrodynamics describing the *RBC* situation in a variable-viscosity dielectric liquid with volumetric heat source using the notation of Chapter II is

$$\nabla . \vec{q} = 0,$$
(4.4.1)

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} . \nabla \right) \vec{q} \right] = -\nabla p - \rho g \, \hat{k} + \left(\vec{P} . \nabla \right) \vec{E}$$
$$+ \nabla . \left[\mu(E,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right],$$
(4.4.2)

$$\rho_0 C_{VE} \left[\frac{\partial T}{\partial t} + \left(\vec{q} . \nabla \right) T \right] = k_1 \nabla^2 T + S,$$
(4.4.3)

$$\rho = \rho_0 \left[1 - \alpha(T - T_a) \right],$$
(4.4.4)

$$\nabla . \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0},$$
(4.4.5)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_0 \left(\varepsilon_r - 1 \right) \vec{E},$$
(4.4.6)

$$\varepsilon_r = (1 + \chi_e) - e(T - T_a),$$
(4.4.7)

where the effective viscosity $\mu(E,T)$ has been assumed to be a function of the temperature and magnitude of the *ac* electric field. We note that the assumed strength of the electric field is such that the dielectric liquid does not exhibit any non-Newtonian characteristics.

By analogy with Eq. (4.2.8), the equation for the variable-viscosity of a dielectric liquid is assumed to be

$$\mu(E,T) = \mu_1\left[1 + \delta_1(E - E_o)^2 - \delta_2(T - T_a)^2\right].$$ (4.4.8)

The electric boundary conditions specify that the normal component of dielectric field $\vec{D}$ and tangential components of electric field $\vec{E}$ are continuous across the boundaries. Taking the components of the electric field in the basic state to be $[0, 0, E_b(z)]$, one obtains the quiescent state solution in the form:

$$\left.\begin{aligned}
&\vec{q}_b = \vec{0}, \quad T_b(z) = T_a - f(z), \quad p = p_b(z), \\
&\mu_b(z) = \mu_1\left[1 - \delta\{f(z)\}^2\right], \quad \rho_b(z) = \rho_o[1 + \alpha f(z)], \\
&\vec{E}_b = \left[\frac{E_o(1 + \chi_e)}{(1 + \chi_e) + ef(z)}\right]\hat{k}, \quad \vec{P}_b = \varepsilon_o E_o(1 + \chi_e)\left[1 - \frac{1}{(1 + \chi_e) + ef(z)}\right]\hat{k},
\end{aligned}\right\}$$ (4.4.9)

where $\delta = \delta_2 - \delta_1\left[eE_o\big/(1 + \chi_e)\right]^2$. In obtaining Eq. (4.2.9), it has been assumed that $e\Delta T \ll (1 + \chi_e)$ and $eSd^2 < (1 + \chi_e)$.

The second of Eq. (4.4.6), upon application of linear analysis, yields

$$\left.\begin{aligned}
&P_i' = \varepsilon_o \chi_e E_i', \quad (i = 1, 2) \\
&P_3' = \varepsilon_o \chi_e E_3' - e\varepsilon_o E_o T'.
\end{aligned}\right\}$$ (4.4.10)

Following exactly the same procedure as in the development for a ferromagnetic liquid, introducing the perturbed electric potential Φ' through the relation $\vec{E}' = \nabla\Phi'$ and non-dimensionalizing the resulting equations using the scaling given in Chapter II, we arrive at the following system of dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left[g(z)\,D^2 g(z) + \{Dg(z)\}^2\right]\left(D^2 + a^2\right)w$$

$$+\,4\,\Gamma g(z)\,Dg(z)\left(D^2 - a^2\right)Dw - \left[1 - \Gamma\{g(z)\}^2\right]\left(D^2 - a^2\right)^2 w \quad (4.4.11)$$

$$+\,R\,a^2 T - R_E\,a^2 Dg(z)\left[D\Phi - T\right] = 0\,,$$

$$\frac{\partial T}{\partial t} = \left(D^2 - a^2\right)T + Dg(z)\,w, \tag{4.4.12}$$

$$(D^2 - a^2)\,\Phi - DT = 0\,. \tag{4.4.13}$$

Equations (4.4.11) – (4.4.13) are solved subject to the boundary conditions on velocity and temperature given in Section 4.2.1. The boundary conditions on the electric potential given in Eq. (2.2.20) relating to the isothermal boundaries become

$$\left.\begin{aligned} D\Phi + \frac{a\Phi}{1 + \chi_e} &= 0 \ \text{ at } \ z = \frac{1}{2}\,, \\[2mm] D\Phi - \frac{a\Phi}{1 + \chi_e} &= 0 \ \text{ at } \ z = -\frac{1}{2}\,. \end{aligned}\right\}$$

Comparing the set of Eqs. (4.2.20) – (4.2.22) and the set of Eqs. (4.4.11) – (4.4.13) together with their boundary conditions, the analogy between "Rayleigh-Bénard ferroconvection" and "Rayleigh-Bénard electroconvection" in the presence of volumetric heat source/sink can easily be recognized. It is clear that one can recover the equations of the latter from those of the former when one replaces $R\,M_1$ by R_E, M_3 by 1 and χ_m by χ_e. Thus it is quite explicit that the study of the problem of electroconvection of the type reported here can be construed from that of ferroconvection. In what follows, we study the effect of internal heat source (sink) on the onset of Marangoni-ferroconvection with variable viscosity.

4.5 MARANGONI-FERROCONVECTION WITH INTERNAL HEAT GENERATION

The system of equations describing the Marangoni instability situation in a thin, variable viscosity ferromagnetic liquid layer (with a free upper surface) with uniform heat source is

$$\nabla \cdot \vec{q} = 0, \tag{4.5.1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{q} \right] = -\nabla p + \mu_0 (\vec{M} \cdot \nabla) \vec{H} + \nabla \cdot \left[\mu(H,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right], \tag{4.5.2}$$

$$\rho_0 C_{VH} \left[\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla \right) T \right] = k_1 \nabla^2 T + S, \tag{4.5.3}$$

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = \vec{0}, \tag{4.5.4}$$

$$\vec{B} = \mu_0 \left(\vec{M} + \vec{H} \right), \tag{4.5.5}$$

$$M = M_0 + \chi_m \left(H - H_0 \right) - K_1 \left(T - T_a \right), \tag{4.5.6}$$

$$\mu(H,T) = \mu_1 \left[1 + \delta_1 \left(H - H_0 \right)^2 - \delta_2 \left(T - T_a \right)^2 \right]. \tag{4.5.7}$$

The interface at the upper boundary has a temperature and magnetic field strength dependent surface-tension

$$\sigma(H,T) = \sigma_0 + \sigma_H \left(H - H_0 \right) - \sigma_T \left(T - T_a \right). \tag{4.5.8}$$

Application of the linear stability analysis discussed in Section 4.2.1 yields the following dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left[g(z)\,D^2g(z) + \{Dg(z)\}^2\right]\left(D^2 + a^2\right)w$$

$$+ \ 4\,\Gamma g(z)\,Dg(z)\left(D^2 - a^2\right)Dw - \left[1 - \Gamma\{g(z)\}^2\right]\left(D^2 - a^2\right)^2 w \qquad (4.5.9)$$

$$- \ R_M\,a^2\,Dg(z)\left[D\Phi - T\right] = 0\,,$$

$$\frac{\partial T}{\partial t} = \left(D^2 - a^2\right)T + Dg(z)\,w, \qquad\qquad (4.5.10)$$

$$(D^2 - M_3\,a^2)\,\Phi - DT \ = \ 0. \qquad\qquad (4.5.11)$$

Absence of overstability has already been shown numerically for both the *MC* and combined *RBC* and *MC* problems of a nonmagnetic fluid with constant viscosity (Vidal and Acrivos, 1966; Takashima, 1970). Weilepp and Brand (1996) substantiated, under experimentally relevant conditions, that Marangoni instability is impossible in constant viscosity ferromagnetic liquids. Selak and Lebon (1993, 1997) excluded, considering a given range of parameters, the possibility of overstability for the combined *RBC* and *MC* problem of a nonmagnetic fluid with temperature-dependent viscosity. However, one can prove, using the procedure adopted in Section 4.2.2, that the *PES* holds good for a thin layer of fluid layer with variable viscosity without resorting to experimentally relevant conditions or to a numerical search. This is explained in what follows.

4.5.1 Oscillatory Instability

We analyze the possibility of the existence of the oscillatory instability by resorting to the Rayleigh – Ritz technique. We investigate the possibility of oscillatory instability by considering Eqs. (4.5.9) – (4.5.11) subject to the following boundary conditions

$$w = \left[1 - \Gamma\{g(z)\}^2\right]D^2 w + a^2 Ma\, T - a^2 Ma_H\, D\Phi \; = \; DT = 0$$

$$\text{and} \quad D\Phi + \frac{a\Phi}{1+\chi_m} - T = 0 \qquad \text{at} \quad z \; = \; \frac{1}{2},$$

$$w \; = \; Dw \; = \; T \; = D\Phi - \frac{a\Phi}{1+\chi_m} = 0 \qquad \text{at} \quad z \; = \; -\frac{1}{2}.$$

$$(4.5.12)$$

The quantities Γ, Ma and Ma_H appearing in the above equation have been defined in Chapter II. Eq. (4.5.12) signifies the use of rigid, thermally-conducting lower boundary and free, thermally-insulating non-deformable upper surface. The boundary conditions on the magnetic potential Φ are similar to those chosen by Finlayson (1970) except for the term T appearing in the boundary condition pertaining to the adiabatic upper surface that would not have been there had the upper surface been isothermal. In arriving at the boundary condition for w on the upper boundary, use has been made of the fact that the surface tension depends linearly on the temperature and strength of the magnetic field. In the numerical calculations pursued we assume $|Ma_H| \ll 1$.

Multiplying Eqs. (4.5.9) – (4.5.11) by w, T and Φ respectively, integrating with respect to z between the limits $z = -1/2$ and $z = 1/2$, taking $w(z,t) = A_1(t)\, w_1(z)$, $T(z,t) = B_1(t)\, T_1(z)$, $\Phi(z,t) = C_1(t)\, \Phi_1(z)$ (in which $w_1(z)$, $T_1(z)$ and $\Phi_1(z)$ are trial functions), and using the boundary conditions in Eq. (4.5.12) give rise to the following system of ordinary differential equations

$$\frac{Q_1}{Pr} \frac{dA_1}{dt} = (-Q_2)A_1 + \left[R_M\, Q_3 - Ma\, Q_4\right]B_1 + (-R_M\, Q_5)C_1, \qquad (4.5.13)$$

$$Q_6 \frac{dB_1}{dt} \; = \; Q_7\, A_1 + (-Q_8)\, B_1, \qquad (4.5.14)$$

$$0 = Q_9\, B_1 + Q_{10}\, C_1, \qquad (4.5.15)$$

where

$$Q_1 = \left\langle (Dw_1)^2 \right\rangle + a^2 \left\langle w_1^2 \right\rangle,$$

$$Q_2 = \left\langle (D^2 w_1)^2 \right\rangle - 2 a^2 \left\langle w_1 \left(1 - \Gamma \{ g(z) \}^2 \right) D^2 w_1 \right\rangle - \Gamma \left\langle w_1 \{ g(z) \}^2 D^4 w_1 \right\rangle$$

$$+ a^4 \left\langle w_1 \left(1 - \Gamma \{ g(z) \}^2 \right) w_1 \right\rangle - 2 \Gamma \left\langle w_1 \left[g(z) D^2 g(z) + \{ Dg(z) \}^2 \right] D^2 w_1 \right\rangle$$

$$- 2 \Gamma a^2 \left\langle w_1 \left[g(z) D^2 g(z) + \{ Dg(z) \}^2 \right] w_1 \right\rangle - 4 \Gamma \left\langle w_1 g(z) Dg(z) D^3 w_1 \right\rangle$$

$$+ 4 \Gamma a^2 \left\langle w_1 g(z) Dg(z) Dw_1 \right\rangle ,$$

$$Q_3 = a^2 \left\langle w_1 Dg(z) T_1 \right\rangle , \qquad Q_4 = \frac{4 a^2}{(4 - \Gamma)} Dw_1 (\tfrac{1}{2}) T_1 (\tfrac{1}{2}),$$

$$Q_5 = a^2 \left\langle w_1 Dg(z) D\Phi_1 \right\rangle , \qquad Q_6 = \left\langle T_1^2 \right\rangle , \qquad Q_7 = \left\langle T_1 Dg(z) w_1 \right\rangle ,$$

$$Q_8 = \left\langle (DT_1)^2 \right\rangle + a^2 \left\langle T_1^2 \right\rangle , \qquad Q_9 = \left\langle \Phi_1 DT_1 \right\rangle - \Phi_1 (\tfrac{1}{2}) T_1 (\tfrac{1}{2}),$$

$$Q_{10} = \left\langle (D\Phi_1)^2 \right\rangle + M_3 a^2 \left\langle \Phi_1^2 \right\rangle + \frac{a}{1 + \chi_m} \left[\left\{ \Phi_1 \left(\tfrac{1}{2} \right) \right\}^2 + \left\{ \Phi_1 \left(\tfrac{-1}{2} \right) \right\}^2 \right].$$

Eliminating C_1 between Eqs. (4.5.13) and (4.5.15), the resulting system of equations can be rearranged into the matrix form

$$\frac{dA}{dt} = F A , \qquad\qquad (4.5.16)$$

where

$$A = \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \quad \text{and} \quad F = \begin{pmatrix} -\dfrac{\text{Pr} \, Q_2}{Q_1} & \dfrac{\text{Pr}}{Q_1} \left[R_M \left\{ Q_3 + \dfrac{Q_5 Q_9}{Q_{10}} \right\} - Ma \, Q_4 \right] \\[3mm] \dfrac{Q_7}{Q_6} & -\dfrac{Q_8}{Q_6} \end{pmatrix} .$$

Following the procedure illustrated in Section 4.2.2, we arrive at the following expression for the thermal Marangoni number Ma

$$Ma = \frac{R_M}{Q_4} \left\{ Q_3 + \frac{Q_5 Q_9}{Q_{10}} \right\} - \frac{1}{Q_4 Q_7} \left\{ Q_2 Q_8 + \frac{Q_1 Q_6}{\text{Pr}} \omega^2 \right\} - i \omega N , \quad (4.5.17)$$

122

where

$$N = \frac{1}{Q_4 Q_7} \left\{ Q_2 Q_6 + \frac{Q_1 Q_8}{Pr} \right\}.$$

Since Ma is a real quantity and since N is independent of ω, we infer that oscillatory convection is not possible in the present problem. So we take $\omega = 0$ in the subsequent analysis. We now proceed to discuss the preferred mode of instability, *viz.*, stationary convection.

4.5.2 Stationary Instability

The system of equations associated with the stationary instability reads as

$$\left[1 - \Gamma \{g(z)\}^2 \right] (D^2 - a^2)^2 w - 2\, \Gamma \left[g(z) D^2 g(z) + \{Dg(z)\}^2 \right] (D^2 + a^2) w$$

$$- 4\, \Gamma g(z) Dg(z) (D^2 - a^2) Dw + R_M\, a^2 Dg(z) [D\Phi - T] = 0, \qquad (4.5.18)$$

$$(D^2 - a^2)\, T + Dg(z) w = 0, \qquad (4.5.19)$$

$$\left(D^2 - M_3 a^2 \right) \Phi - DT = 0. \qquad (4.5.20)$$

The system of Eqs. (4.5.18) – (4.5.20) together with the boundary conditions in Eq. (4.5.12) poses an eigenvalue problem for Ma with Γ, N_S, R_M, M_3 and χ_m as parameters. We employ the Higher Order Rayleigh-Ritz Technique (*HORT*) (Finlayson, 1972) to obtain the eigenvalue and the associated wave number. To this end, we expand $w(z)$, $T(z)$ and $\Phi(z)$ in a series of trial functions

$$w(z) = \sum_{i=1}^{n} \alpha_i w_i(z), \;\; T(z) = \sum_{i=1}^{n} \beta_i T_i(z) \;\; \text{and} \;\; \Phi(z) = \sum_{i=1}^{n} \gamma_i \Phi_i(z), \qquad (4.5.21)$$

where α_i, β_i and γ_i are constants, and $w_i(z)$, $T_i(z)$ and $\Phi_i(z)$ are trial functions. We choose the following trial functions

$$w_i = \left(z - \frac{1}{2} \right)\left(z + \frac{1}{2} \right)^{i+1}, \quad T_i = \left(z(z-1) - \frac{3}{4} \right)^{i+1}, \quad \Phi_i = z^i$$

guided by the boundary conditions in Eq. (4.5.12) and variational considerations. Applying *HORT* to Eqs. (4.5.18) – (4.5.20) leads to a system of homogeneous equations using which we may get the critical eigenvalue Ma_c and the corresponding critical wavenumber a_c.

In Section 4.3 we have clarified that the effects of both uniform internal heat source and heat sink on the threshold of *RBC* in ferromagnetic liquids are identical. In what follows, we investigate the applicability or otherwise of this observation to the *MC* problem in ferromagnetic liquids.

4.5.3 Heat Sink within the Ferromagnetic Liquid

It is interesting to note that, unlike the *RBC* problem where the eigenvalue (thermal Rayleigh number) appears in the differential equations, the eigenvalue (thermal Marangoni number) of the *MC* problem emanates from the boundary condition pertaining to the free upper surface.

In order to perceive the influence of uniform heat sink associated with the problem at hand, we apply the transformations $N_S \rightarrow -N_S$ and $z \rightarrow -z$, as discussed by Watson (1968), to Eqs. (4.5.18) – (4.5.20) and to the boundary conditions in Eq. (4.5.12). It can be seen that, the boundary conditions do not turn out to be identical even though Eqs. (4.5.18) – (4.5.20) are tacitly invariant under the aforementioned transformation. In deed, the above analysis elucidates the fact that buoyancy and surface-tension driven instability problems of a variable viscosity ferromagnetic liquid should not be affected the same way by the introduction of heat sink and that the effect of heat sink is to stabilize (in contrast to the destabilizing effect of heat source) the variable viscosity ferro liquid for the case of *MC*. The above result on the effect of heat source (sink) is also true for the case of *MC* in constant viscosity magnetic and nonmagnetic liquids, as can be seen from Eqs. (4.5.12) and (4.5.18) – (4.5.20) by taking $\Gamma = 0$ and $R_M = \Gamma = 0$ respectively. A more enlightening discussion on the dissimilarity between the heat

source and heat sink problems for *MC* will be deferred until Chapter VII. It is interesting and intriguing to note that several investigators (Lam and Bayazitoglu, 1987; Char and Chiang, 1994; Wilson, 1997) paid attention only to the effect of uniform heat source on the onset of *MC* in nonmagnetic liquids. Implicitly, they perhaps meant that the heat source and heat sink problems are similar!

In what follows we discuss an analogy between the Marangoni problems in variable-viscosity ferro liquids and dielectric liquids in the presence of internal heat source.

4.6 ANALOGY FOR MARANGONI CONVECTION BETWEEN FERROMAGNETIC AND DIELECTRIC LIQUIDS WITH INTERNAL HEAT SOURCE

The system of equations of electrohydrodynamics describing the Marangoni instability situation in a variable-viscosity dielectric liquid with volumetric heat source (sink) reads as

$$\nabla . \vec{q} = 0, \tag{4.6.1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla \right) \vec{q} \right] = -\nabla p + \left(\vec{P}.\nabla \right) \vec{E} + \nabla . \left[\mu(E,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right], \tag{4.6.2}$$

$$\rho_0 C_{VE} \left[\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla \right) T \right] = k_1 \nabla^2 T + S, \tag{4.6.3}$$

$$\nabla . \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0}, \tag{4.6.4}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_0 \left(\varepsilon_r - 1 \right) \vec{E}, \tag{4.6.5}$$

$$\varepsilon_r = (1 + \chi_e) - e(T - T_a), \tag{4.6.6}$$

$$\mu(E,T) = \mu_1 \left[1 + \delta_1 (E - E_0)^2 - \delta_2 (T - T_a)^2 \right]. \tag{4.6.7}$$

The interface at the upper boundary has a temperature and electric field strength dependent surface-tension given by

$$\sigma(E,T) = \sigma_o + \sigma_E(E - E_o) - \sigma_T(T - T_a). \tag{4.6.8}$$

Application of linear stability analysis discussed in a previous section yields the following dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\Gamma\left[g(z)D^2g(z) + \{Dg(z)\}^2\right]\left(D^2 + a^2\right)w$$

$$+ 4\Gamma g(z)Dg(z)\left(D^2 - a^2\right)Dw - \left[1 - \Gamma\{g(z)\}^2\right]\left(D^2 - a^2\right)^2 w \tag{4.6.9}$$

$$- R_E a^2 Dg(z)\left[D\Phi - T\right] = 0,$$

$$\frac{\partial T}{\partial t} = \left(D^2 - a^2\right)T + Dg(z)w, \tag{4.6.10}$$

$$(D^2 - a^2)\Phi - DT = 0. \tag{4.6.11}$$

Equations (4.6.9) – (4.6.11) are solved subject to the boundary conditions

$$\left.\begin{array}{l} w = \left[1 - \Gamma\{g(z)\}^2\right]D^2w + a^2 Ma\,T - a^2 Ma_E\,D\Phi = DT = 0 \\[2mm] \qquad \text{and} \qquad D\Phi + \dfrac{a\Phi}{1+\chi_e} - T = 0 \qquad \text{at } z = +\dfrac{1}{2}, \\[4mm] w = Dw = T = D\Phi - \dfrac{a\Phi}{1+\chi_e} = 0 \qquad \text{at } z = -\dfrac{1}{2}. \end{array}\right\} \tag{4.6.12}$$

The analogy between "Marangoni-ferroconvection" and "Marangoni-electroconvection" in the presence of volumetric heat source/sink can easily be recognized if we compare the set of Eqs. (4.5.9) – (4.5.12) and the set of Eqs. (4.6.9) – (4.6.12). It is apparent that one can recover the equations of the latter from those of the former when one replaces R_M by R_E, Ma_H by Ma_E, M_3 by 1 and χ_m by χ_e. In view of the above analogy, there is no need to study the electroconvection problem in isolation.
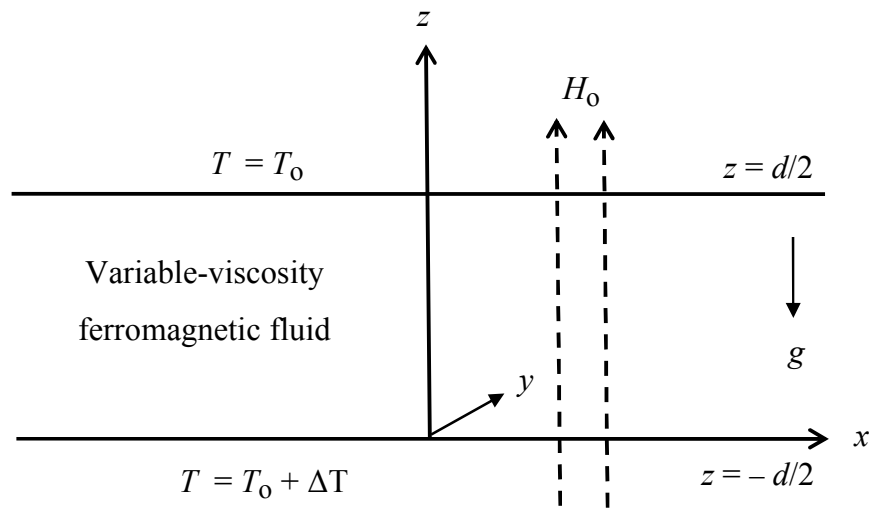
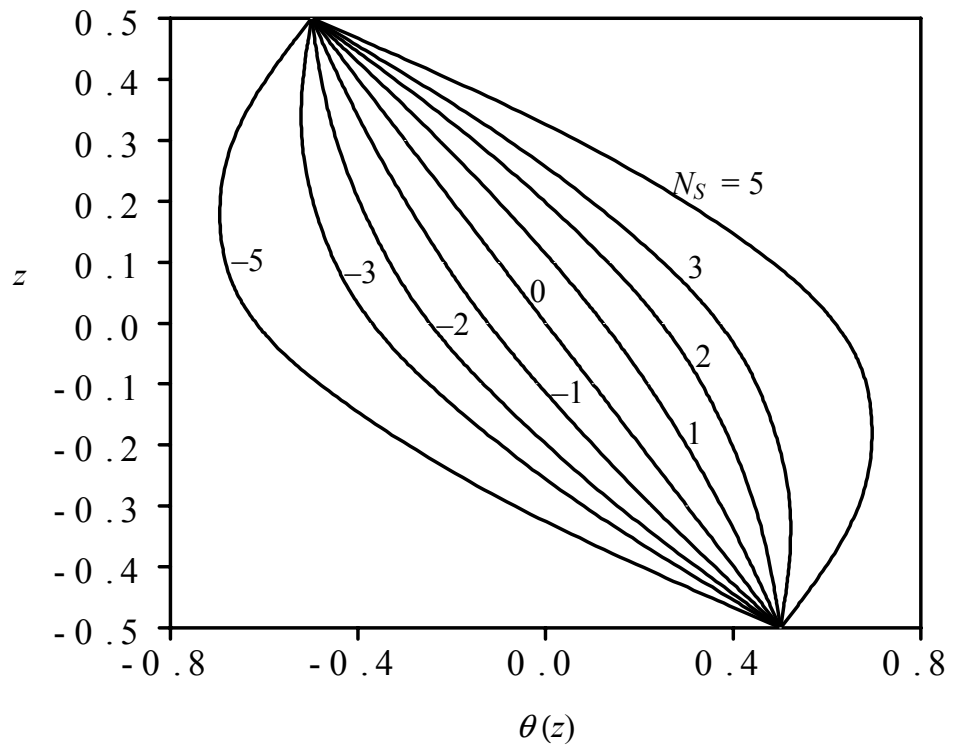**Figure 4.1:** Configuration of the problem.

Figure 4.2: Plot of dimensionless, basic state temperature profile $\theta(z)$ for different values of heat source/sink parameter N_S.
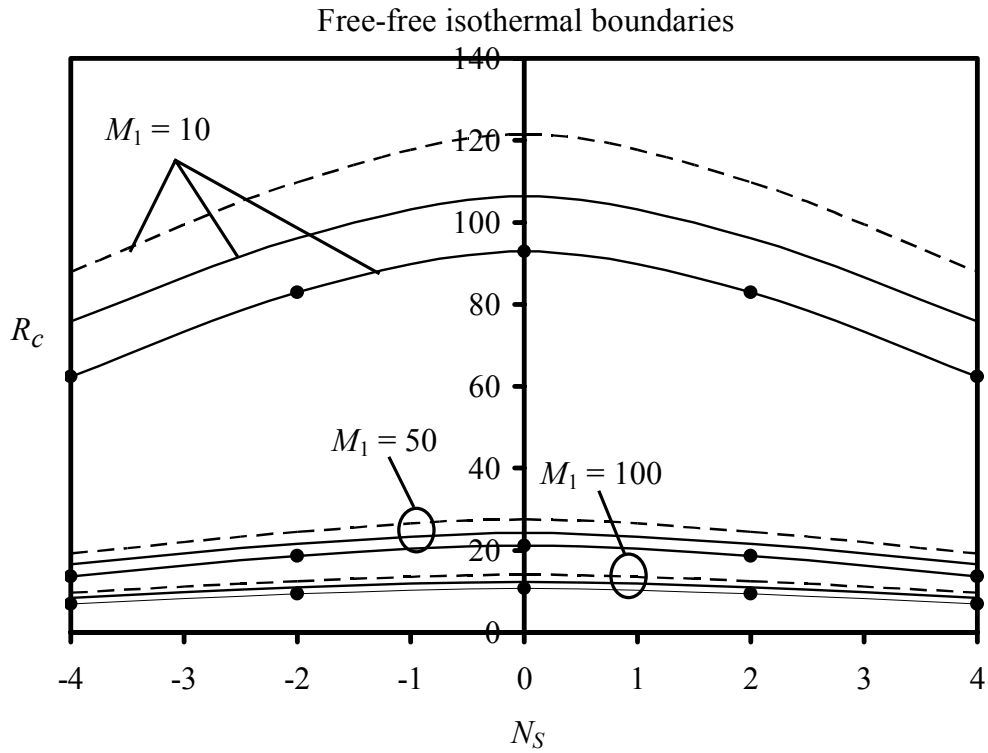
Figure 4.3: Plot of critical thermal Rayleigh number R_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of buoyancy-magnetization parameter M_1 and effective viscosity parameter Γ.

$- - - -\ \Gamma = -1\ ;\ \underline{\hspace{1.5cm}}\ \Gamma = 0\ ;\ \underline{\hspace{0.7cm}}\bullet\underline{\hspace{0.7cm}}\ \Gamma = 1.$

Figure 4.4: Plot of R_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of M_1 and Γ.

$---- \;\; \Gamma = -1 \;\; ; \;\; \underline{\hspace{1.5cm}} \;\; \Gamma = 0 \;\; ; \;\; \underline{\hspace{1cm}}\bullet\underline{\hspace{1cm}} \;\; \Gamma = 1.$
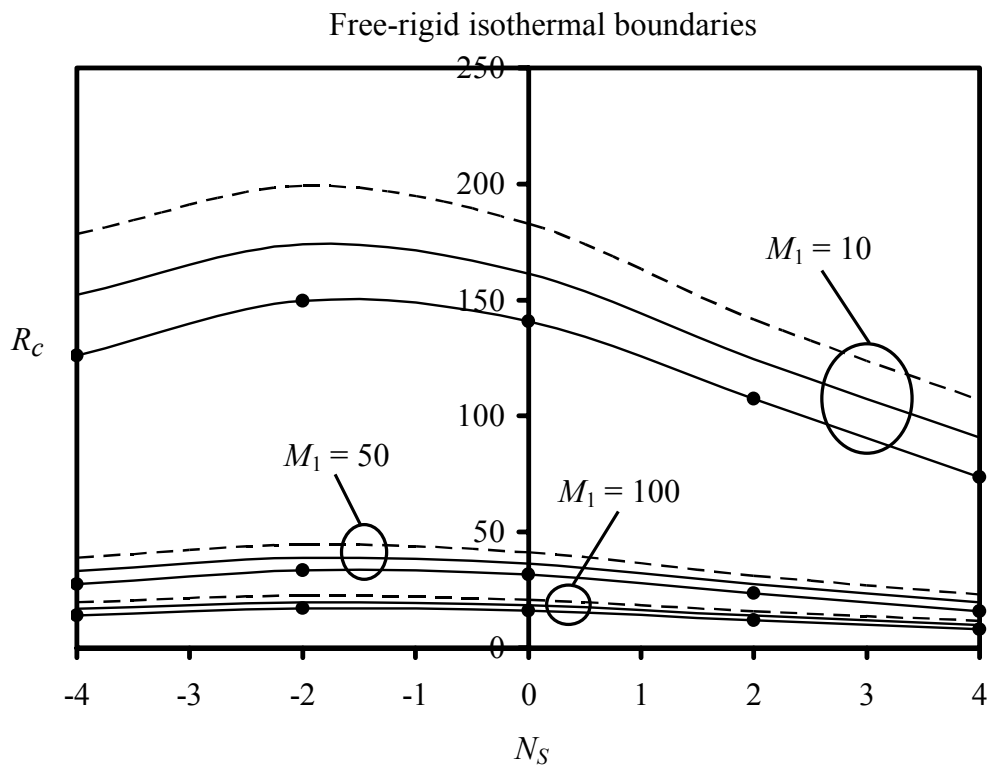
Rigid-rigid isothermal boundaries

Figure 4.5: Plot of R_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of M_1 and Γ.

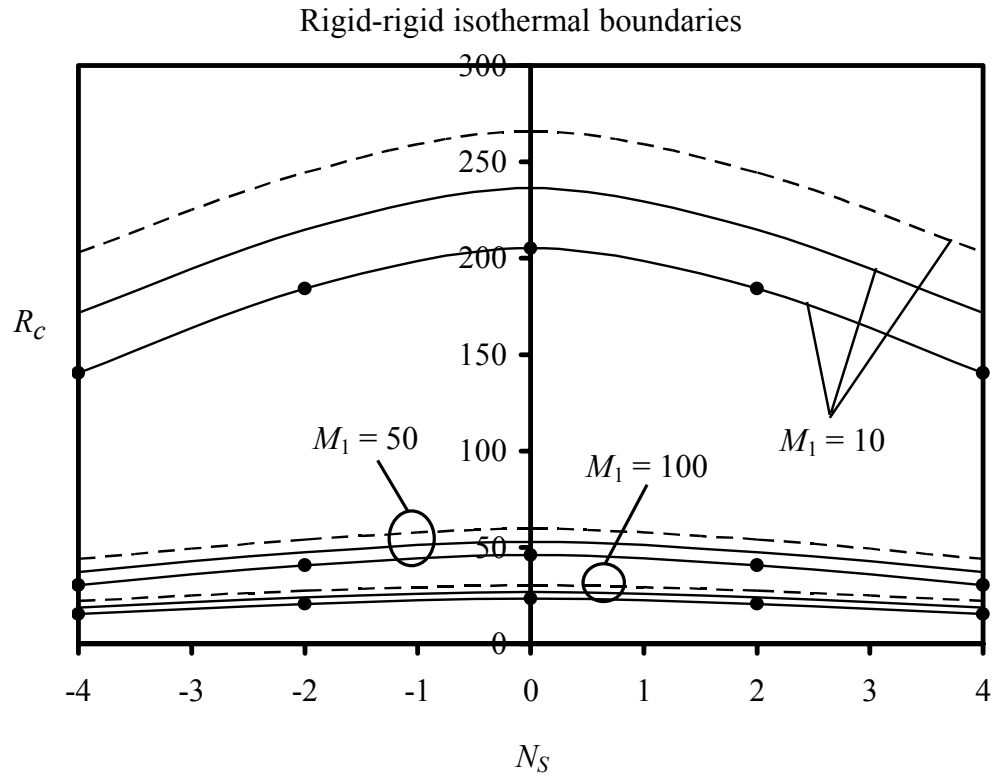$---- \Gamma = -1$; ———— $\Gamma = 0$; ——•—— $\Gamma = 1$.

Figure 4.6: Plot of critical wavenumber a_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of M_1 and Γ.

$\Gamma = -1$; $\Gamma = 0$; $\Gamma = 1$.

Free-rigid isothermal boundaries

Figure 4.7: Plot of critical wavenumber a_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of M_1 and Γ.

$---- \quad \Gamma = -1 \ ; \quad \underline{\hspace{1cm}} \quad \Gamma = 0 \ ; \quad \underline{\hspace{0.6cm}}\bullet\underline{\hspace{0.6cm}} \quad \Gamma = 1.$
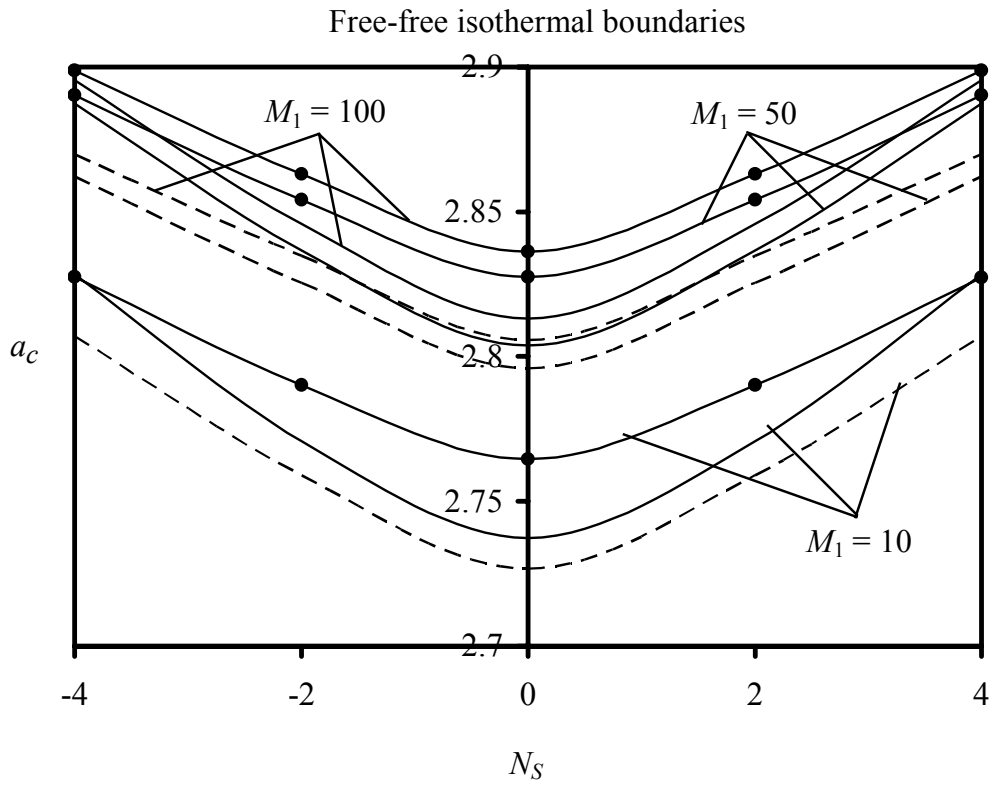
Figure 4.8: Plot of critical wavenumber a_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of M_1 and Γ.

$---- \; \Gamma = -1 \; ; \; \underline{\qquad} \; \Gamma = 0 \; ; \; \underline{\;\bullet\;} \; \Gamma = 1.$

Figure 4.9: Plot of critical Marangoni number Ma_c versus N_S for $M_3 = 1$, $\chi_m = 1$, and for different values of magnetic Rayleigh number R_M and effective viscosity parameter Γ.

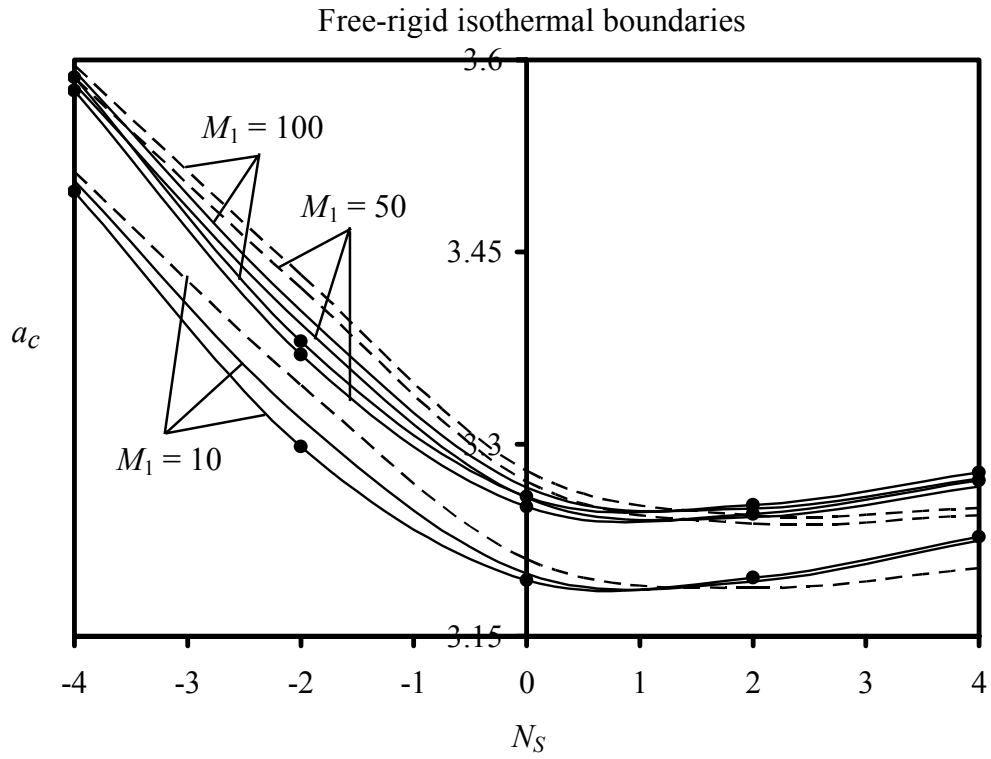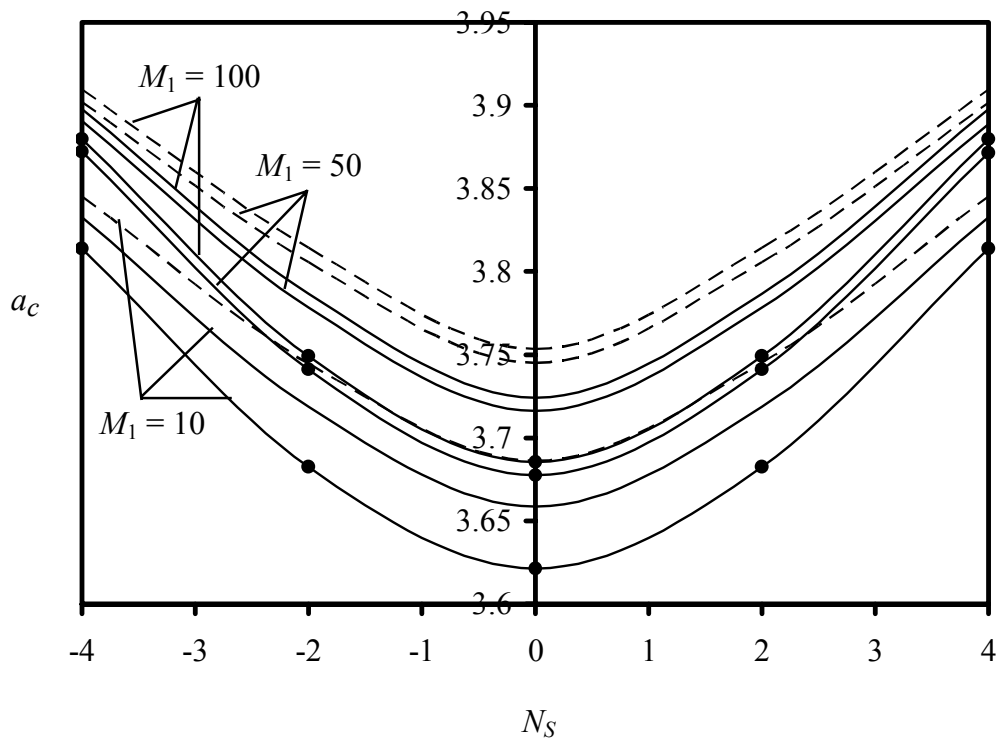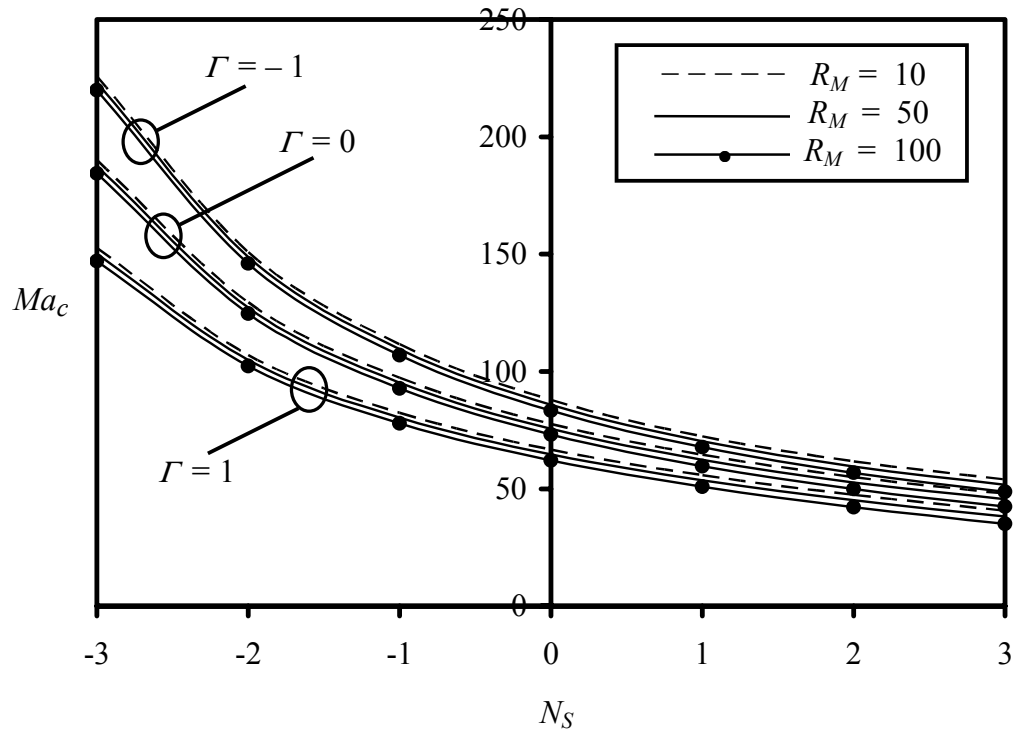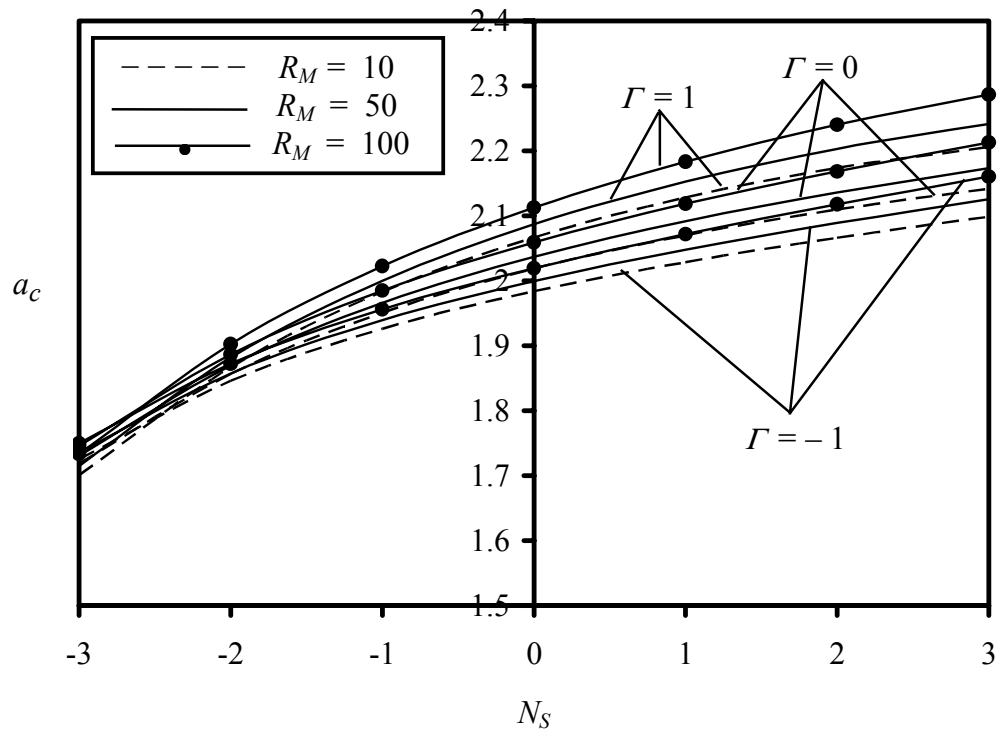**Figure 4.10**: Plot of critical wavenumber a_c versus N_S for $M_3 = 1$, $\chi_m = 1$ and for different values of R_M and Γ.

Table 4.1 : Critical thermal Rayleigh number (R_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with internal heat source/sink when the layer is bounded by free-free boundaries and $M_1 = 10$.

N_S	χ_m	M_3	$\Gamma = -2$		$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$		$\Gamma = 2$	
			R_c	a_c	R_c	a_c	R_c	a_c	R_c	a_c	R_c	a_c
-4	1	1	98.048	2.764	87.838	2.807	75.916	2.828	62.429	2.827	47.688	2.806
		5	76.774	2.566	68.495	2.601	58.853	2.614	48.029	2.605	36.355	2.574
		25	67.256	2.381	60.014	2.413	51.476	2.422	41.852	2.406	31.491	2.362
	5	1	107.399	2.892	95.901	2.933	82.699	2.954	67.928	2.955	51.889	2.938
		5	78.000	2.596	69.503	2.629	59.664	2.641	48.664	2.632	36.827	2.601
		25	67.355	2.384	60.091	2.416	51.535	2.425	41.896	2.408	31.521	2.364
-2	1	1	123.478	2.750	109.721	2.759	96.173	2.771	82.979	2.790	70.286	2.823
		5	91.358	2.527	80.778	2.537	70.524	2.552	60.694	2.576	51.382	2.616
		25	77.908	2.317	68.826	2.331	60.085	2.351	51.773	2.382	43.962	2.431
	5	1	137.311	2.905	121.796	2.908	106.556	2.913	91.734	2.925	77.483	2.949
		5	92.942	2.559	82.107	2.568	71.622	2.581	61.583	2.603	52.081	2.640
		25	78.025	2.320	68.919	2.334	60.158	2.353	51.828	2.385	44.003	2.433
0	1	1	138.192	2.732	121.504	2.727	106.465	2.737	92.907	2.765	80.651	2.809
		5	98.855	2.504	86.645	2.506	75.821	2.523	66.209	2.558	57.629	2.612
		25	83.080	2.286	72.781	2.294	63.783	2.320	55.893	2.367	48.916	2.433
	5	1	155.157	2.897	136.236	2.884	119.111	2.886	103.622	2.902	89.595	2.936
		5	100.636	2.539	88.148	2.538	77.069	2.553	67.227	2.586	58.443	2.636
		25	83.206	2.289	72.882	2.297	63.863	2.323	55.954	2.369	48.961	2.435
2	1	1	123.478	2.750	109.721	2.759	96.173	2.771	82.979	2.790	70.286	2.823
		5	91.358	2.527	80.778	2.537	70.524	2.552	60.694	2.576	51.382	2.616
		25	77.908	2.317	68.826	2.331	60.085	2.351	51.773	2.382	43.962	2.431
	5	1	137.311	2.905	121.796	2.908	106.556	2.913	91.734	2.925	77.483	2.949
		5	92.942	2.559	82.107	2.568	71.622	2.581	61.583	2.603	52.081	2.640
		25	78.025	2.320	68.919	2.334	60.158	2.353	51.828	2.385	44.003	2.433
4	1	1	98.048	2.764	87.838	2.807	75.916	2.828	62.429	2.827	47.688	2.806
		5	76.774	2.566	68.495	2.601	58.853	2.614	48.029	2.605	36.355	2.574
		25	67.256	2.381	60.014	2.413	51.476	2.422	41.852	2.406	31.491	2.362
	5	1	107.399	2.892	95.901	2.933	82.699	2.954	67.928	2.955	51.889	2.938
		5	78.000	2.596	69.503	2.629	59.664	2.641	48.664	2.632	36.827	2.601
		25	67.355	2.384	60.091	2.416	51.535	2.425	41.896	2.408	31.521	2.364

Table 4.2 : Critical thermal Rayleigh number (R_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with internal heat source/sink when the layer is bounded by free-rigid boundaries and $M_1 = 10$.

N_S	χ_m	M_3	$\Gamma = -2$		$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$		$\Gamma = 2$	
			R_c	a_c	R_c	a_c	R_c	a_c	R_c	a_c	R_c	a_c
−4	1	1	204.111	3.516	178.379	3.512	152.359	3.506	135.122	4.339	99.003	3.489
		5	162.048	3.231	141.312	3.221	120.354	3.208	99.104	3.189	77.529	3.165
		25	146.474	3.043	127.566	3.029	108.456	3.009	89.089	2.981	69.461	2.942
	5	1	215.269	3.632	240.114	5.369	160.693	3.624	169.690	5.368	104.434	3.616
		5	162.968	3.248	142.111	3.238	121.032	3.224	99.662	3.206	77.966	3.183
		25	146.506	3.044	127.594	3.029	108.479	3.010	89.108	2.982	69.476	2.943
−2	1	1	225.087	3.374	199.280	3.346	174.058	3.319	149.578	3.298	126.011	3.285
		5	169.534	3.085	149.598	3.061	130.258	3.041	111.650	3.027	93.910	3.024
		25	149.659	2.881	131.807	2.859	114.576	2.842	98.092	2.832	82.479	2.833
	5	1	241.109	3.515	213.652	3.484	186.756	3.455	160.582	3.429	135.314	3.411
		5	170.833	3.106	150.754	3.082	131.270	3.062	112.515	3.047	94.629	3.043
		25	149.716	2.882	131.856	2.860	114.618	2.843	98.127	2.833	82.508	2.835
0	1	1	205.394	3.228	182.748	3.210	161.242	3.199	140.778	3.194	121.191	3.193
		5	153.424	2.971	136.085	2.959	119.757	2.954	104.338	2.956	89.668	2.965
		25	134.556	2.772	119.187	2.764	104.804	2.764	91.290	2.773	78.479	2.790
	5	1	222.858	3.374	198.319	3.351	174.969	3.333	152.722	3.322	131.421	3.315
		5	154.991	2.996	137.456	2.983	120.939	2.977	105.342	2.978	90.505	2.985
		25	134.644	2.774	119.263	2.766	104.866	2.767	91.341	2.775	78.519	2.792
2	1	1	158.988	3.182	141.637	3.187	124.482	3.193	107.434	3.195	90.331	3.192
		5	123.885	2.943	109.916	2.951	96.213	2.959	82.681	2.966	69.161	2.970
		25	110.127	2.756	97.624	2.767	85.387	2.779	73.318	2.791	61.256	2.801
	5	1	172.456	3.320	153.444	3.320	134.710	3.320	116.159	3.317	97.623	3.308
		5	125.369	2.970	111.172	2.976	97.261	2.982	83.541	2.988	69.851	2.991
		25	110.233	2.759	97.709	2.769	85.456	2.781	73.371	2.793	61.297	2.803
4	1	1	121.359	3.167	106.839	3.203	90.884	3.224	73.534	3.227	54.799	3.200
		5	98.676	2.952	86.564	2.984	73.308	3.002	58.968	3.003	43.572	2.976
		25	89.108	2.783	78.183	2.815	66.169	2.834	53.135	2.835	39.115	2.805
	5	1	131.077	3.293	115.099	3.325	97.714	3.343	78.952	3.343	58.825	3.313
		5	99.941	2.979	87.592	3.009	74.121	3.026	59.585	3.026	44.010	2.997
		25	89.213	2.786	78.264	2.818	66.230	2.836	53.179	2.837	39.144	2.807

Table 4.3 : Critical thermal Rayleigh number (R_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with internal heat source/sink when the layer is bounded by rigid-rigid boundaries and $M_1 = 10$.

N_S	χ_m	M_3	$\Gamma = -2$ R_c	a_c	$\Gamma = -1$ R_c	a_c	$\Gamma = 0$ R_c	a_c	$\Gamma = 1$ R_c	a_c	$\Gamma = 2$ R_c	a_c
−4	1	1	233.754	3.853	202.884	3.845	171.803	3.833	140.359	3.814	108.169	3.779
		5	191.106	3.558	165.606	3.550	139.925	3.539	113.929	3.519	87.281	3.482
		25	175.923	3.381	152.347	3.373	128.596	3.360	104.542	3.339	79.852	3.297
	5	1	245.007	3.966	212.672	3.957	180.118	3.945	147.186	3.926	113.484	3.891
		5	192.069	3.574	166.437	3.566	140.624	3.554	114.496	3.535	87.713	3.498
		25	175.965	3.382	152.383	3.374	128.626	3.361	104.566	3.340	79.869	3.298
−2	1	1	273.905	3.765	244.523	3.745	214.678	3.719	184.062	3.683	151.993	3.628
		5	214.066	3.459	190.874	3.445	167.302	3.426	143.096	3.399	117.696	3.357
		25	193.629	3.268	172.545	3.256	151.108	3.241	129.079	3.218	105.937	3.180
	5	1	288.759	3.889	257.992	3.867	226.746	3.839	194.702	3.801	161.156	3.744
		5	215.163	3.476	191.871	3.461	168.199	3.442	143.890	3.415	118.386	3.373
		25	193.670	3.269	172.583	3.257	151.142	3.242	129.110	3.219	105.964	3.181
0	1	1	294.625	3.707	266.055	3.686	236.401	3.659	205.183	3.621	171.579	3.567
		5	224.474	3.408	202.577	3.394	179.838	3.375	155.881	3.348	130.060	3.308
		25	201.147	3.215	181.454	3.204	160.994	3.189	139.422	3.168	116.148	3.134
	5	1	311.739	3.835	281.840	3.812	250.803	3.782	218.125	3.742	182.937	3.684
		5	225.636	3.425	203.658	3.410	180.836	3.391	156.792	3.365	130.876	3.325
		25	201.187	3.215	181.492	3.205	161.029	3.190	139.456	3.169	116.179	3.135
2	1	1	273.905	3.765	244.523	3.745	214.678	3.719	184.062	3.683	151.993	3.628
		5	214.066	3.459	190.874	3.445	167.302	3.426	143.096	3.399	117.696	3.357
		25	193.629	3.268	172.545	3.256	151.108	3.241	129.079	3.218	105.937	3.180
	5	1	288.759	3.889	257.992	3.867	226.746	3.839	194.702	3.801	161.156	3.744
		5	215.163	3.476	191.871	3.461	168.199	3.442	143.890	3.415	118.386	3.373
		25	193.670	3.269	172.583	3.257	151.142	3.242	129.110	3.219	105.964	3.181
4	1	1	233.754	3.853	202.884	3.845	171.803	3.833	140.359	3.814	108.169	3.779
		5	191.106	3.558	165.606	3.550	139.925	3.539	113.929	3.519	87.281	3.482
		25	175.923	3.381	152.347	3.373	128.596	3.360	104.542	3.339	79.852	3.297
	5	1	245.007	3.966	212.672	3.957	180.118	3.945	147.186	3.926	113.484	3.891
		5	192.069	3.574	166.437	3.566	140.624	3.554	114.496	3.535	87.713	3.498
		25	175.965	3.382	152.383	3.374	128.626	3.361	104.566	3.340	79.869	3.298

Table 4.4: Critical Marangoni number (Ma_c) and critical wavenumber (a_c) for a variable-viscosity ferromagnetic liquid with internal heat source/sink and $R_M = 10$.

N_S	χ_m	M_3	$\Gamma = -2$		$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$		$\Gamma = 2$	
			Ma_c	a_c	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
-4	1	1	469.928	1.548	407.082	1.532	340.560	1.503	267.896	1.447	175.693	1.309
		5	469.161	1.549	406.331	1.533	339.833	1.504	267.213	1.448	175.137	1.311
		25	468.376	1.549	405.552	1.533	339.062	1.504	266.455	1.448	174.441	1.311
	5	1	470.187	1.548	407.338	1.532	340.811	1.503	268.136	1.447	175.896	1.309
		5	469.245	1.549	406.416	1.533	339.919	1.504	267.299	1.448	175.219	1.311
		25	468.381	1.549	405.558	1.533	339.068	1.504	266.462	1.448	174.448	1.311
-2	1	1	171.437	1.833	150.945	1.846	129.410	1.857	106.909	1.867	83.433	1.878
		5	171.094	1.834	150.613	1.846	129.089	1.857	106.599	1.867	83.134	1.878
		25	170.810	1.834	150.340	1.845	128.827	1.856	106.347	1.866	82.892	1.877
	5	1	171.561	1.833	151.065	1.846	129.527	1.857	107.024	1.867	83.544	1.878
		5	171.129	1.834	150.647	1.846	129.122	1.857	106.632	1.867	83.166	1.878
		25	170.813	1.834	150.343	1.845	128.829	1.856	106.349	1.866	82.895	1.877
0	1	1	98.033	1.958	87.892	1.984	77.664	2.019	66.726	2.067	53.828	2.132
		5	97.760	1.958	87.622	1.984	77.397	2.019	66.463	2.067	53.579	2.132
		25	97.540	1.956	87.407	1.981	77.188	2.016	66.264	2.063	53.398	2.128
	5	1	98.152	1.958	88.010	1.984	77.780	2.019	66.839	2.067	53.934	2.132
		5	97.798	1.958	87.659	1.984	77.433	2.019	66.497	2.067	53.610	2.132
		25	97.546	1.956	87.412	1.981	77.193	2.016	66.268	2.063	53.402	2.128
2	1	1	68.257	2.041	61.733	2.066	55.022	2.110	47.373	2.174	37.531	2.257
		5	67.999	2.040	61.467	2.065	54.751	2.110	47.104	2.174	37.281	2.255
		25	67.787	2.037	61.250	2.061	54.536	2.105	46.899	2.168	37.099	2.248
	5	1	68.388	2.041	61.868	2.066	55.158	2.110	47.505	2.174	37.650	2.259
		5	68.045	2.040	61.512	2.066	54.795	2.110	47.146	2.174	37.317	2.256
		25	67.795	2.037	61.258	2.061	54.543	2.105	46.906	2.168	37.104	2.248
4	1	1	53.122	2.104	48.313	2.128	42.402	2.168	35.042	2.225	25.709	2.288
		5	52.866	2.103	48.034	2.127	42.107	2.166	34.744	2.222	25.429	2.283
		25	52.646	2.097	47.801	2.121	41.869	2.160	34.512	2.213	25.219	2.272
	5	1	53.268	2.104	48.468	2.128	42.561	2.168	35.199	2.225	25.852	2.291
		5	52.919	2.103	48.089	2.127	42.162	2.166	34.796	2.222	25.474	2.286
		25	52.656	2.097	47.811	2.121	41.879	2.160	34.522	2.213	25.227	2.272

Table 4.5: Comparison of critical values obtained from the present study with those obtained by Chandrasekhar (1961) for the limiting case of a constant-viscosity, nonmagnetic liquid with no internal heat source / sink.

Order of Rayleigh-Ritz technique	Free - free boundaries		Free - rigid boundaries		Rigid - rigid boundaries	
	R_c	a_c	R_c	a_c	R_c	a_c
1	664.52	2.227	1138.70	2.669	1749.97	3.117
2	664.52	2.227	1116.87	2.686	1749.97	3.117
3	657.64	2.221	1101.37	2.683	1708.55	3.116
4	657.64	2.221	1100.84	2.682	1708.55	3.116
5	657.51	2.221	1100.66	2.682	1707.76	3.116
6	**657.51**	**2.221**	**1100.66**	**2.682**	**1707.76**	**3.116**
Chandrasekhar (1961)	**657.51**	**2.221**	**1100.66**	**2.682**	**1707.76**	**3.117**

Table 4.6: Comparison of the present results with those of earlier works for the limiting case of a constant-viscosity, nonmagnetic liquid with no internal heat source/sink.

Order of Rayleigh - Ritz technique	Present study		Pearson (1958) [Exact solution]		Nield (1964) [Fourier Series]		Siddheshwar and Pranesh (2002) [Single term Rayleigh – Ritz technique]	
	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
1	64.59	2.34						
2	79.06	2.09						
3	79.70	2.00	**80.00**	**2.00**	**79.61**	**1.99**	**78.44**	**2.43**
4	79.99	2.00						
5	**79.99**	**2.00**						

Table 4.7: Critical values corresponding to a constant-viscosity, nonmagnetic liquid with uniform internal heat source.

N_S	Lam and Bayazitoglu (1987) (Sequential gradient restoration algorithm)		Char and Chiang (1994) (Shooting technique)		Present study (*HORT*)	
	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
0	79.61	1.99	79.61	1.99	79.99	2.00
1	56.91	2.08	56.91	2.08	56.71	2.09
2	44.21	2.13	--	--	43.85	2.14
3	36.12	2.16	--	--	35.69	2.18
4	30.52	2.19	--	--	30.12	2.21
5	26.42	2.21	26.43	2.21	26.02	2.23

Note: The N_S in the above table for the present study is taken to be $2N_S$ to match with the definition of N_S of the other two.

Table 4.8: Critical values corresponding to a nonmagnetic liquid with linear viscosity (temperature-dependent) variation in the absence of internal heat source.

Γ_L	Free - free boundaries		Free - rigid boundaries		Rigid - rigid boundaries	
	R_c	a_c	R_c	a_c	R_c	a_c
0.5	661.79	2.232	1081.86	2.654	1686.25	3.113
	(616.34)	(2.246)	(1029.38)	(2.688)	(1595.68)	(3.108)
1	673.83	2.261	1028.39	2.614	1614.85	3.099
	(577.71)	(2.275)	(959.54)	(2.695)	(1479.21)	(3.097)
1.5	692.64	2.309	919.14	2.539	1478.23	3.068
	(541.45)	(2.309)	(891.61)	(2.705)	(1358.27)	(3.083)
2	716.60	2.370	720.56	2.358	1238.43	2.986
	(507.29)	(2.350)	(825.25)	(2.716)	(1231.64)	(3.066)

Note: The values in parenthesis are those obtained by using the quadratic viscosity law.

Table 4.9: Critical values corresponding to a nonmagnetic liquid with linear viscosity (temperature-dependent) variation for a non-deformable upper surface in the absence of internal heat source.

Γ_L	Lam and Bayazitoglu (1987) [$\Gamma_L = 0$ in their upper boundary condition: see Eq. (7.2)]		Present study [with $\Gamma_L = 0$ in Eq. (7.2)]		Present study [with $\Gamma_L \neq 0$ in Eq. (7.2)]	
	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
0.1	77.06 (77.06)	1.98 (1.99)	77.34	2.01	81.21	2.03
0.3	72.98 (72.97)	1.97 (1.95)	72.54	1.99	83.42	1.99
0.5	69.85 (69.85)	1.95 (1.95)	68.24	1.97	85.30	1.97

Note: The values in parenthesis are those obtained by Cloot and Lebon (1985) by means of a series solution.

CHAPTER V

THERMAL RADIATION EFFECTS ON FERROCONVECTION

5.1 INTRODUCTION

The convective instability problems for radiating fluids received great attention in the past due to their implications in astrophysical and geophysical applications, and in other applications such as solar collectors (Bdeoui and Soufiani, 1997). The Rayleigh-Bénard and Marangoni instability problems involve only two modes of heat transfer, *viz.*, conduction and convection. Radiative heat transfer is important in physical systems that have less convective motions because of its stabilizing effect (Siegel and Howell, 1992; Modest, 1993; Howell and Menguc, 1998). Heat transfer problems involving conduction, convection and radiation are difficult to solve since the momentum and heat transport equations are coupled and the latter equation is an integro-differential equation.

Goody (1956) estimated the radiative transfer effects in the natural convection problem with free boundaries using a variational method. He solved the problem for optically thin and optically thick cases and showed that there could be very large variations near the boundaries. Goody's radiative transfer model has been extended and modified by many researchers by taking into account the effects of magnetic field, rotation and fluid non-grayness (Spiegel, 1960; Murgai and Khosla, 1962; Khosla and Murgai, 1963; Christophorides and Davis, 1970; Arpaci and Gozum, 1973; Yang, 1990; Bdeoui and Soufiani, 1997; Yan and Li, 2001 and references therein). Motivated by the meteorological applications, Larson (2001) studied the linear and nonlinear stability of an idealized radiative-convective model due to Goody (1956). More recently, Lan *et al*. (2003) analyzed, assuming the boundaries to be black and the fluid medium to be gray, the stability of a Newtonian fluid subject to combined natural convection and radiation using a spectral method.

Thermal process control of ferromagnetic and dielectric fluids through radiative heat transfer is also important in quite a few engineering applications (Luminosu *et al.*, 1987; Lloyd and Radcliffe, 1994; Hargrove *et al.*, 1998; Pode and Minea, 2000). In this chapter we have sought to investigate the effect of thermal radiation on the onset of both *RBC* and *MC* problems of a ferromagnetic fluid. Further, thermorheological and magnetorheological effects are given attention by treating the effective viscosity of the ferromagnetic fluid as a variable. An analogy for both *RBC* and *MC* problems between ferromagnetic and dielectric fluids is also presented.

5.2 MATHEMATICAL FORMULATION

Consider a Boussinesq ferromagnetic fluid confined between two parallel, infinite, isothermal plane boundaries heated from below. A uniform magnetic field $\vec{H}_o$ acts parallel to the vertical *z*-axis (Figure 5.1). The lower boundary is in the *xy*-plane. The ferromagnetic fluid is assumed to have a variable effective viscosity μ that depends on the temperature as well as on the magnitude of the magnetic field. The fluid between the isothermal boundaries absorbs and emits thermal radiation. We treat the two isothermal boundaries as either rigid or stress-free and also as black bodies. The absorption coefficient of the fluid is assumed to be the same at all wavelengths and to be independent of the physical state.

The system of equations describing the radiation-affected Rayleigh-Bénard instability in a variable-viscosity ferromagnetic fluid in the notation of Chapter II is

$$\nabla . \vec{q} = 0 , \tag{5.2.1}$$

$$\rho_o \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} . \nabla \right) \vec{q} \right] = -\nabla p - \rho g \, \hat{k} + \mu_o \, (\vec{M} . \nabla) \vec{H}$$
$$+ \nabla . \left[\mu(H,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right] , \tag{5.2.2}$$

$$\rho_{o}C_{VH}\left[\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T\right] = k_1 \nabla^2 T + \rho_{o}C_{VH}\frac{G}{s_r}, \tag{5.2.3}$$

$$\rho = \rho_{o}\left[1 - \alpha\left(T - T_a\right)\right], \tag{5.2.4}$$

$$\nabla.\vec{B} = 0, \quad \nabla\times\vec{H} = \vec{0}, \tag{5.2.5}$$

$$\vec{B} = \mu_{o}\left(\vec{M} + \vec{H}\right), \tag{5.2.6}$$

$$M = M_{o} + \chi_m\left(H - H_{o}\right) - K_1\left(T - T_a\right), \tag{5.2.7}$$

$$\mu\left(H,T\right) = \mu_1\left[1 + \delta_1\left(H - H_{o}\right)^2 - \delta_2\left(T - T_a\right)^2\right]. \tag{5.2.8}$$

In writing the energy equation (5.2.3), the viscous dissipation term and the volumetric heat source term have been neglected.

The equation of radiative heat transfer (Kourganoff, 1952) is

$$\frac{dI(\vec{r})}{ds} = K_a\left[P_B - I(\vec{r})\right], \tag{5.2.9}$$

where $I(\vec{r})$ is the intensity of radiation along the direction of the vector $\vec{r}$ and ds is an infinitesimal displacement in the $\vec{r}$ direction. The radiative heating rate is

$$G = -\int\frac{dI(\vec{r})}{ds}\,d\omega_s, \tag{5.2.10}$$

where the integral is taken over the solid angle 4π and ω_s is the element of solid angle. In the quiescent basic state, the equation of radiative transfer (5.2.9) takes the form

$$\mu_3\frac{dI}{dz} = K_a\left[P_B - I\right], \tag{5.2.11}$$

where μ_3 is the directional cosine of $\vec{r}$ in the z-direction. Equation (5.2.11) is indicative of the fact that the intensity of radiation is increased by emission and decreased by absorption.

Iterative solutions of one-dimensional radiative equilibrium problems all show that remarkably accurate results can be obtained by assuming a simple from for the angular distribution of radiative intensity. One of the simplest assumptions is the Milne-Eddington approximation :

$$\left.\begin{aligned} I(\mu_3, z) = I^+(z), \quad \text{for} \quad 0 < \mu_3 \leq 1, \\ I(\mu_3, z) = I^-(z), \quad \text{for} \quad -1 \leq \mu_3 \leq 0. \end{aligned}\right\} \tag{5.2.12}$$

The energy equation (5.2.3) in the basic state becomes

$$\frac{G_b}{s_r} + \kappa \frac{d^2 T_b}{dz^2} = 0. \tag{5.2.13}$$

Equation (5.2.13) is suggestive of the fact that the heat transfer in the basic state is essentially by conduction and radiation.

If F_z is the z-component of the radiative heat flux, then we have

$$G_b = -\frac{dF_z}{dz} \tag{5.2.14}$$

and we may write Eq. (5.2.13) in the integrated form

$$F_z + \kappa s_r \beta = C, \tag{5.2.15}$$

where $\beta = -\dfrac{dT_b}{dz}$ and C is the constant of integration. Taking the components of magnetization and magnetic field in the basic state to be $[0, 0, M_b(z)]$ and $[0, 0, H_b(z)]$, we obtain the quiescent state solution in the form

$$\vec{q}_b = \vec{0}, \qquad T_b(z) = T_a - \beta z, \qquad p = p_b(z),$$

$$\mu_b(z) = \mu_1 \left[1 - \delta \beta^2 z^2 \right], \qquad \rho_b(z) = \rho_o \left[1 + \alpha \beta z \right], \tag{5.2.16}$$

$$\vec{H}_b = \left[H_o - \frac{K_1 \beta z}{1 + \chi_m} \right] \hat{k}, \qquad \vec{M}_b = \left[M_o + \frac{K_1 \beta z}{1 + \chi_m} \right] \hat{k},$$

where $\delta = \delta_2 - \delta_1 \left[K_1 / (1 + \chi_m) \right]^2$. In arriving at the above basic state solution we have assumed that

$$T = T_o \qquad\qquad \text{at } z = d/2$$
$$T = T_1 \, (= T_o + \Delta T) \quad \text{at } z = -d/2.$$

The dominance of magnetic dependency over temperature dependency of viscosity is implied by $\delta < 0$ while $\delta > 0$ signifies dominance of temperature dependency.

Assuming the Milne-Eddington approximation, given by Eq. (5.2.12), and using the radiative heat transfer equation (5.2.11), the differential equation associated with the heat flux F_z can be obtained in the form (Goody, 1956)

$$\frac{d^2 F_z}{dz*^2} - \tau^2 F_z = -\tau^2 \frac{\chi}{1 + \chi} C, \tag{5.2.17}$$

where $z* = \dfrac{z}{d}$, $\tau^2 = 3 K_a^2 d^2 (1 + \chi)$, $\chi = \dfrac{4 \pi Q_1}{3 \kappa K_a s_r}$ and $Q_1 = \dfrac{4 S_c}{\pi} T_a^3$. Solving Eq. (5.2.17) using the following dimensionless radiative boundary conditions

$$\frac{dF_z}{dz*} = -2 K_a F_z d \quad \text{at } z* = +1/2,$$
$$\frac{dF_z}{dz*} = +2 K_a F_z d \quad \text{at } z* = -1/2, \tag{5.2.18}$$

we obtain

$$f(z*) = \frac{\beta}{\bar{\beta}} = L_1 \cosh(\tau z*) + L_2 \tag{5.2.19}$$

where

$$L_1 = \chi \left[\frac{2\chi}{\tau} + \frac{1}{2} \sqrt{3+3\chi} \sinh\left(\tau/2 \right) + \cosh\left(\tau/2 \right) \right]^{-1},$$

$$L_2 = \frac{L_1}{\chi} \left[\frac{1}{2} \sqrt{3+3\chi} \sinh\left(\tau/2 \right) + \cosh\left(\tau/2 \right) \right]$$

and $\overline{\beta}$ is the mean value of β throughout the medium. The radiative boundary conditions in Eq. (5.2.18) are obtained using the fact that the molecular conduction ensures continuity of temperature at the two surfaces. It is advantageous mentioning that $f(z^*)$ tends to unity if either τ or χ tends to zero independently. Moreover, if τ and χ are both greater than unity, the variation of the basic state temperature is exponential. In other words, the basic state temperature is no longer linear if the radiation effect is accounted for. In what follows we study the stability of the quiescent state within the framework of the linear theory.

5.2.1 Linear Stability Analysis

Let the components of the perturbed magnetization and the magnetic field be $\left(M_1', M_2', M_b(z) + M_3' \right)$ and $\left(H_1', H_2', H_b(z) + H_3' \right)$ respectively. The temperature T is taken to be $T_b(z) + T'$ with T' being the perturbation from the quiescent state. The effective viscosity μ, likewise, is taken to be $\mu_b(z) + \mu'$. Using these in Eq. (5.2.7) and linearizing the resulting equation, we obtain

$$\left. \begin{aligned} H_i' + M_i' &= \left(1 + \frac{M_0}{H_0} \right) H_i', \quad (i = 1, 2) \\ H_3' + M_3' &= (1 + \chi_m) H_3' - K_1 T', \end{aligned} \right\} \tag{5.2.20}$$

where it has been assumed that $K_1 \beta d \ll (1 + \chi_m) H_0$. The second of Eq. (5.2.5) suggests that one can write $\vec{H}' = \nabla \Phi'$, where Φ' is the perturbed magnetic scalar potential.

Introducing the magnetic potential Φ', eliminating the pressure p in Eq. (5.2.2) and incorporating the quiescent state solution, we obtain the vorticity transport equation in the form

$$
\rho_0 \frac{\partial}{\partial t}\left(\nabla^2 w'\right) - 2\delta\mu_1\beta^2\left(\nabla_1^2 - D^2\right)w' + 4\delta\mu_1\beta^2 z\nabla^2\left(Dw'\right)
$$

$$
- \mu_1\left[1 - \delta\beta^2 z^2\right]\nabla^4 w' - \alpha\rho_0 g\nabla_1^2 T'
$$

$$
- \frac{\mu_0 K_1^2 \beta}{1+\chi_m}\nabla_1^2 T' + \mu_0 K_1\beta\nabla_1^2\left(D\Phi'\right) = 0 , \tag{5.2.21}
$$

where $\nabla^2 = \nabla_1^2 + D^2$, $\nabla_1^2 = \left(\partial^2/\partial x^2\right) + \left(\partial^2/\partial y^2\right)$ and $D = \partial/\partial z$. The linear form of equation (5.2.3) in the perturbed state, on incorporation of the quiescent state solution, becomes

$$
\frac{\partial T'}{\partial t} - \beta w' = \kappa\nabla^2 T' + \frac{G'}{s_r} . \tag{5.2.22}
$$

Using Eq. (5.2.20) in the first of Eq. (5.2.5), we obtain

$$
(1+\chi_m) D^2\Phi' + \left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi' - K_1 DT' = 0 . \tag{5.2.23}
$$

As is customary in the linear stability analysis we make use of the normal mode technique. The infinitesimal perturbations w', T' and Φ' are supposed to be periodic waves that lead to a separable solution to Eqs. (5.2.21) – (5.2.23) in the form

$$
\begin{bmatrix} w' \\ T' \\ \Phi' \end{bmatrix} = \begin{bmatrix} w(z,t) \\ T(z,t) \\ \Phi(z,t) \end{bmatrix} \exp\left[i\left(k_x x + k_y y\right)\right] . \tag{5.2.24}
$$

Substituting Eq. (5.2.24) into Eq. (5.2.21), we obtain

$$\rho_o \left(D^2 - k^2\right)\frac{\partial w}{\partial t} + 2\delta\mu_1\beta^2\left(D^2 + k^2\right)w + 4\delta\mu_1\beta^2 z\left(D^2 - k^2\right)Dw$$

$$- \mu_1\left[1 - \delta\beta^2 z^2\right]\left(D^2 - k^2\right)^2 w + \alpha\rho_o g k^2 T$$

$$- \frac{\mu_o K_1 \beta}{1 + \chi_m}k^2\left[(1 + \chi_m)D\Phi - K_1 T\right] = 0 , \tag{5.2.25}$$

where $k = \sqrt{k_x^2 + k_y^2}$ is the wavenumber in the horizontal direction. Since Eq. (5.2.22) is an integro-differential equation, we adopt two approximations, one is valid when the fluid medium is optically thin (known as *transparent approximation*) and the other is applicable when the fluid medium is optically thick (known as *opaque approximation*). For the transparent approximation (Goody, 1956), we have the relation

$$\nabla_1^2 G' = -4\pi Q K_a \nabla_1^2 T' \tag{5.2.26}$$

and for the optically thick one (Goody, 1956), we have

$$\nabla_1^2 G' = \frac{4\pi Q}{3K_a}\nabla^2\left(\nabla_1^2 T'\right). \tag{5.2.27}$$

Equation (5.2.22) corresponding to the transparent approximation, after making use of Eqs. (5.2.24) and (5.2.26), becomes

$$\frac{\partial T}{\partial t} - \beta w = \kappa\left(D^2 - k^2\right)T - \frac{4\pi Q K_a}{s_r} T . \tag{5.2.28}$$

Similarly, Eq. (5.2.22) relating to the opaque approximation, turns out to be

$$\frac{\partial T}{\partial t} - \beta w = \left[\kappa + \frac{4\pi Q}{3K_a s_r}\right]\left(D^2 - k^2\right)T . \tag{5.2.29}$$

It is apparent from the above equation that the effect of radiation in the case of optically thick fluid medium is to enhance the thermal diffusivity.

Using Eq. (5.2.24) in Eq. (5.2.23), we obtain

$$(1+\chi_m)\,D^2\Phi \,-\, \left(1+\frac{M_o}{H_o}\right)k^2\Phi \,-\, K_1\,DT \,=\, 0. \tag{5.2.30}$$

We next make Eqs. (5.2.25) and (5.2.28) – (5.2.30) dimensionless by considering the following definition

$$z^* = \frac{z}{d}, \quad t^* = \frac{\kappa}{d^2}t, \quad w^* = \frac{d}{\kappa}w, \quad T^* = \frac{T}{\bar{\beta}d}, \quad \Phi^* = \frac{(1+\chi_m)}{K_1\bar{\beta}d^2}\Phi, \quad a^* = kd, \tag{5.2.31}$$

where the quantities with asterisks are dimensionless. Equations (5.2.25), (5.2.28) and (5.2.29), using (5.2.31), can be written (after dropping the asterisks) as

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left(D^2 + a^2\right)w + 4\,\Gamma z\left(D^2 - a^2\right)Dw$$
$$-\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w + R\,a^2 T - R\,M_1\,a^2[D\Phi - T] = 0, \tag{5.2.32}$$

$$\frac{\partial T}{\partial t} = \left(D^2 - a^2 - \frac{\tau^2\chi}{1+\chi}\right)T + f(z)\,w, \tag{5.2.33}$$

$$\frac{\partial T}{\partial t} = (1+\chi)\left(D^2 - a^2\right)T + f(z)\,w, \tag{5.2.34}$$

where $R = \dfrac{\alpha\rho_o g\,\bar{\beta}d^4}{\mu_1\kappa}$ is the thermal Rayleigh number and the expression for

$f(z)$ is given by Eq. (5.2.19) with z^* replaced by z. It is of interest to note that for $\Gamma = 0$ and as χ or $\tau \to 0$, the above equations reduce to those obtained by Finlayson (1970). Equations (5.2.33) and (5.2.34) may be combined into a single equation of the form

$$\frac{\partial T}{\partial t} = \left((D^2 - a^2) - \chi\left\{\delta_{k1}\frac{\tau^2}{1+\chi} - \delta_{k2}(D^2 - a^2)\right\}\right)T + f(z)w, \tag{5.2.35}$$

where the Kronecker delta, δ_{km}, is defined as

$$\delta_{km} = 1, \ \text{if} \ k = m, \atop 0, \ \text{if} \ k \neq m.$$

In Eq. (5.2.35), values of $k = 1$ and 2 respectively yields the expression for transparent and opaque medium.

The dimensionless form of Eq. (5.2.30) is

$$(D^2 - M_3 a^2)\, \Phi - DT \ = \ 0. \tag{5.2.36}$$

Equations (5.2.32), (5.2.35) and (5.2.36) are solved subject to the following boundary combinations

    (a)   Free – free, isothermal
$$w = D^2 w = T = 0 \quad \text{at} \ \ z = \pm \, 1/2 \ ,$$

    (b)   Free – rigid, isothermal
$$w = D^2 w = T = 0 \quad \text{at} \ \ z = + \, 1/2$$
$$w = Dw = T = 0 \quad \text{at} \ \ z = - \, 1/2 \ ,$$

    (c)   Rigid – rigid, isothermal
$$w = Dw = T = 0 \quad \text{at} \ \ z = \pm \, 1/2 .$$

The above boundary conditions on w and T are not affected by radiation (Khosla and Murgai, 1963). For isothermal boundaries as taken above, the boundary conditions on the magnetic potential Φ are given by (see Eq. 2.2.19)

$$D\Phi + \frac{a\Phi}{1+\chi_m} = 0 \qquad \text{at} \ \ z = +1/2, \atop D\Phi - \frac{a\Phi}{1+\chi_m} = 0 \qquad \text{at} \ \ z = -1/2 .$$

In the succeeding section, we focus on the validity or otherwise of the principle of exchange of stabilities (*PES*) for the problem under consideration.

5.2.2 Oscillatory Instability

We analyze the possibility of the existence of overstable motions by means of the Rayleigh-Ritz technique as discussed in Chapter IV. Multiplying Eqs. (5.2.32), (5.2.35) and (5.2.36) by w, T and Φ respectively, integrating with respect to z between the limits $z = -1/2$ and $z = 1/2$, taking $w(z,t) = A_1(t)\,w_1(z)$, $T(z,t) = B_1(t)\,T_1(z)$, $\Phi(z,t) = C_1(t)\,\Phi_1(z)$ (in which $w_1(z)$, $T_1(z)$ and $\Phi_1(z)$ are trial functions), and using the boundary conditions mentioned in Section 5.2.1 give rise to the following system of ordinary differential equations

$$\frac{Q_1}{Pr}\frac{dA_1}{dt} = -Q_2\,A_1 + R(1+M_1)\,Q_3 B_1 - RM_1 Q_4 C_1, \tag{5.2.37}$$

$$Q_5\,\frac{dB_1}{dt} = Q_6\,A_1 - Q_7\,B_1, \tag{5.2.38}$$

$$0 = Q_8\,B_1 + Q_9\,C_1, \tag{5.2.39}$$

where

$$Q_1 = \left\langle (Dw_1)^2 \right\rangle + a^2 \left\langle w_1^2 \right\rangle,$$

$$Q_2 = \left\langle w_1\left(1 - \Gamma z^2\right)D^4 w_1 \right\rangle - 2\,a^2 \left\langle w_1\left(1 - \Gamma z^2\right)D^2 w_1 \right\rangle$$
$$+ a^4 \left\langle w_1\left(1 - \Gamma z^2\right)w_1 \right\rangle + 2\,\Gamma \left\langle (Dw_1)^2 \right\rangle - 2\,\Gamma a^2 \left\langle w_1^2 \right\rangle$$
$$- 4\,\Gamma \left\langle w_1 z D^3 w_1 \right\rangle + 4\,\Gamma a^2 \left\langle w_1 z Dw_1 \right\rangle,$$

$$Q_3 = a^2 \left\langle w_1 T_1 \right\rangle, \quad Q_4 = a^2 \left\langle w_1 D\Phi_1 \right\rangle, \quad Q_5 = \left\langle T_1^2 \right\rangle,$$

$$Q_6 = \left\langle T_1 f(z) w_1 \right\rangle,$$

$$Q_7 = \left(1 + \delta_{k2}\chi\right)\left\langle (DT_1)^2 \right\rangle + \left[a^2 + \chi\left\{ \delta_{k1}\frac{\tau^2}{1+\chi} + \delta_{k2}a^2 \right\}\right]\left\langle T_1^2 \right\rangle,$$

$$Q_8 = \left\langle \Phi_1 DT_1 \right\rangle,$$

$$Q_9 = \left\langle (D\Phi_1)^2 \right\rangle + M_3\, a^2 \left\langle \Phi_1^2 \right\rangle + \frac{a}{1+\chi_m}\left[\left\{\Phi_1\!\left(\frac{1}{2}\right)\right\}^2 + \left\{\Phi_1\!\left(\frac{-1}{2}\right)\right\}^2\right]$$

and $\langle uv \rangle = \int_{-1/2}^{1/2} uv\, dz$. Eliminating C_1 between Eqs. (5.2.37) and (5.2.39), the resulting system of equations can be rearranged into the matrix form

$$\frac{\mathrm{d}A}{\mathrm{d}t} = F\,A, \tag{5.2.40}$$

where

$$A = \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \quad \text{and} \quad F = \begin{pmatrix} -\dfrac{Pr\, Q_2}{Q_1} & \dfrac{R\,Pr}{Q_1}\left[(1+M_1)Q_3 + \dfrac{Q_4\,Q_8}{Q_9}M_1\right] \\ \dfrac{Q_6}{Q_5} & -\dfrac{Q_7}{Q_5} \end{pmatrix}.$$

Following the procedure illustrated in Chapter IV, we arrive at the following expression for the thermal Rayleigh number R

$$R = \frac{1}{Q_6\,N_1}\left\{ Q_2\,Q_7 - \frac{Q_1\,Q_5}{Pr}\,\omega^2 \right\} + i\,\omega N_2, \tag{5.2.41}$$

where

$$N_1 = (1+M_1)Q_3 + \frac{Q_4\,Q_8}{Q_9}M_1 \quad \text{and} \quad N_2 = \frac{Pr\,Q_2\,Q_5 + Q_1\,Q_7}{Pr\,Q_6\,N_1}.$$

We note that R is a real quantity and N_2 is independent of ω. This means, following the analysis given in Chapter IV, that the possibility of the existence of overstable motions can be ruled out and that the *PES* holds good. So we take $\omega = 0$ in the subsequent analysis. We now proceed to discuss the preferred mode of instability, *viz.*, stationary convection.

5.2.3 Stationary Instability

The system of equations associated with the stationary instability reads as

$$\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w - 2\,\Gamma\left(D^2 + a^2\right)w - 4\,\Gamma z\left(D^2 - a^2\right)Dw$$
$$- R\left(1 + M_1\right)a^2 T + R M_1 a^2 D\Phi = 0 , \tag{5.2.42}$$

$$\left(\left(D^2 - a^2\right) - \chi\left\{\delta_{k1}\frac{\tau^2}{1 + \chi} - \delta_{k2}\left(D^2 - a^2\right)\right\}\right)T + f(z)w = 0, \tag{5.2.43}$$

$$(D^2 - M_3 a^2)\Phi - DT = 0 . \tag{5.2.44}$$

The system of Eqs. (5.2.42) – (5.2.44) together with the boundary conditions specified in Section 5.2.1 poses an eigenvalue problem for R with Γ, M_1, M_3, τ, χ and χ_m as parameters. We employ the Higher Order Rayleigh-Ritz Technique (*HORT*) to obtain the eigenvalue and the associated wavenumber. To this end, we expand $w(z)$, $T(z)$ and $\Phi(z)$ in a series of trial functions

$$w(z) = \sum_{i=1}^{n} \alpha_i w_i(z),\ T(z) = \sum_{i=1}^{n} \beta_i T_i(z)\ \text{and}\ \Phi(z) = \sum_{i=1}^{n} \gamma_i \Phi_i(z), \tag{5.2.45}$$

where α_i, β_i and γ_i are constants, and $w_i(z)$, $T_i(z)$ and $\Phi_i(z)$ are trial functions. We now choose the following trial functions guided by the boundary conditions given in Section 5.2.1 and variational considerations

    (a)   Free – free, isothermal
$$w_i = \left[\left(z^2 - \frac{1}{4}\right)^2 - \left(z^2 - \frac{1}{4}\right)\right]z^{i-1},\quad T_i = \left(z^2 - \frac{1}{4}\right)z^{i-1},\quad \Phi_i = z^{2i-1}$$

    (b)   Free – rigid, isothermal
$$w_i = \left(2z^2 - z - 1\right)\left(z^2 - \frac{1}{4}\right)z^{i-1},\quad T_i = \left(z^2 - \frac{1}{4}\right)z^{i-1},\quad \Phi_i = z^{2i-1}$$

    (c)   Rigid – rigid, isothermal
$$w_i = \left(z^2 - \frac{1}{4}\right)^2 z^{i-1},\quad T_i = \left(z^2 - \frac{1}{4}\right)z^{i-1},\quad \Phi_i = z^{2i-1}$$

Applying *HORT* to Eqs. (5.2.42) – (5.2.44) leads to a system of homogeneous equations using which we get the critical eigenvalue R_c and the corresponding critical wavenumber a_c. In the following section, we explore an analogy for the Rayleigh-Bénard instability problems between ferromagnetic and dielectric liquids in the presence of thermal radiation.

5.3 ANALOGY FOR RADIATION-AFFECTED RAYLEIGH-BÉNARD CONVECTION BETWEEN FERROMAGNETIC AND DIELECTRIC LIQUIDS

The system of equations of electrohydrodynamics describing the radiation-affected Rayleigh-Bénard instability in a variable-viscosity dielectric liquid in the notation of Chapter II is

$$\nabla . \vec{q} = 0, \tag{5.3.1}$$

$$\rho_{0}\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q}\right] = -\nabla p - \rho g \, \hat{k} + \left(\vec{P}.\nabla\right)\vec{E}$$
$$+ \nabla.\left[\mu(E,T)\left(\nabla\vec{q} + \nabla\vec{q}^{Tr}\right)\right], \tag{5.3.2}$$

$$\rho_{0}C_{VE}\left[\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T\right] = k_{1}\nabla^{2}T + \rho_{0}C_{VE}\frac{G}{s_{r}}, \tag{5.3.3}$$

$$\rho = \rho_{0}\left[1 - \alpha\left(T - T_{a}\right)\right], \tag{5.3.4}$$

$$\nabla . \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0}, \tag{5.3.5}$$

$$\vec{D} = \varepsilon_{0}\vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_{0}\left(\varepsilon_{r} - 1\right)\vec{E}, \tag{5.3.6}$$

$$\varepsilon_{r} = (1 + \chi_{e}) - e\left(T - T_{a}\right), \tag{5.3.7}$$

$$\mu\left(E,T\right) = \mu_{1}\left[1 + \delta_{1}\left(E - E_{0}\right)^{2} - \delta_{2}\left(T - T_{a}\right)^{2}\right]. \tag{5.3.8}$$

$$\frac{dI(\vec{r})}{ds} = K_a \left[P_B - I(\vec{r}) \right], \tag{5.3.9}$$

$$G = - \int \frac{dI(\vec{r})}{ds} \, d\omega_s \,, \tag{5.3.10}$$

where the effective viscosity $\mu(E,T)$ has been assumed to be a function of the temperature and magnitude of the *ac* electric field.

The electric boundary conditions specify that the normal component of dielectric field $\vec{D}$ and tangential components of electric field $\vec{E}$ are continuous across the boundaries. Taking the components of the electric field in the basic state to be $[0,\, 0,\, E_b(z)]$, one obtains the quiescent state solution in the form

$$\left.\begin{array}{l} \vec{q}_b = \vec{0}, \qquad T_b(z) = T_a - \beta z, \qquad p = p_b(z), \\[2mm] \mu_b(z) = \mu_1 \left[1 - \delta \, \beta^2 z^2 \right], \qquad \rho_b(z) = \rho_{\mathrm{o}} \left[1 + \alpha \beta z \right], \\[2mm] \vec{E}_b = \left[\dfrac{E_{\mathrm{o}}(1+\chi_e)}{(1+\chi_e) + e\beta z} \right] \hat{k}, \quad \vec{P}_b = \varepsilon_{\mathrm{o}} E_{\mathrm{o}}(1+\chi_e) \left[1 - \dfrac{1}{(1+\chi_e)+e\beta z} \right] \hat{k}, \end{array}\right\} \tag{5.3.11}$$

where $\delta = \delta_2 - \delta_1 \left[e E_{\mathrm{o}} / (1+\chi_e) \right]^2$. In obtaining Eq. (5.3.11), it has been assumed that $e\beta d \ll (1+\chi_e)$. It should be noted that the expression for $f(z^*)$ in Eq. (5.2.19) remains the same for dielectric liquid convection in the presence of radiation.

The second of Eq. (5.3.6), upon application of linear analysis, yields

$$\left.\begin{array}{l} P_i' = \varepsilon_{\mathrm{o}} \chi_e E_i', \qquad (i = 1, 2) \\[2mm] P_3' = \varepsilon_{\mathrm{o}} \chi_e E_3' - e\varepsilon_{\mathrm{o}} E_{\mathrm{o}} T'. \end{array}\right\} \tag{5.3.12}$$

Following exactly the same procedure as in the development for a ferromagnetic liquid, introducing the perturbed electric potential Φ' through the relation $\vec{E}' = \nabla \Phi'$ and taking $eE_0\,\bar{\beta}\,d^2/(1+\chi_m)$ as the unit electric potential, we arrive at the following system of dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left(D^2 + a^2\right)w + 4\,\Gamma z\left(D^2 - a^2\right)Dw$$
$$-\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w + R\,a^2 T - R_E\,a^2\left[D\Phi - T\right] = 0\,, \qquad (5.3.13)$$

$$\frac{\partial T}{\partial t} = \left((D^2 - a^2) - \chi\left\{\delta_{k1}\frac{\tau^2}{1+\chi} - \delta_{k2}(D^2 - a^2)\right\}\right)T + f(z)\,w\,, \qquad (5.3.14)$$

$$(D^2 - a^2)\,\Phi - DT = 0\,. \qquad (5.3.15)$$

Equations (5.3.13) – (5.3.15) are solved subject to the boundary conditions on velocity and temperature given in Section 5.2.1. The boundary conditions on the electric potential given in Eq. (2.2.20) relating to the isothermal boundaries become

$$\left.\begin{array}{l}
D\Phi + \dfrac{a\Phi}{1 + \chi_e} = 0 \ \ \text{at} \ \ z = \dfrac{1}{2}\,, \\[4mm]
D\Phi - \dfrac{a\Phi}{1 + \chi_e} = 0 \ \ \text{at} \ \ z = -\dfrac{1}{2}\,.
\end{array}\right\}$$

It is clear, comparing the set of Eqs. (5.2.32), (5.2.35) and (5.2.36) and the set of Eqs. (5.3.13) – (5.3.15) together with the boundary conditions, that one can recover the equations of the latter from those of the former when one replaces RM_1 by R_E, M_3 by 1 and χ_m by χ_e. In view of this analogy, there is no need to study the radiation-affected RBC problem of dielectric liquids in isolation. In what follows, we study the effect of thermal radiation on the onset of Marangoni-ferroconvection with variable viscosity.

5.4 MARANGONI FERROCONVECTION WITH THERMAL RADIATION

The system of equations describing the Marangoni instability situation in a thin, variable-viscosity ferromagnetic liquid layer (with a free upper surface) with thermal radiation is

$$\nabla . \vec{q} = 0, \tag{5.4.1}$$

$$\rho_o \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} . \nabla \right) \vec{q} \right] = -\nabla p + \mu_o \, (\vec{M} . \nabla) \vec{H} + \nabla . \left[\mu(H,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right], \tag{5.4.2}$$

$$\rho_o C_{VH} \left[\frac{\partial T}{\partial t} + \left(\vec{q} . \nabla \right) T \right] = k_1 \, \nabla^2 T + \rho_o C_{VH} \, \frac{G}{s_r}, \tag{5.4.3}$$

$$\nabla . \vec{B} = 0, \quad \nabla \times \vec{H} = \vec{0}, \tag{5.4.4}$$

$$\vec{B} = \mu_o \left(\vec{M} + \vec{H} \right), \tag{5.4.5}$$

$$M = M_o + \chi_m \left(H - H_o \right) - K_1 \left(T - T_a \right), \tag{5.4.6}$$

$$\mu \, (H,T) \; = \; \mu_1 \left[1 + \delta_1 \left(H - H_o \right)^2 - \delta_2 \left(T - T_a \right)^2 \right], \tag{5.4.7}$$

$$\sigma(H,T) = \sigma_o + \sigma_H (H - H_o) - \sigma_T \, (T - T_a), \tag{5.4.8}$$

$$\frac{dI(\vec{r})}{ds} = K_a \left[P_B - I(\vec{r}) \right], \tag{5.4.9}$$

$$G = -\int \frac{dI(\vec{r})}{ds} \, d\omega_s . \tag{5.4.10}$$

Application of the linear stability analysis discussed in Section 5.2.1 yields the following dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\Gamma\left(D^2 + a^2\right)w + 4\Gamma z\left(D^2 - a^2\right)Dw$$

$$-\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w - R_M a^2\left[D\Phi - T\right] = 0, \tag{5.4.11}$$

$$\frac{\partial T}{\partial t} = \left((D^2 - a^2) - \chi\left\{\delta_{k1}\frac{\tau^2}{1+\chi} - \delta_{k2}(D^2 - a^2)\right\}\right)T + f(z)w, \tag{5.4.12}$$

$$(D^2 - M_3 a^2)\Phi - DT = 0, \tag{5.4.13}$$

where $R_M = \dfrac{\mu_0 K_1^2 \beta\bar{\beta} d^4}{\mu_1 \kappa(1+\chi_m)}$ is the magnetic Rayleigh number and the remaining

parameters are as defined in Section 5.2.1. We have shown, in Section 5.2.2, that stationary instability is preferred to oscillatory one in the case of Rayleigh-Bénard ferroconvection with thermal radiation. We now move over to examine the preferred mode of convection relating to the Marangoni ferroconvection.

5.4.1 Oscillatory Instability

We make use of the single-term Rayleigh-Ritz technique discussed in Chapter IV to analyze the possibility of the existence of overstable motions. To this end, we consider Eqs. (5.4.11) – (5.4.13) subject to the following boundary conditions

$$\left.\begin{array}{l} w = \left[1 - \Gamma z^2\right]D^2 w + a^2 Ma\,T - a^2 Ma_H\,D\Phi = 0 \\[4mm] DT = -Bi\,T \quad \text{and} \quad D\Phi + \dfrac{a\Phi}{1+\chi_m} - T = 0 \quad \text{at} \quad z = \dfrac{1}{2}, \\[4mm] w = Dw = T = D\Phi - \dfrac{a\Phi}{1+\chi_m} = 0 \qquad \text{at} \quad z = -\dfrac{1}{2}, \end{array}\right\} \tag{5.4.14}$$

where $Ma = \dfrac{\sigma_T \bar{\beta} d^2}{\mu_1 \kappa}$ is the thermal Marangoni number, $Ma_H = \dfrac{K_1 \sigma_H \bar{\beta} d^2}{\mu_1 \kappa(1+\chi_m)}$ is

the magnetic Marangoni number and the rest of the parameters are as defined in Chapter II. In the pursued numerical calculations we assume $|Ma_H| \ll 1$.

Multiplying Eqs. (5.4.11) – (5.4.13) by w, T and Φ respectively, integrating with respect to z between the limits $z = -1/2$ and $z = 1/2$, taking $w(z,t) = A_1(t)\,w_1(z)$, $T(z,t) = B_1(t)\,T_1(z)$, $\Phi(z,t) = C_1(t)\,\Phi_1(z)$ (in which $w_1(z)$, $T_1(z)$ and $\Phi_1(z)$ are trial functions), and using the boundary conditions in Eq. (5.4.14) give rise to the following system of ordinary differential equations

$$\frac{Q_1}{\mathrm{Pr}}\frac{\mathrm{d}A_1}{\mathrm{d}t} = (-Q_2)A_1 + \left[R_M\,Q_3 - Ma\,Q_4\right]B_1 + (-R_M\,Q_5)C_1, \qquad (5.4.15)$$

$$Q_6\frac{\mathrm{d}B_1}{\mathrm{d}t} = Q_7\,A_1 + (-Q_8)B_1, \qquad (5.4.16)$$

$$0 = Q_9\,B_1 + Q_{10}\,C_1, \qquad (5.4.17)$$

where

$$Q_1 = \left\langle (Dw_1)^2 \right\rangle + a^2\left\langle w_1^2 \right\rangle,$$

$$\begin{aligned}
Q_2 &= \left\langle (D^2 w_1)^2 \right\rangle - 2\,a^2\left\langle w_1\left(1 - \Gamma z^2\right)D^2 w_1\right\rangle - \Gamma\left\langle w_1 z^2 D^4 w_1\right\rangle \\
&\quad + a^4\left\langle w_1\left(1 - \Gamma z^2\right)w_1\right\rangle + 2\Gamma\left\langle (Dw_1)^2 \right\rangle - 2\,\Gamma a^2\left\langle w_1^2 \right\rangle \\
&\quad - 4\Gamma\left\langle w_1 z D^3 w_1\right\rangle + 4\Gamma a^2\left\langle w_1 z\,Dw_1\right\rangle,
\end{aligned}$$

$$Q_3 = a^2\left\langle w_1 T_1\right\rangle, \qquad Q_4 = \frac{4\,a^2}{(4 - \Gamma)}\,Dw_1(\tfrac{1}{2})\,T_1(\tfrac{1}{2}),$$

$$Q_5 = a^2\left\langle w_1 D\Phi_1\right\rangle, \qquad Q_6 = \left\langle T_1^2 \right\rangle, \qquad Q_7 = \left\langle T_1 f(z) w_1\right\rangle,$$

$$\begin{aligned}
Q_8 &= \left(1 + \delta_{k2}\chi\right)\left\langle (DT_1)^2 \right\rangle \\
&\quad + \left[a^2 + \chi\left\{\delta_{k1}\frac{\tau^2}{1+\chi} + \delta_{k2}\,a^2\right\}\right]\left\langle T_1^2 \right\rangle + Bi\left\{T_1\left(\tfrac{1}{2}\right)\right\}^2,
\end{aligned}$$

$$Q_9 = \left\langle \Phi_1 DT_1\right\rangle - \Phi_1(\tfrac{1}{2})\,T_1(\tfrac{1}{2}),$$

$$Q_{10} = \left\langle (D\Phi_1)^2 \right\rangle + M_3\, a^2 \left\langle \Phi_1^2 \right\rangle + \frac{a}{1+\chi_m} \left[\left\{ \Phi_1\left(\frac{1}{2}\right) \right\}^2 + \left\{ \Phi_1\left(\frac{-1}{2}\right) \right\}^2 \right].$$

Eliminating C_1 between Eqs. (5.4.15) and (5.4.17), the resulting system of equations can be rearranged into the matrix form

$$\frac{\mathrm{d}A}{\mathrm{d}t} = FA, \tag{5.4.18}$$

where

$$A = \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \quad \text{and} \quad F = \begin{pmatrix} -\dfrac{Pr\,Q_2}{Q_1} & \dfrac{Pr}{Q_1}\left[R_M\left\{ Q_3 + \dfrac{Q_5\,Q_9}{Q_{10}} \right\} - Ma\,Q_4 \right] \\[3ex] \dfrac{Q_7}{Q_6} & -\dfrac{Q_8}{Q_6} \end{pmatrix}.$$

Following the procedure illustrated in Section 5.2.2, we arrive at the following expression for the thermal Marangoni number Ma

$$Ma = \frac{R_M}{Q_4}\left\{ Q_3 + \frac{Q_5\,Q_9}{Q_{10}} \right\} - \frac{1}{Q_4\,Q_7}\left\{ Q_2 Q_8 + \frac{Q_1 Q_6}{Pr}\,\omega^2 \right\} - i\omega N, \tag{5.4.19}$$

where

$$N = \frac{1}{Q_4\,Q_7}\left\{ Q_2 Q_6 + \frac{Q_1 Q_8}{Pr} \right\}.$$

Since Ma is a real quantity and since N is independent of ω, we infer that oscillatory convection is not possible in the present problem. So we take $\omega = 0$ in the subsequent analysis. We now proceed to discuss the preferred mode of instability, viz., stationary convection.

146

5.4.2 Stationary Instability

The system of equations associated with the stationary instability reads as

$$\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w - 2\Gamma\left(D^2 + a^2\right)w - 4\Gamma z\left(D^2 - a^2\right)Dw$$
$$+ R_M\, a^2\left[D\Phi - T\right] = 0, \tag{5.4.20}$$

$$\left((D^2 - a^2) - \chi\left\{\delta_{k1}\frac{\tau^2}{1 + \chi} - \delta_{k2}(D^2 - a^2)\right\}\right)T + f(z)w = 0, \tag{5.4.21}$$

$$(D^2 - M_3\, a^2)\Phi - DT = 0. \tag{5.4.22}$$

The system of Eqs. (5.4.20) – (5.4.22) together with the boundary conditions in Eq. (5.4.14) poses an eigenvalue problem for Ma with Γ, R_M, M_3, τ, χ and χ_m as parameters. We employ the Higher Order Rayleigh-Ritz Technique (*HORT*) discussed in Chapter IV to obtain the eigenvalue and the associated wavenumber. To this end, we expand $w(z)$, $T(z)$ and $\Phi(z)$ in a series of trial functions

$$w(z) = \sum_{i=1}^{n} \alpha_i w_i(z), \quad T(z) = \sum_{i=1}^{n} \beta_i T_i(z) \quad \text{and} \quad \Phi(z) = \sum_{i=1}^{n} \gamma_i \Phi_i(z), \tag{5.4.23}$$

where α_i, β_i and γ_i are constants, and $w_i(z)$, $T_i(z)$ and $\Phi_i(z)$ are trial functions. We choose the following trial functions

$$w_i = \left(z - \frac{1}{2}\right)\left(z + \frac{1}{2}\right)^{i+1}, \qquad T_i = \left(z + \frac{1}{2}\right)^{i}, \qquad \Phi_i = z^i$$

guided by the boundary conditions in Eq. (5.4.14) and variational considerations. In the succeeding section, we discuss an analogy for Marangoni instability problems between variable-viscosity ferromagnetic and dielectric liquids in the presence of thermal radiation.

5.5 ANALOGY FOR RADIATION-AFFECTED MARANGONI CONVECTION BETWEEN FERROMAGNETIC AND DIELECTRIC LIQUIDS

The system of equations of electrohydrodynamics delineating the radiation-affected Marangoni instability situation in a variable-viscosity dielectric liquid reads as

$$\nabla \cdot \vec{q} = 0,$$ (5.5.1)

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{q} \right] = -\nabla p + \left(\vec{P} \cdot \nabla \right) \vec{E} + \nabla \cdot \left[\mu(E,T) \left(\nabla \vec{q} + \nabla \vec{q}^{Tr} \right) \right],$$ (5.5.2)

$$\rho_0 C_{VE} \left[\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla \right) T \right] = k_1 \nabla^2 T + \rho_0 C_{VE} \frac{G}{s_r},$$ (5.5.3)

$$\nabla \cdot \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0},$$ (5.5.4)

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad \vec{P} = \varepsilon_0 \left(\varepsilon_r - 1 \right) \vec{E},$$ (5.5.5)

$$\varepsilon_r = (1 + \chi_e) - e(T - T_a),$$ (5.5.6)

$$\mu(E,T) = \mu_1 \left[1 + \delta_1 \left(E - E_0 \right)^2 - \delta_2 \left(T - T_a \right)^2 \right],$$ (5.5.7)

$$\sigma(E,T) = \sigma_0 + \sigma_E (E - E_0) - \sigma_T (T - T_a),$$ (5.5.8)

$$\frac{dI(\vec{r})}{ds} = K_a \left[P_B - I(\vec{r}) \right],$$ (5.5.9)

$$G = -\int \frac{dI(\vec{r})}{ds} d\omega_s.$$ (5.5.10)

Application of linear stability analysis yields the following dimensionless equations

$$\frac{1}{Pr}\left(D^2 - a^2\right)\frac{\partial w}{\partial t} + 2\,\Gamma\left(D^2 + a^2\right)w + 4\,\Gamma z\left(D^2 - a^2\right)Dw$$

$$-\left[1 - \Gamma z^2\right]\left(D^2 - a^2\right)^2 w - R_E a^2\left[D\Phi - T\right] = 0\,, \tag{5.5.11}$$

$$\frac{\partial T}{\partial t} = \left((D^2 - a^2) - \chi\left\{\delta_{k1}\frac{\tau^2}{1+\chi} - \delta_{k2}(D^2 - a^2)\right\}\right)T + f(z)w, \tag{5.5.12}$$

$$(D^2 - a^2)\Phi - DT = 0\,, \tag{5.5.13}$$

where $R_E = \dfrac{\varepsilon_0 e^2 E_0^2 \beta \bar{\beta} d^4}{\mu_1 \kappa (1 + \chi_e)}$ is the electric Rayleigh number and the remaining

parameters are as defined in Chapter II. Equations (5.5.11) – (5.5.13) are solved subject to the boundary conditions

$$\left.\begin{aligned}
w &= \left[1 - \Gamma z^2\right]D^2 w + a^2 Ma\,T - a^2 Ma_E\,D\Phi = 0 \\[1mm]
DT &= -Bi\,T \quad \text{and} \quad D\Phi + \frac{a\Phi}{1+\chi_e} - T = 0 \quad \text{at} \quad z = \frac{1}{2}, \\[1mm]
w &= Dw = T = D\Phi - \frac{a\Phi}{1+\chi_e} = 0 \qquad \text{at} \quad z = -\frac{1}{2},
\end{aligned}\right\} \tag{5.5.14}$$

where $Ma_E = \dfrac{e E_0 \sigma_E \bar{\beta} d^2}{\mu_1 \kappa (1 + \chi_m)}$ is the electric Marangoni number which is assumed

negligible. The analogy between "Marangoni-ferroconvection" and "Marangoni-electroconvection" in the presence of thermal radiation can easily be understood if we compare the set of Eqs. (5.4.11) – (5.4.14) and the set of Eqs. (5.5.11) – (5.5.14). It is perceptible that one can recover the equations of the latter from those of the former when one replaces R_M by R_E, Ma_H by Ma_E, M_3 by 1 and χ_m by χ_e. In view of this analogy, there is no need to study the latter problem in isolation.
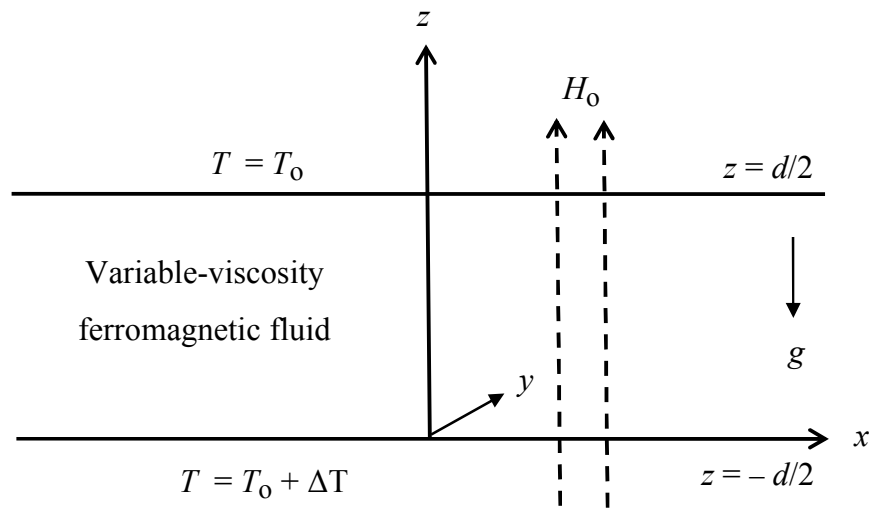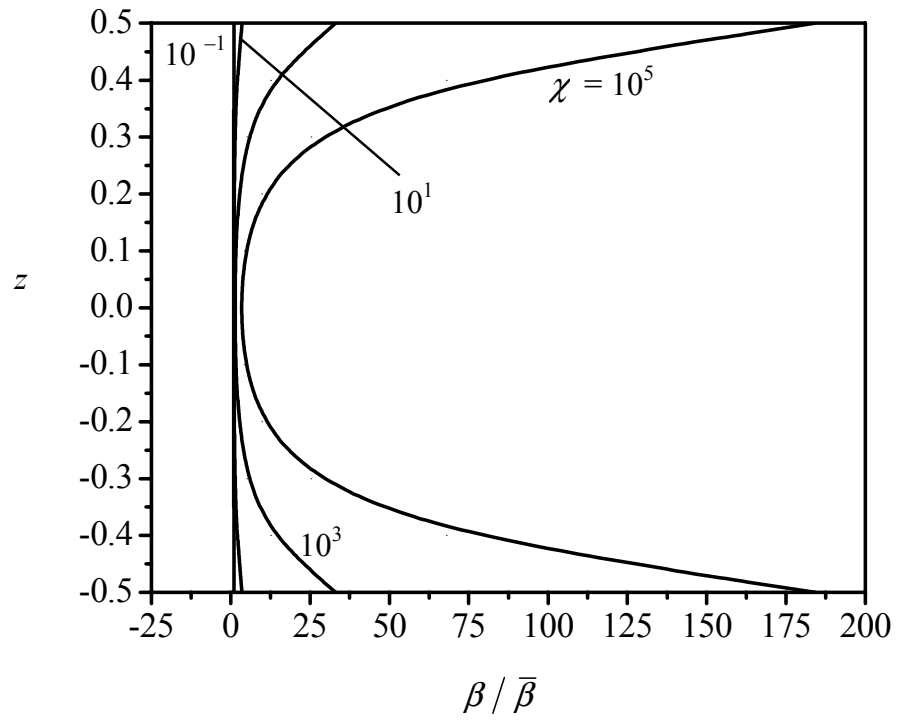
Figure 5.1: Configuration of the problem.

Figure 5.2: $\beta / \bar{\beta}$ as a function of the vertical coordinate z for different values of the conduction-radiation parameter χ and for $\tau = 10$.
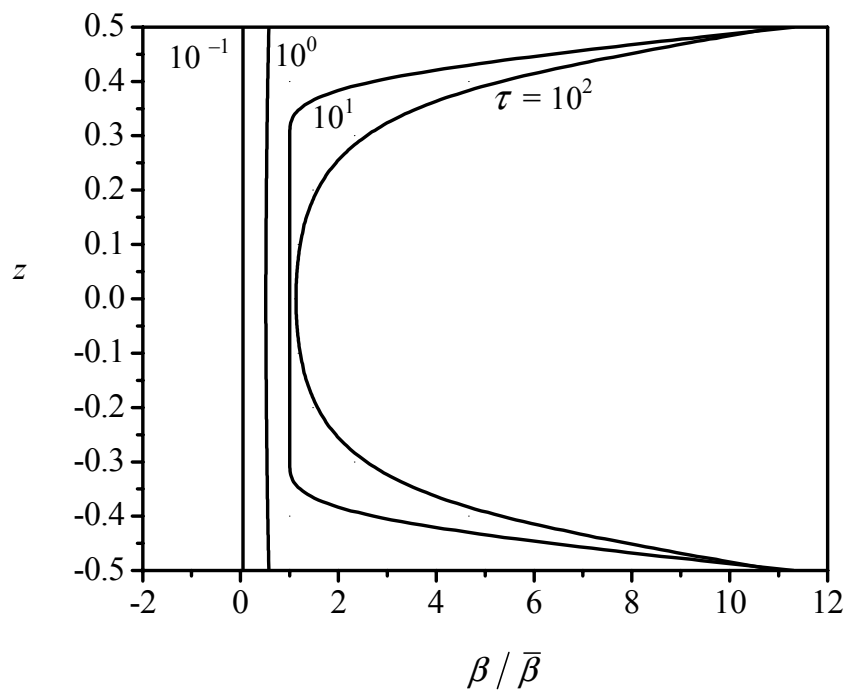
Figure 5.3: $\beta / \bar{\beta}$ as a function of the vertical coordinate z for different values of the absorptivity parameter τ and for $\chi = 10^2$.
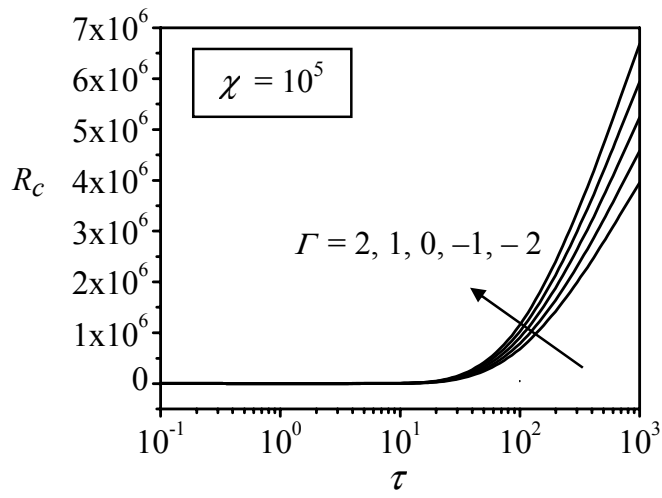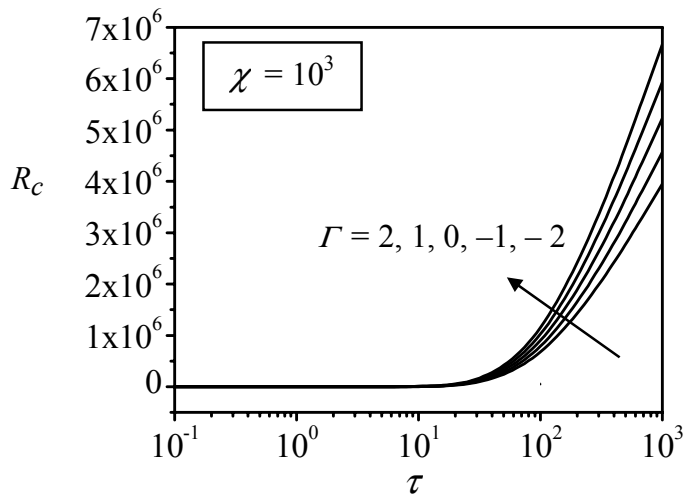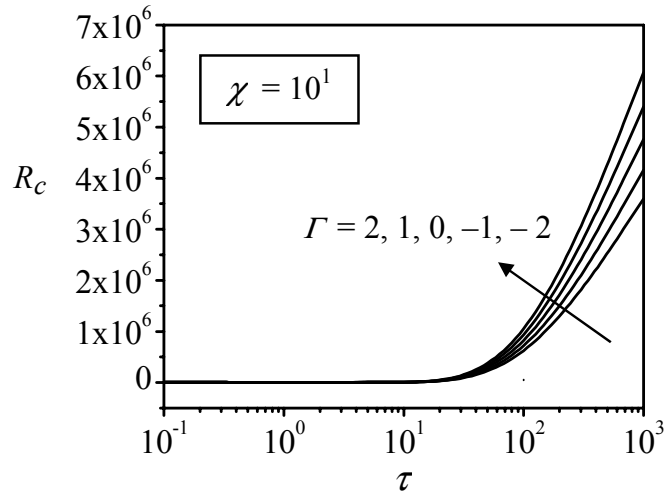
Figure 5.4: Plot of critical thermal Rayleigh number R_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and effective viscosity parameter Γ (transparent case).
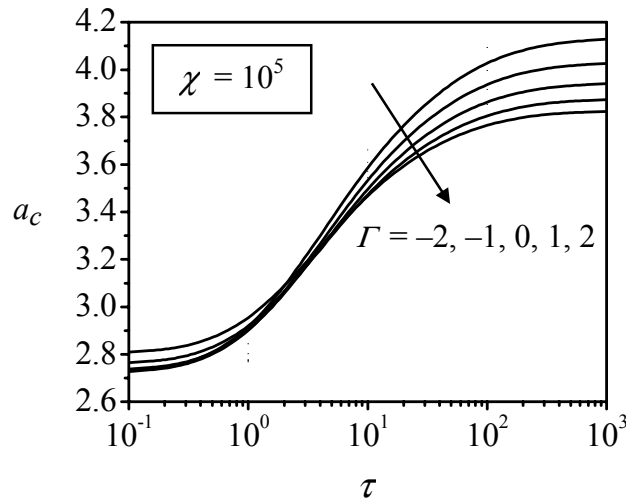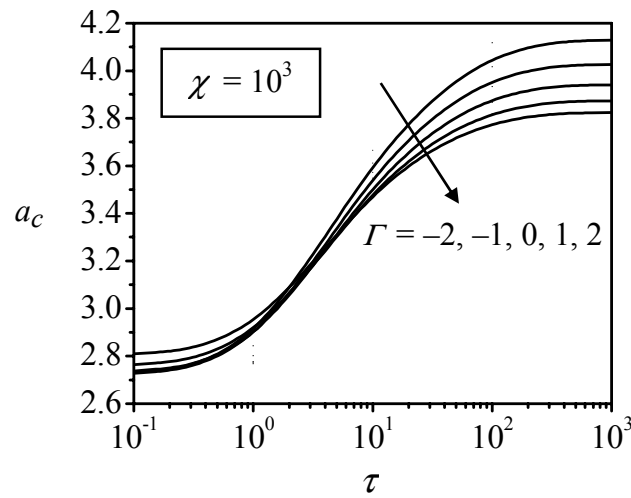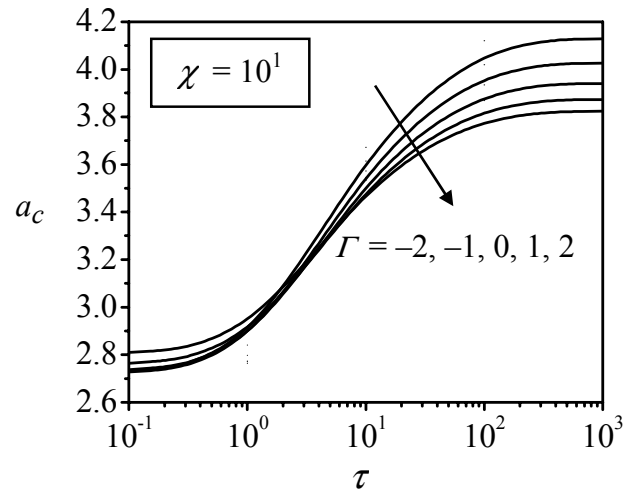
Figure 5.5: Plot of critical wavenumber a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and Γ (transparent case).
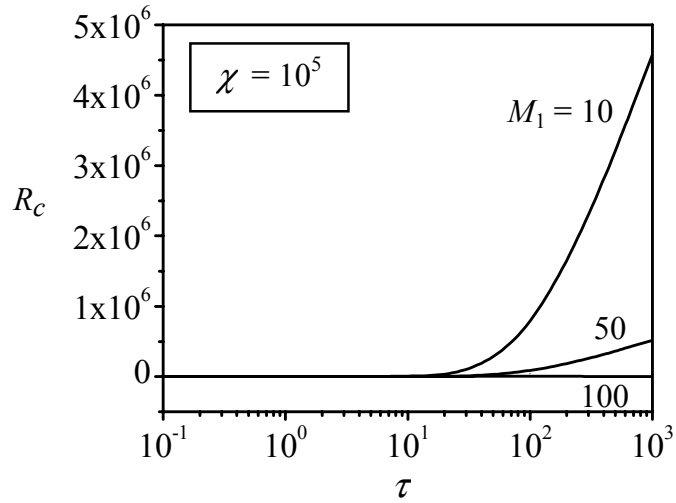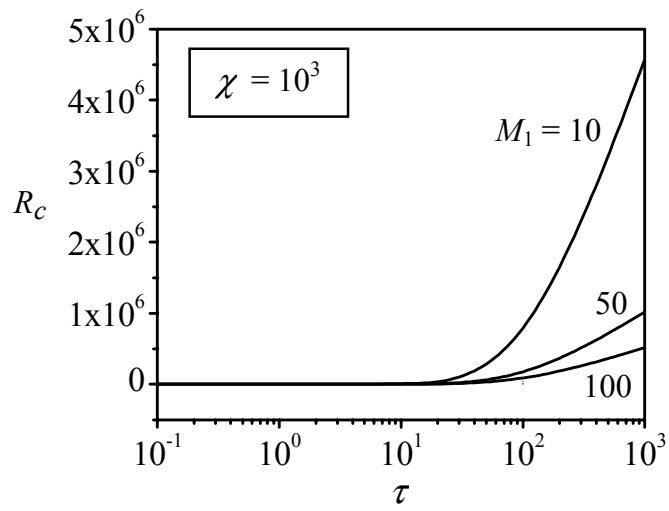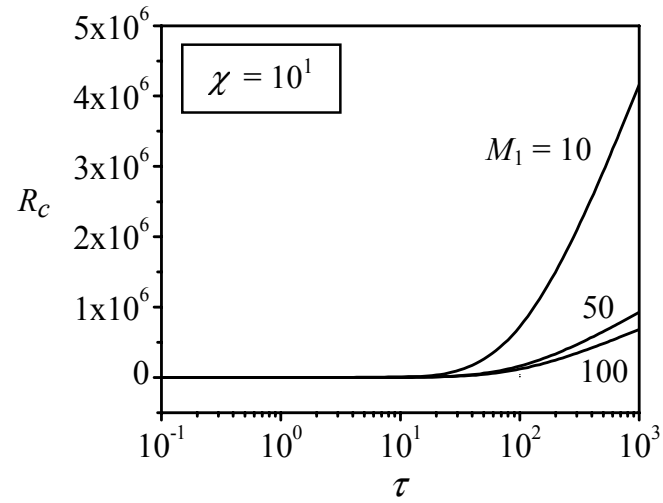
Figure 5.6: Plot of R_c versus τ for $\Gamma = 1$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and buoyancy-magnetization parameter M_1 (transparent case).

Figure 5.7: Plot of a_c versus τ for $\Gamma = 1$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and M_1 (transparent case).
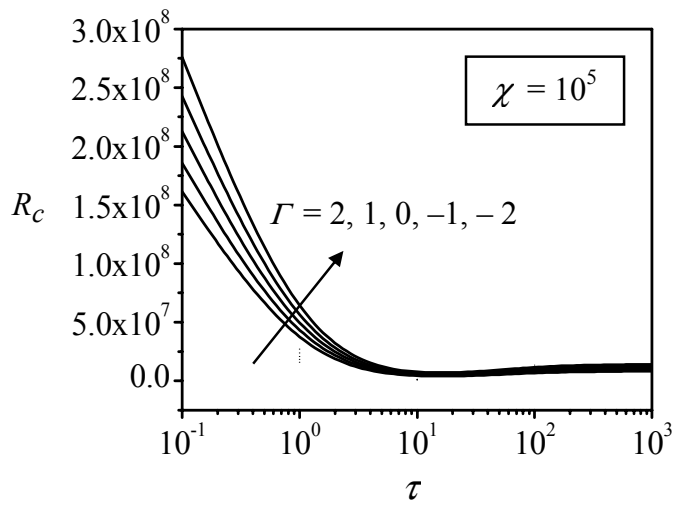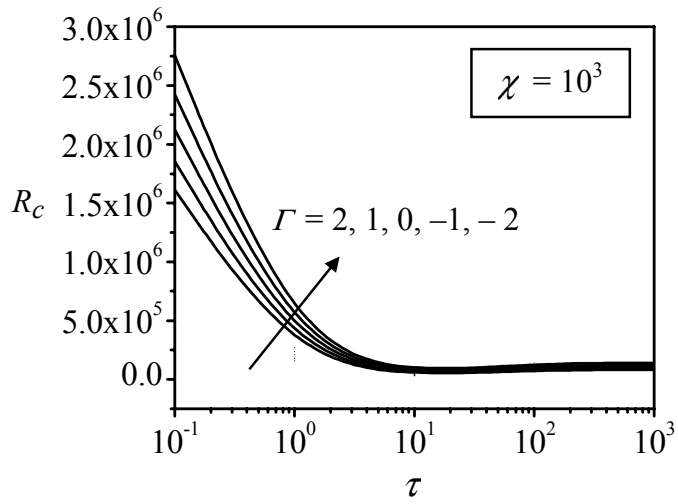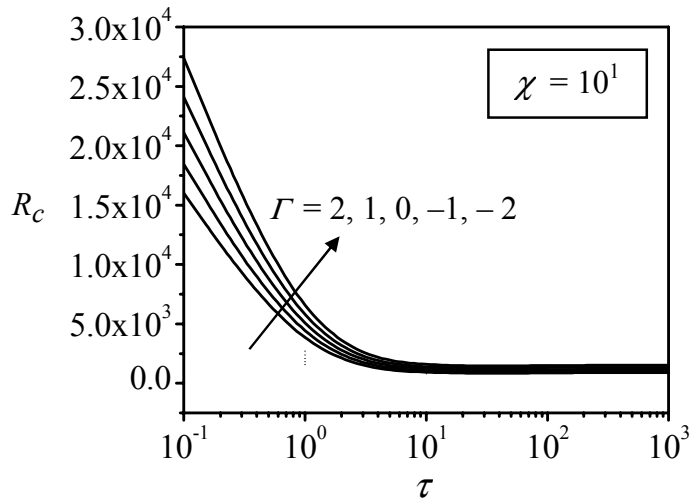
Figure 5.8: Plot of R_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and Γ (opaque case).
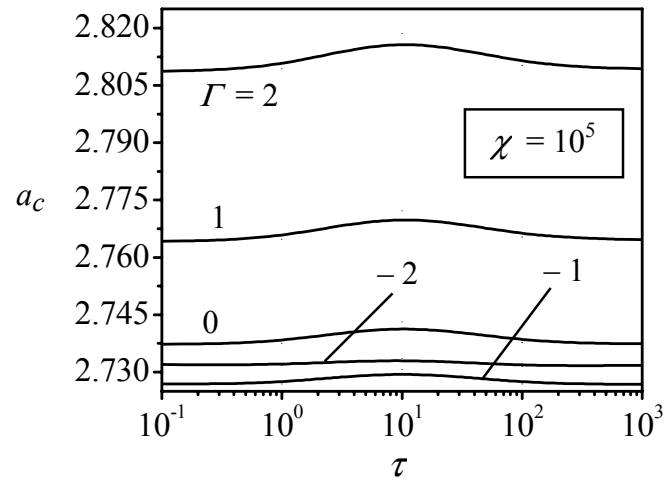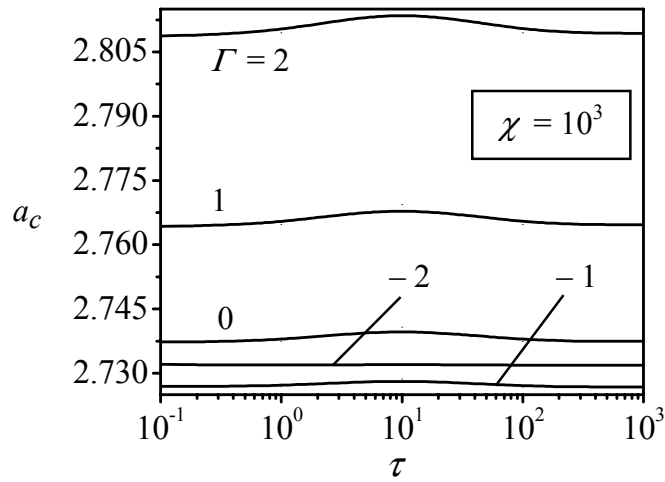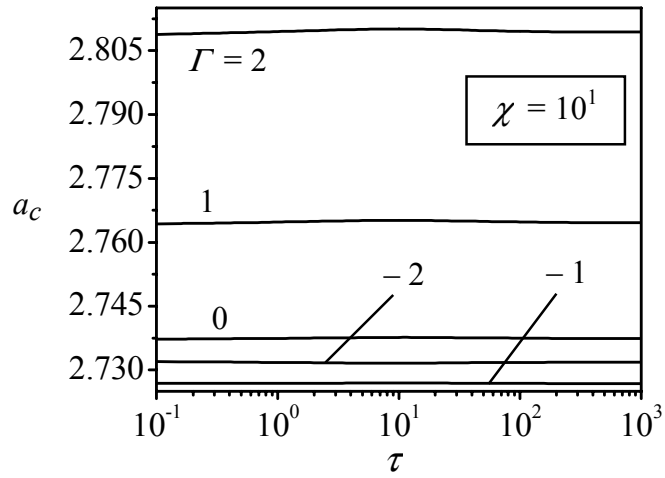
Figure 5.9: Plot of a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and Γ (opaque case).
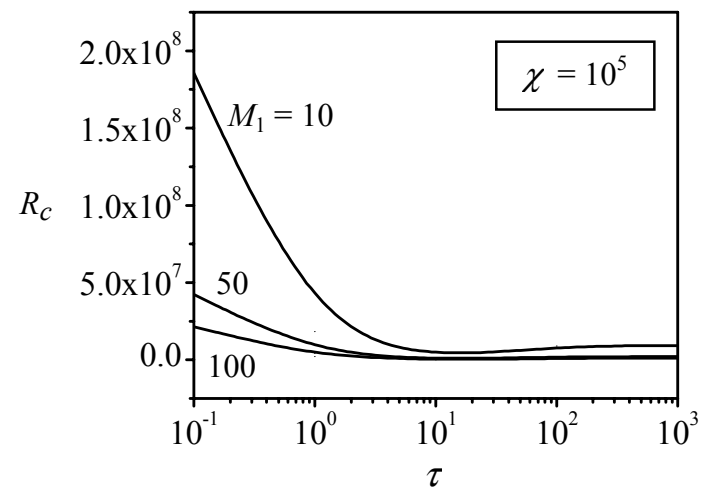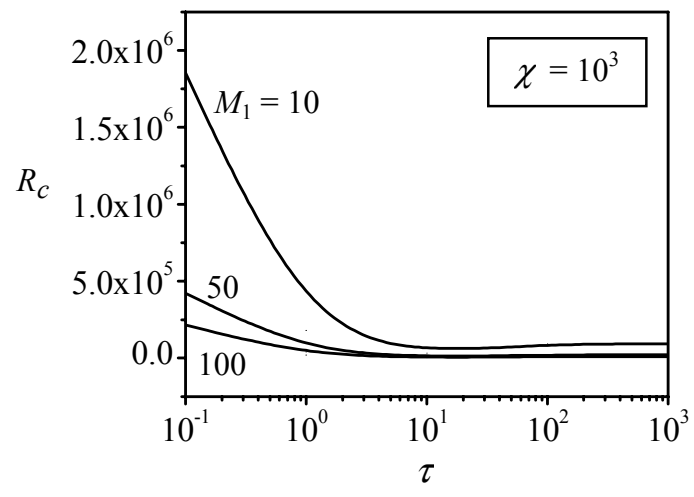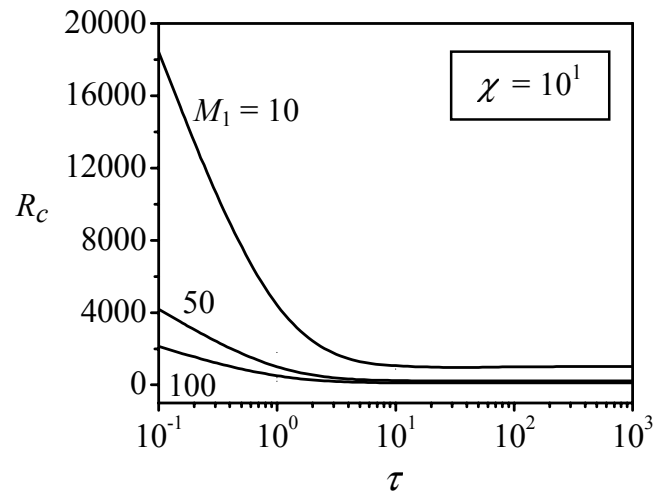
Figure 5.10: Plot of R_c versus τ for $\Gamma = 1$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and M_1 (opaque case).
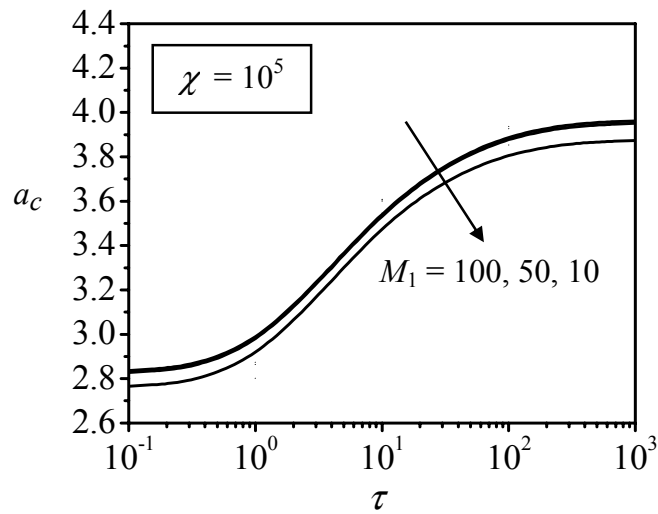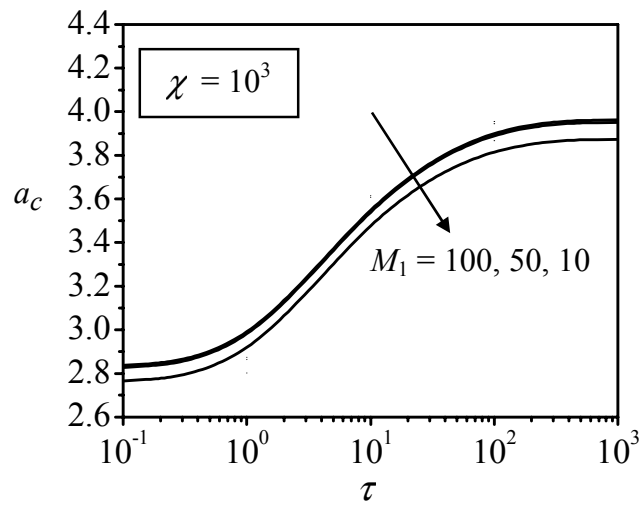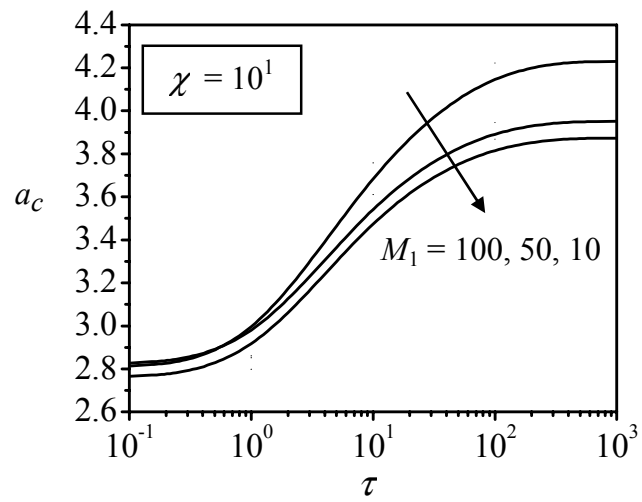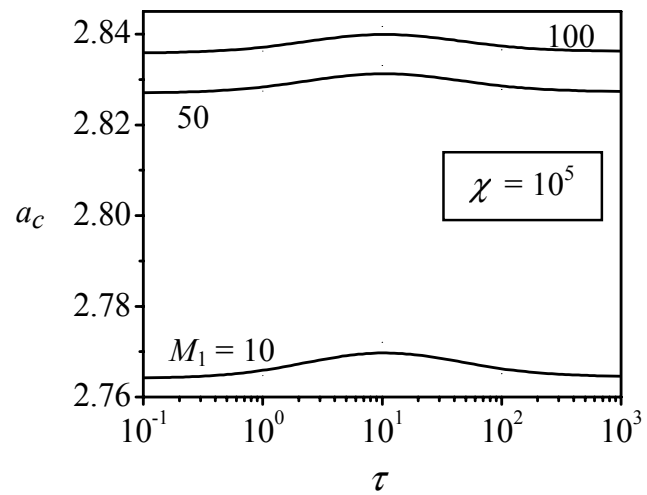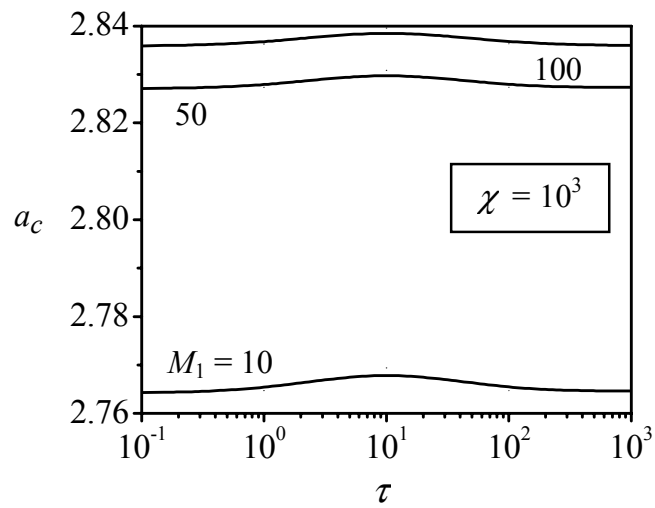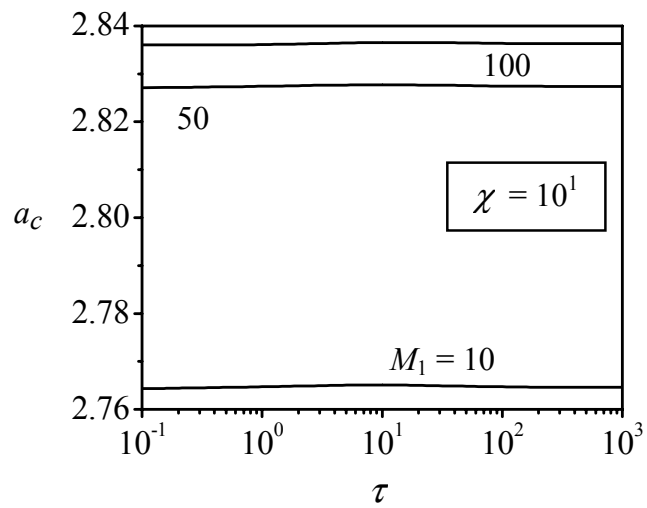
Figure 5.11: Plot of a_c versus τ for $\Gamma = 1$, $M_3 = 1$, $\chi_m = 1$, and for different values of χ and M_1 (opaque case).
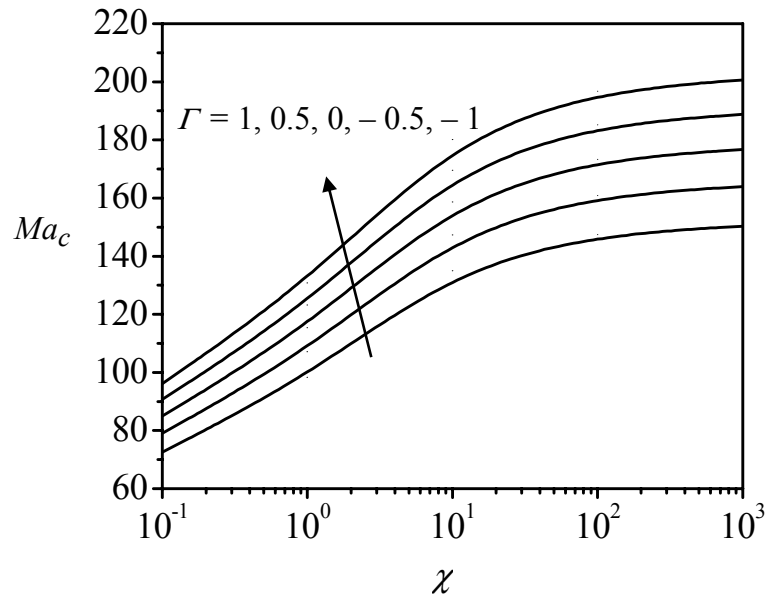
Figure 5.12: Plot of critical thermal Marangoni number Ma_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\tau = 1$ and for different values of Γ (transparent case).
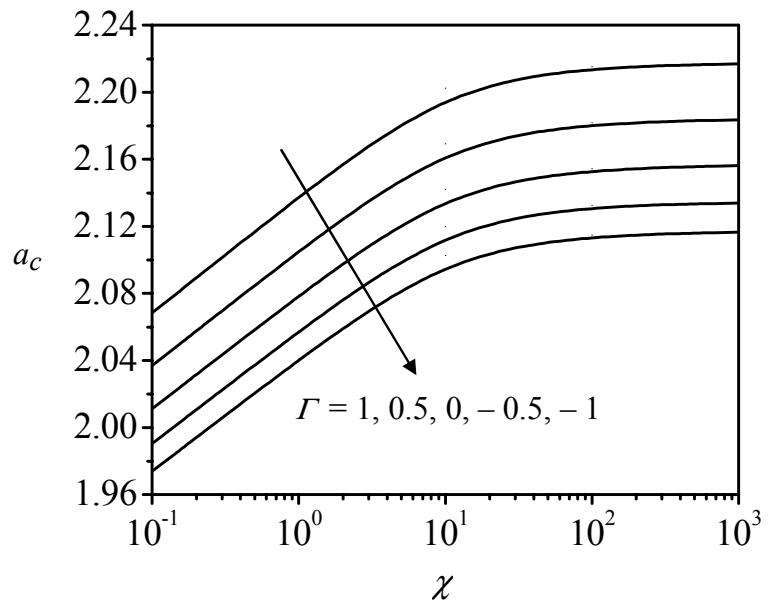
Figure 5.13: Plot of a_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\tau = 1$ and for different values of Γ (transparent case).

Figure 5.14: Plot of Ma_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\tau = 1$ and for different values of Bi (transparent case).

Figure 5.15: Plot of a_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\tau = 1$ and for different values of Bi (transparent case).
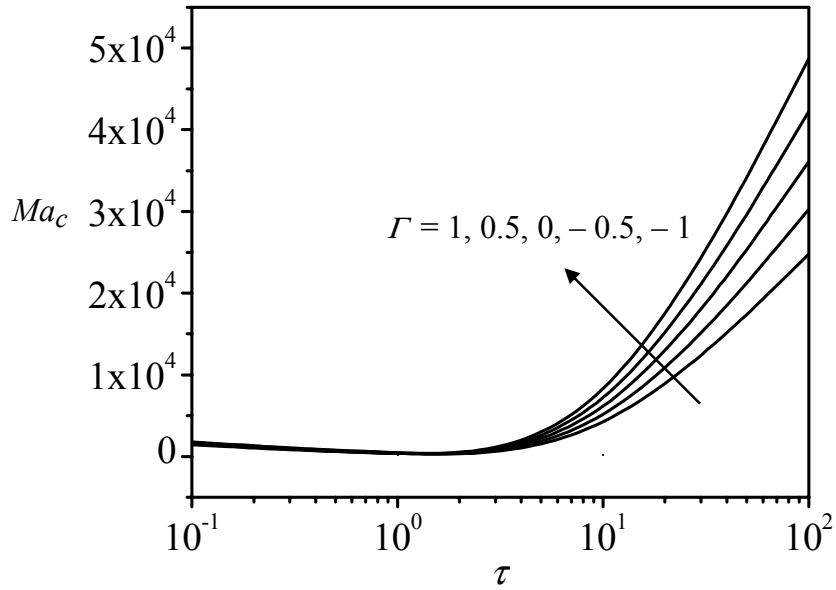
Figure 5.16: Plot of Ma_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\chi = 100$ and for different values of Γ (transparent case).
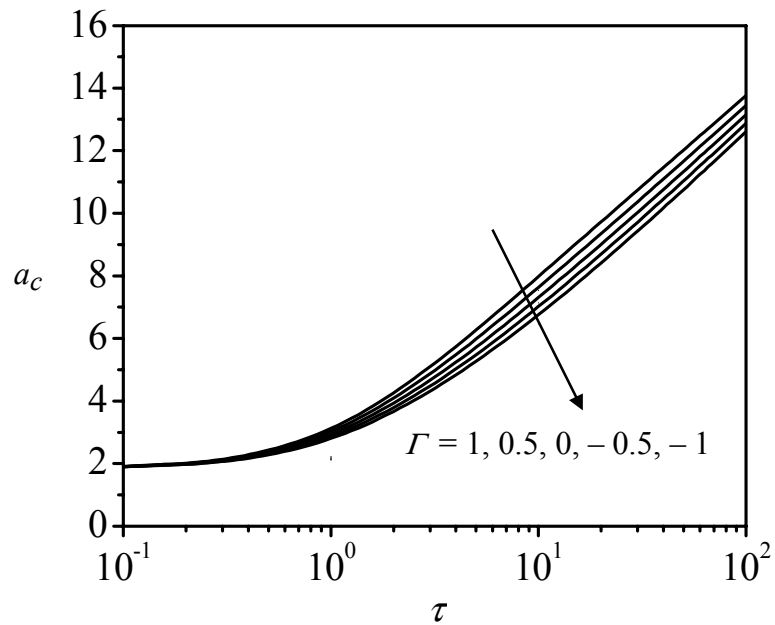
Figure 5.17: Plot of a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\chi = 100$ and for different values of Γ (transparent case).
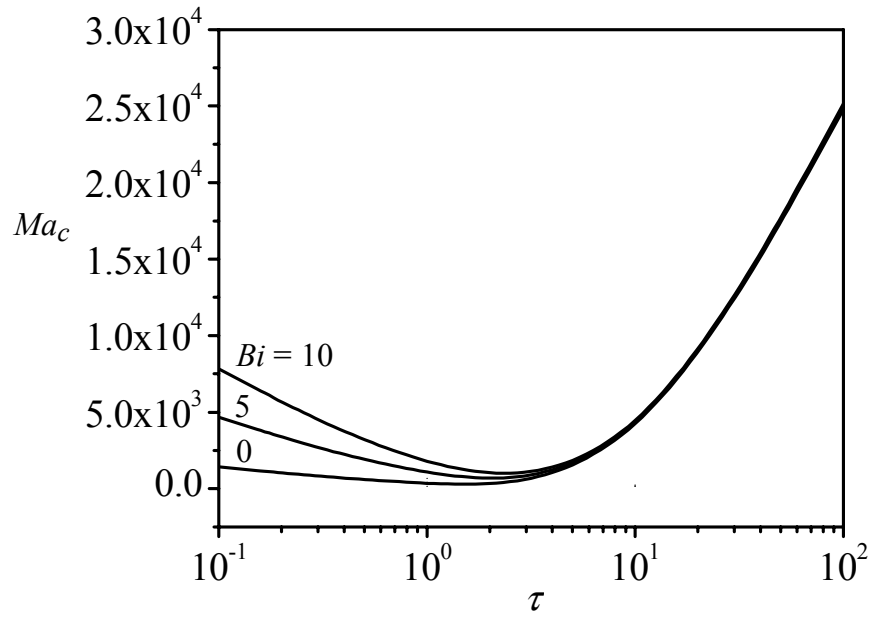
Figure 5.18: Plot of Ma_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\chi = 100$ and for different values of Bi (transparent case).
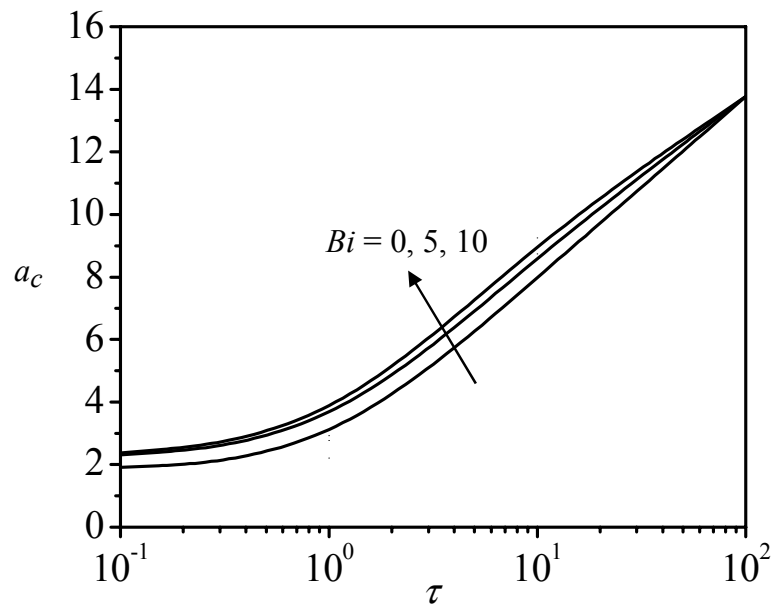
Figure 5.19: Plot of a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\chi = 100$ and for different values of Bi (transparent case).
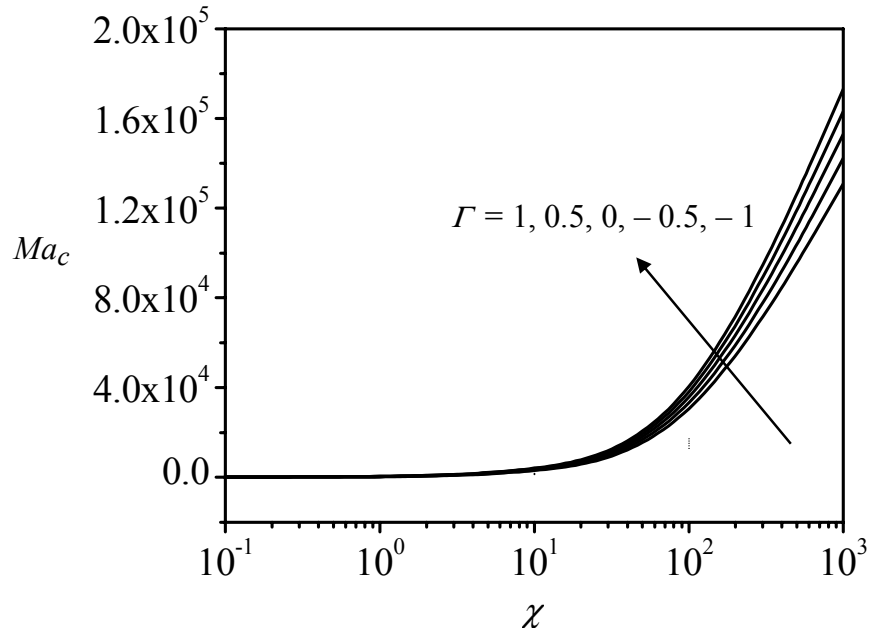
Figure 5.20: Plot of Ma_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\tau = 1$ and for different values of Γ (opaque case).
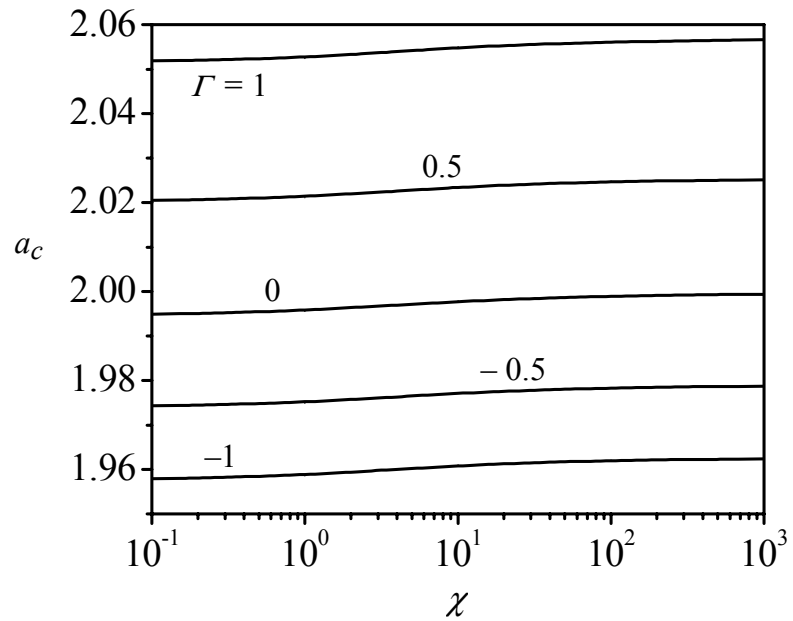
Figure 5.21: Plot of a_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\tau = 1$ and for different values of Γ (opaque case).
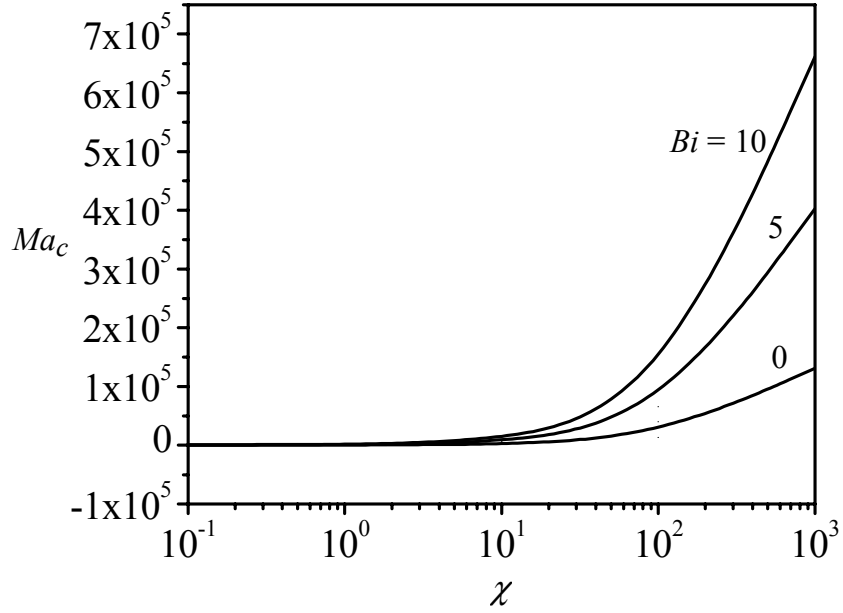
Figure 5.22: Plot of Ma_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\tau = 1$ and for different values of Bi (opaque case).
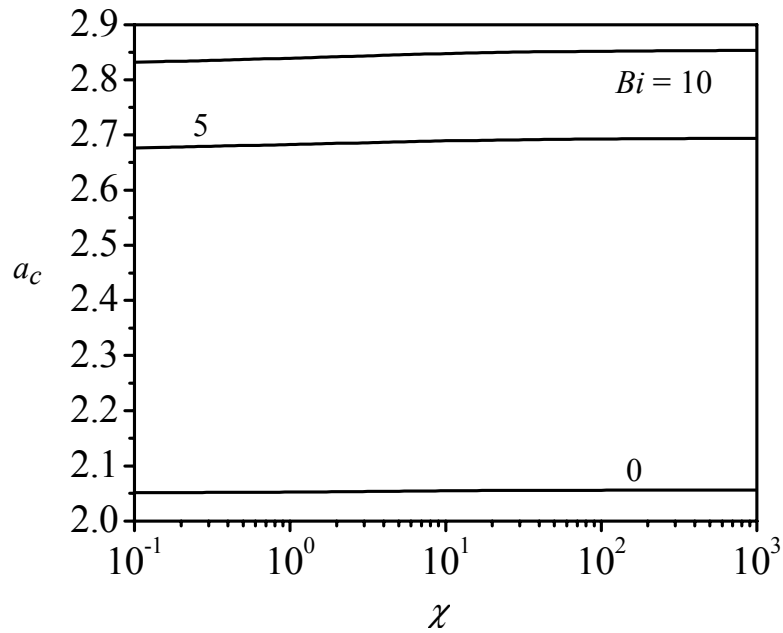
Figure 5.23: Plot of a_c versus χ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\tau = 1$ and for different values of Bi (opaque case).
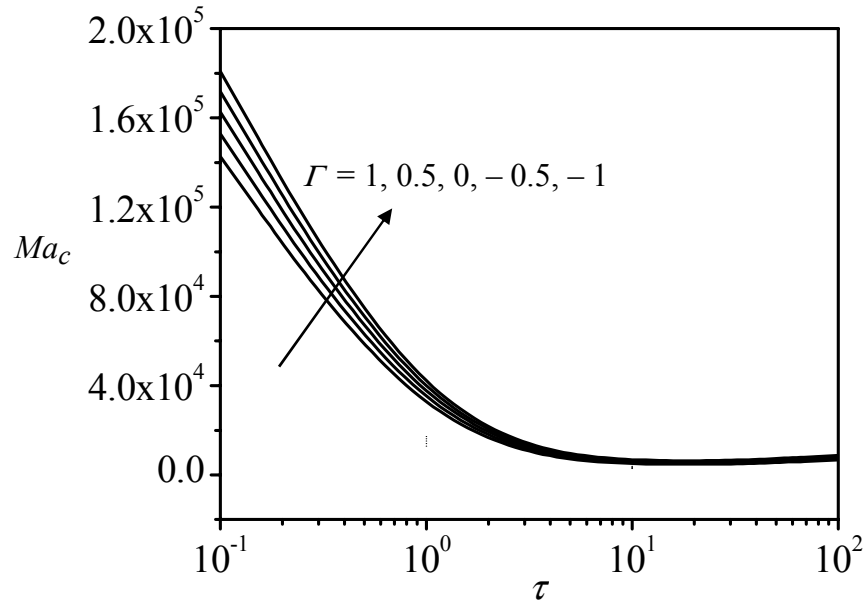
Figure 5.24: Plot of Ma_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\chi = 100$ and for different values of Γ (opaque case).
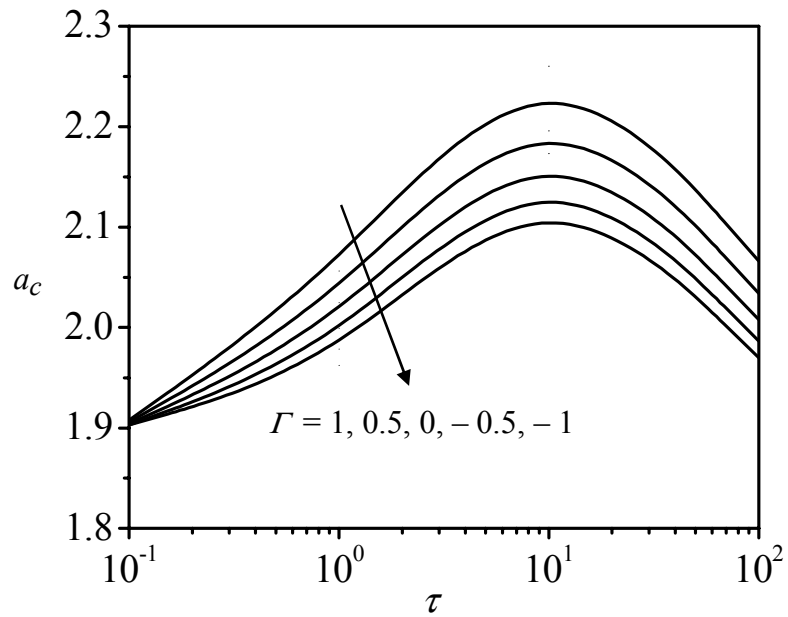
Figure 5.25: Plot of a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $Bi = 0$, $\chi = 100$ and for different values of Γ (opaque case).
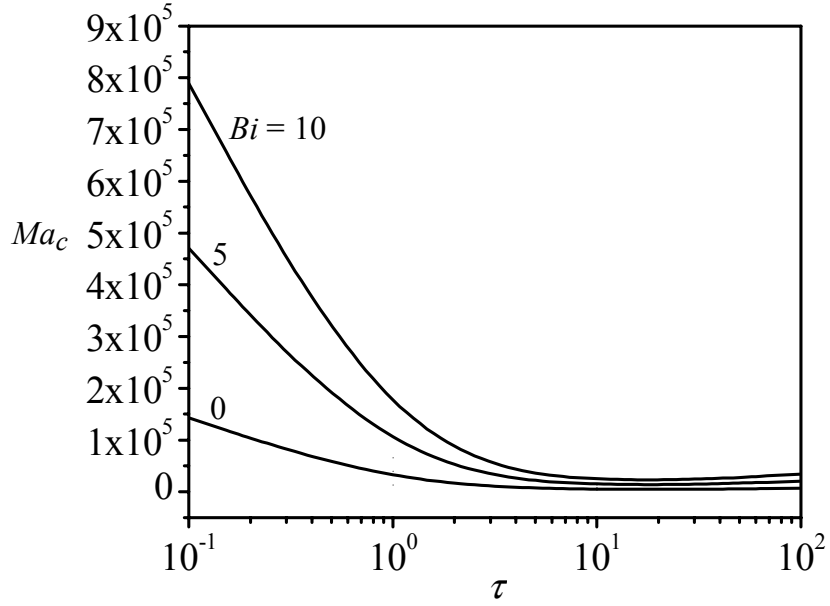
Figure 5.26: Plot of Ma_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\chi = 100$ and for different values of Bi (opaque case).

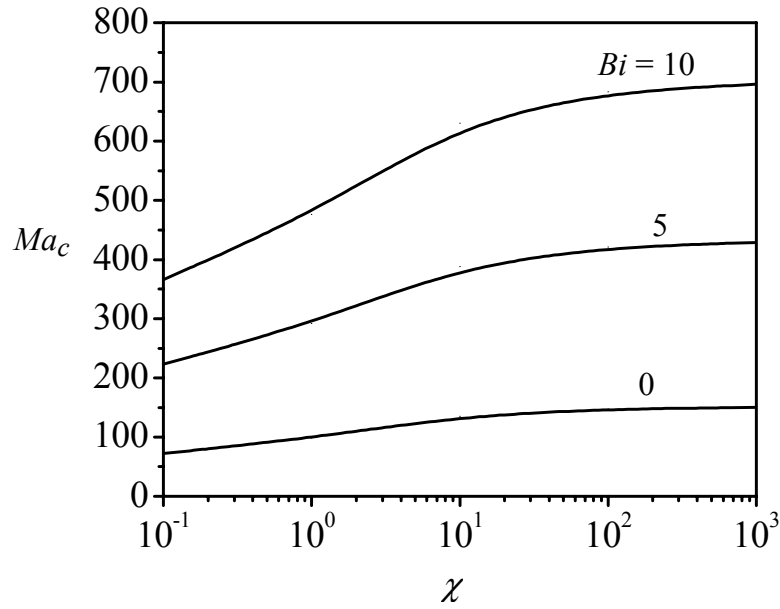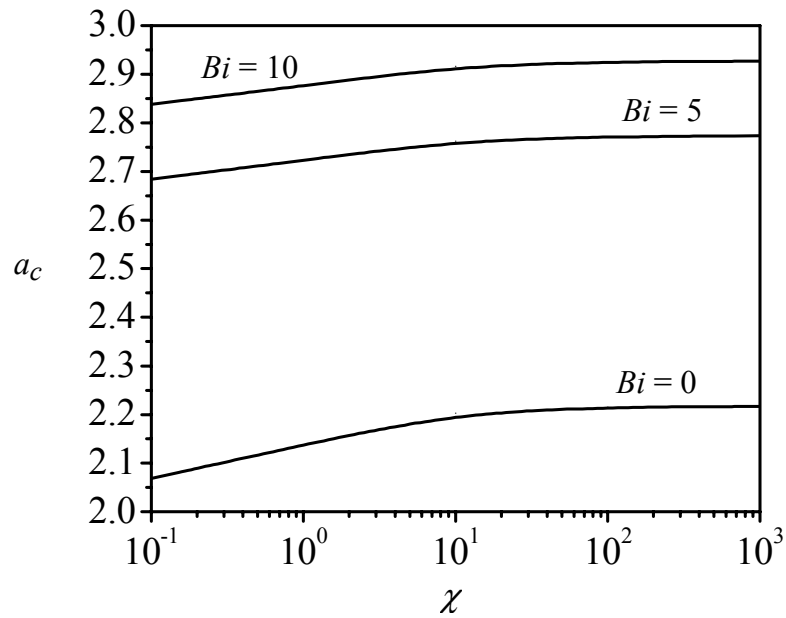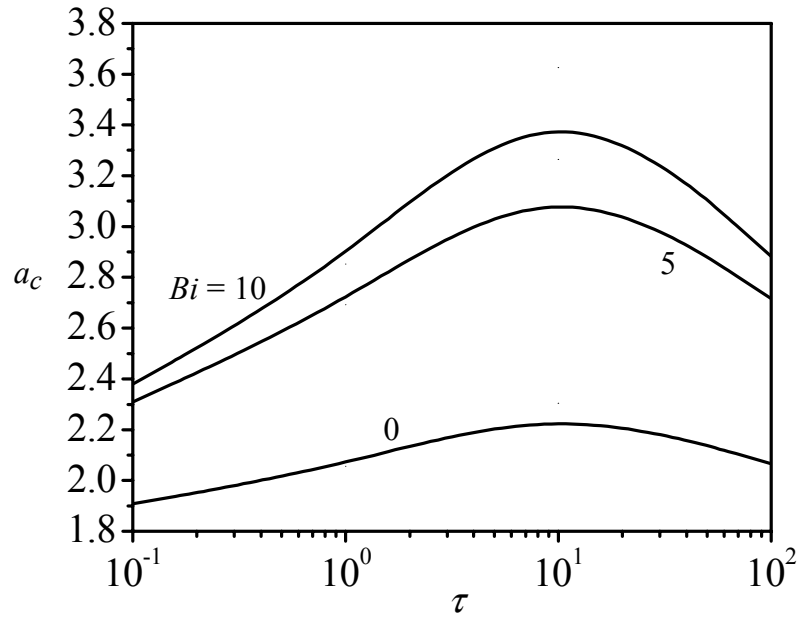**Figure 5.27**: Plot of a_c versus τ for $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$, $\Gamma = 1$, $\chi = 100$ and for different values of Bi (opaque case).

Table 5.1 : Critical thermal Rayleigh number (R_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with thermal radiation relating to the transparent case when the layer is bounded by free-free boundaries and $M_1 = 10$, $\tau = 100$.

χ	χ_m	M_3	$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$	
			R_c	a_c	R_c	a_c	R_c	a_c
10^1	1	1	54126.29	4.019	47733.17	3.935	41705.02	3.868
		5	42707.96	3.551	37425.19	3.499	32536.97	3.469
		25	38690.65	3.273	33817.71	3.239	29361.27	3.231
	5	1	57269.43	4.234	50599.07	4.136	44272.31	4.052
		5	43011.09	3.583	37692.42	3.528	32766.93	3.495
		25	38709.87	3.275	33833.91	3.242	29374.48	3.233
10^2	1	1	58882.03	4.019	51927.21	3.935	45369.55	3.868
		5	46471.71	3.551	40724.52	3.499	35406.41	3.469
		25	42102.39	3.273	36801.81	3.239	31953.92	3.231
	5	1	62311.299	4.235	55033.97	4.136	48170.55	4.053
		5	46803.507	3.583	41017.03	3.528	35658.14	3.495
		25	42123.54	3.276	36819.64	3.242	31968.47	3.233
10^3	1	1	59235.96	4.017	52238.57	3.933	45641.51	3.867
		5	46782.93	3.551	41000.57	3.499	35649.56	3.469
		25	42388.65	3.273	37058.51	3.239	32182.57	3.231
	5	1	62721.23	4.234	55415.84	4.135	48487.48	4.052
		5	47123.55	3.583	41300.87	3.528	35908.05	3.495
		25	42410.72	3.275	37077.11	3.242	32197.75	3.233

Table 5.2 : Critical thermal Rayleigh number (R_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with thermal radiation relating to the opaque case when the layer is bounded by free-free boundaries and $M_1 = 10$, $\tau = 100$.

χ	χ_m	M_3	$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$	
			R_c	a_c	R_c	a_c	R_c	a_c
10^1	1	1	1336.25	2.727	1170.85	2.737	1021.74	2.765
		5	952.90	2.506	833.86	2.523	728.15	2.558
		25	800.43	2.294	701.47	2.320	614.69	2.367
	5	1	1498.29	2.884	1309.94	2.886	1139.59	2.902
		5	969.43	2.538	847.59	2.553	739.34	2.586
		25	801.54	2.297	702.35	2.323	615.36	2.369
10^2	1	1	12261.04	2.727	10743.26	2.737	9375.03	2.765
		5	8743.99	2.506	7651.61	2.523	6681.53	2.558
		25	7344.98	2.294	6436.88	2.320	5640.56	2.367
	5	1	13748.39	2.884	12020.01	2.886	10456.78	2.902
		5	8895.76	2.538	7777.63	2.553	6784.29	2.586
		25	7355.18	2.297	6444.92	2.323	5646.73	2.369
10^3	1	1	121258.18	2.727	106244.08	2.737	92710.09	2.765
		5	86489.46	2.506	75682.46	2.523	66085.73	2.558
		25	72654.09	2.294	63670.68	2.320	55793.21	2.367
	5	1	135984.32	2.884	118884.83	2.886	103420.09	2.902
		5	87993.94	2.538	76931.69	2.553	67104.59	2.586
		25	72755.45	2.297	63750.51	2.323	55854.54	2.369

Table 5.3: Critical thermal Marangoni number (Ma_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with thermal radiation and $\chi = 100$, $\chi_m = 1$, $M_3 = 1$, $Bi = 0$, $\Gamma = 1$.

M_1	τ	Transparent case		Opaque case	
		Ma_c	a_c	Ma_c	a_c
	10^{-1}	1413.95	1.909	142593.69	1.908
10	10^{0}	147.46	2.214	12962.55	2.056
	10^{1}	145.56	7.973	2793.49	2.304
	10^{2}	24785.51	13.769	6685.034	2.066
	10^{-1}	1411.85	1.910	142591.59	1.908
50	10^{0}	145.39	2.224	12960.61	2.056
	10^{1}	145.28	7.954	2791.98	2.304
	10^{2}	24785.42	13.769	6683.10	2.066
	10^{-1}	1409.22	1.912	142588.97	1.908
50	10^{0}	142.79	2.234	12958.169	2.056
	10^{1}	144.93	7.931	2790.09	2.305
	10^{2}	24785.30	13.769	6680.69	2.066

Table 5.4 : Critical thermal Marangoni number (Ma_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with thermal radiation relating to the transparent case and $M_1 = 10$, $\tau = 1$, $Bi = 0$.

χ	χ_m	M_3	$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$	
			Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
10^1	1	1	180.12	2.103	158.69	2.142	135.05	2.202
		5	179.86	2.103	158.42	2.142	134.79	2.202
		25	179.64	2.101	158.22	2.140	134.60	2.200
	5	1	180.24	2.103	158.81	2.142	135.15	2.202
		5	179.89	2.103	158.46	2.142	134.82	2.202
		25	179.65	2.101	158.22	2.140	134.60	2.200
10^2	1	1	196.80	2.115	173.34	2.154	147.46	2.215
		5	196.53	2.115	173.07	2.154	147.20	2.215
		25	196.33	2.114	172.87	2.153	147.01	2.213
	5	1	196.92	2.115	173.46	2.155	147.56	2.216
		5	196.57	2.115	173.11	2.155	147.23	2.215
		25	196.33	2.114	172.88	2.153	147.02	2.213
10^3	1	1	200.67	2.116	176.74	2.156	150.34	2.217
		5	200.40	2.116	176.48	2.156	150.09	2.217
		25	200.19	2.115	176.27	2.155	149.90	2.215
	5	1	200.79	2.117	176.86	2.156	150.45	2.217
		5	200.44	2.116	176.51	2.156	150.12	2.217
		25	200.20	2.115	176.28	2.155	149.90	2.215

Table 5.5 : Critical thermal Marangoni number (Ma_c) and critical wavenumber (a_c) for a variable viscosity ferromagnetic liquid with thermal radiation relating to the opaque case and $M_1 = 10$, $\tau = 1$, $Bi = 0$.

χ	χ_m	M_3	$\Gamma = -1$		$\Gamma = 0$		$\Gamma = 1$	
			Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
10^1	1	1	1728.27	1.961	1527.69	1.998	1306.38	2.055
		5	1727.99	1.961	1527.43	1.998	1306.12	2.055
		25	1727.77	1.961	1527.21	1.998	1305.92	2.055
	5	1	1728.38	1.961	1527.81	1.998	1306.49	2.055
		5	1728.04	1.961	1527.46	1.998	1306.15	2.055
		25	1727.78	1.961	1527.21	1.998	1305.92	2.055
10^2	1	1	17149.64	1.962	15159.14	1.999	12962.55	2.056
		5	17149.37	1.962	15158.84	1.999	12962.29	2.056
		25	17149.15	1.962	15158.62	1.999	12962.09	2.056
	5	1	17149.76	1.962	15159.22	1.999	12962.66	2.056
		5	17149.41	1.962	15158.87	1.999	12962.33	2.056
		25	17149.15	1.962	15158.62	1.999	12962.09	2.056
10^3	1	1	173103.92	1.962	153010.53	1.999	130837.33	2.056
		5	173103.65	1.962	153010.26	1.999	130837.07	2.056
		25	173103.43	1.962	153010.05	1.999	130836.87	2.056
	5	1	173104.04	1.962	153010.65	1.999	130837.44	2.056
		5	173103.69	1.962	153010.30	1.999	130837.10	2.056
		25	173103.43	1.962	153010.05	1.999	130836.87	2.056

Table 5.6: Comparison of the present results with those of earlier works for the limiting case of a constant-viscosity, nonmagnetic liquid in the absence of thermal radiation.

Order of Rayleigh - Ritz technique	Present study		Pearson (1958) [Exact solution]		Nield (1964) [Fourier Series]		Siddheshwar and Pranesh (2002) [Single term Rayleigh – Ritz technique]	
	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c	Ma_c	a_c
1	59.29	2.34						
2	78.13	2.03						
3	79.68	1.99	80.00	2.00	79.61	1.99	78.44	2.43
4	79.61	1.99						
5	**79.61**	**1.99**						

CHAPTER VI

FERROCONVECTION IN VISCOELASTIC LIQUIDS

6.1 INTRODUCTION

Ferromagnetic/dielectric liquids, which are both thermally and magnetically/ electrically responding, are now well known from the viewpoint of smart liquid applications (Popplewell, 1984; Rosensweig, 1986; Berkovsky *et al.*, 1993; Tao and Roy, 1994; Zhakin, 1997; Kamath and Wereley, 1998; Kamath *et al.*, 1999; Zahn, 2001). Exhaustive literature on Rayleigh-Bénard and Marangoni instabilities in Newtonian ferromagnetic/dielectric liquids is presently available and the same is discussed in Chapter I. The viscoelastic behaviour in commercial ferromagnetic/dielectric fluids is particularly important since they exhibit long-term stability thereby allowing reproducibility of the experiments. Some recent works (Takashima and Ghosh 1979; Agrait and Castellanos, 1986; Kamiyama and Satoh, 1989; Bacri *et al.,* 1993; Odenbach *et al.*, 1999; Othman, 2001; Othman and Zaki, 2003) suggest that magnetic/dielectric liquids demonstrate a non-Newtonian behaviour in the presence of a magnetic/electric field. Odenbach (1999) substantiated that microgravity conditions can amplify the viscoelastic effects in magnetic fluids. The non-Newtonian characteristic can also be there in the ferromagnetic/dielectric liquid due to the carrier liquid being intrinsically non-Newtonian. Motivated by this, Siddheshwar (1998; 1999; 2002a; 2002b, 2005a; 2005b) studied stationary/oscillatory convection in Boussinesq, Oldroyd/Rivlin-Ericksen, third-grade and Careau-Bird ferromagnetic/dielectric liquids.

The most famous influence of magnetic fields on magnetic fluids is the increase in effective viscosity: the so-called magnetorheological effect. Kamiyama *et al.*, (1987) studied experimentally the rheological characteristics of water-, hydrocarbon- and diester-based magnetic fluids in the presence of a magnetic field. They observed that, unlike the hydrocarbon-based magnetic fluids that behave as a

Newtonian fluid, water- and diester-based magnetic fluids exhibit a pseudo-plastic characteristic. Recently, Balau *et al.* (2002) have corroborated the finding of Kamiyama *et al.* (1987) through their experiments that the magnetorheological effect is of significant importance in water-based and kerosene-based solutions, and in physiological-solution-based magnetic liquids even for moderate strengths of applied magnetic field. The works of Kamiyama *et al.* (1987) and Balau *et al.* (2002) lay emphasis on the need to resort to the use of general equations that encompass both Newtonian and viscoelastic descriptions of magnetic fluids if one wishes to account for the rheological aspects. The electric field analog of magnetorheological effect is known as electrorheological effect (Sun and Rao, 1996; Klingenberg, 1998; Otsubo and Edamura, 1998; Hanaoka *et al.*, 2002) which essentially signifies an upsurge in effective viscosity due to the applied electric field.

Another fact about the viscosity of any carrier liquid decreasing with temperature is also well known (Stengel *et al.*, 1982; Gebhart *et al.,* 1988, Platten and Legros 1984, Severin and Herwig 1999) and is referred to as thermorheological effect. Recently, Siddheshwar (2004) studied the thermorheological effect on magnetoconvection in fluids with weak electrical conductivity under 1g and μg conditions. It is imperative therefore to investigate the problems of both *RBC* and *MC* in Newtonian/viscoelastic ferromagnetic/dielectric liquids involving the dependency of effective viscosity on temperature and on the magnitude of magnetic/electric fields.

In this chapter Newtonian as well as three viscoelastic descriptions, *viz.*, Jeffrey, Maxwell and Rivlin-Ericksen, for both ferromagnetic and dielectric liquids are chosen for demonstrating an analogy between the two smart liquids with the aforementioned rheological effects. The work also emphasizes on the need to seek unification of several works, wherever possible, in making a general study so that limiting cases need not be done in isolation. The analogy that we seek to demonstrate is between *dc* ferroconvection and *ac* electroconvection.

151

6.2 MATHEMATICAL FORMULATION

We consider ferromagnetic and dielectric liquids of viscoelastic type (Jeffrey, Maxwell and Rivlin-Ericksen) and of Newtonian type. Each liquid is discussed below using the notation of Chapter II and related standard works.

6.2.1 Jeffrey Ferromagnetic Liquid

We consider an infinite horizontal layer of a Jeffrey ferromagnetic liquid of thickness d. The upper plane at $z = d/2$ and the lower one at $z = -d/2$ are maintained at constant temperatures T_0 and $T_1 (= T_0 + \Delta T)$ respectively. In addition to a temperature gradient, a vertical uniform *dc* magnetic field, $\vec{H}_0$, is also imposed across the layer (Figure 6.1). The Jeffrey liquid is assumed to have an effective variable viscosity μ that depends on the temperature as well as on the magnetic field strength.

The governing equations for the viscoelastic ferromagnetic fluid of Jeffrey type with variable-viscosity in the notation of Chapter II and Chapter IV are

$$\nabla \cdot \vec{q} = 0, \tag{6.2.1}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left[\rho_0 \frac{D\vec{q}}{Dt} + \rho g \hat{k} + \nabla p - \mu_0 (\vec{M}.\nabla)\vec{H}\right]$$
$$= \left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\nabla.\left[\mu(H,T)\left(\nabla\vec{q} + \nabla\vec{q}^{Tr}\right)\right] \tag{6.2.2}$$

$$\rho_0 C_{VH}\left[\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T\right] = k_1 \nabla^2 T, \tag{6.2.3}$$

$$\rho = \rho_0\left[1 - \alpha(T - T_a)\right], \tag{6.2.4}$$

$$\nabla.\vec{B} = 0, \quad \nabla\times\vec{H} = \vec{0}, \tag{6.2.5}$$

$$\vec{B} = \mu_o \left(\vec{M} + \vec{H} \right), \tag{6.2.6}$$

$$M = M_o + \chi_m \left(H - H_o \right) - K_1 \left(T - T_a \right), \tag{6.2.7}$$

$$\mu \left(H, T \right) = \mu_1 \left[1 + \delta_1 \left(H - H_o \right)^2 - \delta_2 \left(T - T_a \right)^2 \right]. \tag{6.2.8}$$

The effect of internal heat source and thermal radiation are neglected in writing the energy equation (6.2.3). The fluid layer is initially quiescent. Let this state be slightly disturbed. Following the classical lines of linear stability theory discussed in the earlier chapters, the non-dimensional equations governing infinitesimal perturbations, on application of the normal mode technique discussed in Chapter III, may be written as

$$(1 - \Gamma z^2)(D^2 - a^2)^2 w - 4\Gamma z (D^2 - a^2) Dw - 2\Gamma (D^2 + a^2) w$$
$$- \frac{G_1 \sigma}{Pr} (D^2 - a^2) w - R(1 + M_1) G_1 a^2 T + R M_1 G_1 a^2 D\Phi = 0, \tag{6.2.9}$$

$$(D^2 - a^2 - \sigma) T + w = 0, \tag{6.2.10}$$

$$\left(D^2 - M_3 a^2 \right) \Phi - DT = 0, \tag{6.2.11}$$

where $D = \dfrac{d}{dz}$, $G_1 = \dfrac{1 + \Gamma_V \sigma}{1 + \Gamma_V \eta \sigma}$ and the remaining quantities are as defined earlier. Equations (6.2.9) – (6.2.11) are solved subject to the boundary conditions

$$\left. \begin{array}{c} w = \left[1 - \Gamma z^2 \right] D^2 w + a^2 Ma\, T - a^2 Ma_H\, D\Phi = DT = 0 \\[2mm] \text{and} \quad D\Phi + \dfrac{a\Phi}{1 + \chi_m} - T = 0 \quad \text{on} \quad z = \dfrac{1}{2}, \\[3mm] w = Dw = T = D\Phi - \dfrac{a\Phi}{1 + \chi_m} = 0 \quad \text{on} \quad z = -\dfrac{1}{2}. \end{array} \right\} \tag{6.2.12}$$

In the numerical calculations pursued later we assume $\left| Ma_H \right| \ll 1$. It should be noted that in the case of *RBC* (*Ma* being zero), the thermal Rayleigh number *R* and

the buoyancy-magnetization parameter M_1 come into play along with other parameters arising in the problem. On the other hand, in the case of MC (R being zero), the thermal Marangoni number Ma and the magnetic Rayleigh number R_M $(=RM_1)$ come into picture. Equations (6.2.9) – (6.2.12) constitute an eigenvalue system for the present problem and take care of most non-isothermal situations of the RBC and MC in different continuums describing ferromagnetic liquids.

6.2.2 Analogy Between Ferromagnetic and Dielectric Liquids of Jeffrey Type

The eigenvalue system for the Jeffrey dielectric liquid (with the effective viscosity being thermally and electrically responding), following the analysis of Chapter II and Chapter IV, can be written as

$$(1 - \Gamma z^2)(D^2 - a^2)^2 w - 4\,\Gamma z (D^2 - a^2) Dw - 2\Gamma(D^2 + a^2)w$$
$$- \frac{G_1 \sigma}{Pr}(D^2 - a^2)w - (R + R_E)G_1 a^2\,T + R_E\,G_1\,a^2\,D\Phi \;=\; 0\,, \quad (6.2.13)$$

$$(D^2 - a^2 - \sigma)\,T + w \;=\; 0\,, \tag{6.2.14}$$

$$\left(D^2 - a^2\right)\Phi - DT = 0\,, \tag{6.2.15}$$

where the physical quantities appearing in the above equations have their usual meaning outlined in the context of variable-viscosity dielectric liquids. Equations (6.2.13) – (6.2.15) are solved subject to the boundary conditions

$$\left.\begin{array}{l} w = \left[1 - \Gamma z^2\right] D^2 w + a^2 Ma\,T - a^2 Ma_E\,D\Phi \;=\; DT = 0 \\[2mm] \quad \text{and} \quad D\Phi + \dfrac{a\Phi}{1 + \chi_e} - T = 0 \quad \text{on} \;\; z \;=\; \dfrac{1}{2}, \\[4mm] w \;=\; Dw \;=\; T \;= D\Phi - \dfrac{a\Phi}{1 + \chi_e} = 0 \quad \text{on} \;\; z \;=\; -\dfrac{1}{2}. \end{array}\right\} \tag{6.2.16}$$

We now discuss the analogy between *RBC* and *MC* in Jeffrey ferromagnetic and Jeffrey dielectric liquids. Comparing the set of Eqs. (6.2.9) – (6.2.12) with that of Eqs. (6.2.13) – (6.2.16), it is quite evident that the latter set can be obtained from the former by setting $M_3 = 1$, and replacing the product RM_1 by R_E, Ma_H by Ma_E and χ_m by χ_e.

We now consider Maxwell and Rivlin-Ericksen liquids that are essentially limiting cases of the rather general equations considered above for both ferromagnetic and dielectric liquids.

6.2.3 Maxwell Ferromagnetic/Dielectric Liquid

The eigenvalue equations for this liquid are Eqs. (6.2.10) – (6.2.12) together with the following equation that can be obtained from Eq. (2.1.20) in the limit of $\lambda_2 \to 0$

$$(1 - \Gamma z^2)(D^2 - a^2)^2 w - 4\,\Gamma z (D^2 - a^2) Dw - 2\,\Gamma (D^2 + a^2) w$$

$$- \frac{(1 + \Gamma_V \sigma)\sigma}{Pr}(D^2 - a^2) w - R(1 + M_1)(1 + \Gamma_V \sigma) a^2\, T \tag{6.2.17}$$

$$+ RM_1(1 + \Gamma_V \sigma) a^2\, D\Phi = 0 \,.$$

It should be noted that the equations for Maxwell ferromagnetic liquids can easily be obtained by taking $\eta = 0$ in the corresponding equations for the Jeffrey liquids. Further, the equations for Maxwell dielectric liquids can be obtained by adopting the procedure explained in Section 6.2.2.

6.2.4 Rivlin-Ericksen Ferromagnetic/Dielectric Liquid

The eigenvalue equations for this liquid are Eqs. (6.2.10) – (6.2.12) together with the following equation that can be obtained from Eq. (2.1.20) in the limit of $\lambda_1 \to 0$ and $\lambda_2 \to \mu_2/\mu_1$

$$\left(1+\frac{Q\sigma}{Pr}\right)\left[(1-\Gamma z^2)(D^2-a^2)^2 w - 4\,\Gamma z(D^2-a^2)Dw - 2\,\Gamma(D^2+a^2)w\right]$$

$$-\frac{\sigma}{Pr}(D^2-a^2)w - R(1+M_1)a^2\,T + RM_1\,a^2\,D\Phi \;=\; 0\,, \qquad (6.2.18)$$

where $Q=\mu_2\big/\rho_{\mathrm{o}}d^2$ and other quantities are as defined earlier. We note that the quantity Q characterizes viscoelasticity of the Rivlin-Ericksen liquid (Siddheshwar 1999; 2002b; Siddheshwar and Srikrishna 2002). It is seen from Eqs. (6.2.9) and (6.2.18) that the equations for the Rivlin-Ericksen ferromagnetic liquid can be obtained from those of Jeffrey ferromagnetic liquid by the limiting process $\Gamma_V \to 0,\ \eta \to \infty$ and $\Gamma_V\,\eta \to \dfrac{Q}{Pr}$. In view of the analogy discussed earlier, we may write down the governing equations for the Rivlin-Ericksen dielectric liquid from the corresponding equations for the ferromagnetic liquids.

6.2.5 Newtonian Ferromagnetic/Dielectric Liquids

The eigenvalue equations for the Newtonian ferromagnetic liquid are Eqs. (6.2.10) – (6.2.12) together with the following equation that can be obtained from Eq. (2.1.20) in the limit of $\lambda_1 \to \lambda_2$

$$\left[(1-\Gamma z^2)(D^2-a^2)^2 w - 4\,\Gamma z(D^2-a^2)Dw - 2\,\Gamma(D^2+a^2)w\right]$$

$$-\frac{\sigma}{Pr}(D^2-a^2)w - R(1+M_1)a^2\,T + RM_1\,a^2\,D\Phi \;=\; 0\,, \qquad (6.2.19)$$

It is unambiguously clear from Eqs. (6.2.9) and (6.2.19) that, the assumption $G=1$, *i.e.*, both Γ_V and η are zero or alternatively $\eta=1$, leads to the latter equation and the analogy discussed in Section 6.2.2 makes it possible to extend the results of Newtonian ferromagnetic liquid to those of Newtonian dielectric liquid. The eigenvalue equations pertaining to the work of Finlayson (1970), who studied the onset of RBC in Newtonian ferromagnetic fluids, can be obtained by taking $\Gamma=0$ in Eq. (6.2.19). From Sections (6.2.1) – (6.2.5), it is obvious that if one makes a study of the RBC and MC in Jeffrey ferromagnetic liquids, then one

essentially has made the study in Maxwell / Rivlin-Ericksen / Newtonian ferromagnetic liquids. Further, in view of the analogy the corresponding studies in dielectric liquids are redundant.

6.3 STABILITY ANALYSIS

In the light of the foregoing discussions, it would suffice to examine the thermo- and magneto-rheological effects on *RBC* and *MC* in Jeffrey ferromagnetic liquids. In the following Sections 6.3.1 and 6.3.2, we shall discuss the stationary and oscillatory instabilities of the problem under consideration with the help of Eqs. (6.2.9) – (6.2.12).

6.3.1 Stationary Instability

In this section, we discuss the method of solution pertaining to the stationary instability ($\sigma = 0$). Since the presence of space varying coefficients in Eq. (6.2.9) and the asymmetric boundary conditions given in Eq. (6.2.12) render the problem analytically intractable, we employ higher order Rayleigh-Ritz technique (*HORT*) to compute the critical values. To this end, we expand $w(z)$, $T(z)$ and $\Phi(z)$ in a series of trial functions as

$$w(z) = \sum_{i=1}^{n} \alpha_i w_i(z), \quad T(z) = \sum_{i=1}^{n} \beta_i T_i(z) \quad \text{and} \quad \Phi(z) = \sum_{i=1}^{n} \gamma_i \Phi_i(z), \qquad (6.3.1)$$

where α_i, β_i and γ_i are constants, and $w_i(z), T_i(z)$ and $\Phi_{\text{i}}(z)$ are trial functions. We choose the following trial functions

$$w_i = \left(z - \frac{1}{2}\right)\left(z + \frac{1}{2}\right)^{i+1}, \quad T_i = \left(z(z-1) - \frac{3}{4}\right)^{i+1}, \quad \Phi_i = z^{\text{i}}$$

guided by the boundary conditions in Eq. (6.2.12) and variational considerations. Application of *HORT* to Eqs. (6.2.9) – (6.2.11) leads to a system of homogeneous equations using which we get the critical eigenvalues.

157

6.3.2 Oscillatory Instability

We now discuss the possibility of the existence of overstable motions. Assuming $\sigma = i\omega$ in Eqs. (6.2.9) – (6.2.11) and applying the first order Rayleigh-Ritz technique ($i = j = 1$) discussed in Chapter IV to the resulting equations, we obtain (after some heavy algebra) the following expression for the thermal Rayleigh number R

$$R = \frac{X_9 \left[\left\{ X_1 X_6 N_1 + \left(X_1 N_2 - \dfrac{X_2}{Pr} \right) X_7 \, \omega^2 + Ma X_3 X_4 N_1 \right\} + i\omega N_3 \right]}{a^2 X_4 \left[(1+M_1) X_4 X_9 + M_1 X_5 X_8 \right]},$$

$$(6.3.2)$$

where

$$X_1 = \left\langle (D^2 w_1)^2 \right\rangle - 2\,a^2 \left\langle w_1 \left(1 - \Gamma z^2\right) D^2 w_1 \right\rangle - \Gamma \left\langle w_1 z^2 D^4 w_1 \right\rangle$$
$$+ a^4 \left\langle w_1 \left(1 - \Gamma z^2\right) w_1 \right\rangle + 2\,\Gamma \left\langle (Dw_1)^2 \right\rangle - 2\,\Gamma a^2 \left\langle w_1^2 \right\rangle$$
$$- 4\,\Gamma \left\langle w_1 z D^3 w_1 \right\rangle + 4\,\Gamma a^2 \left\langle w_1 z Dw_1 \right\rangle,$$

$$X_2 = \left\langle (Dw_1)^2 \right\rangle + a^2 \left\langle w_1^2 \right\rangle, \qquad X_3 = \frac{4\,a^2}{(4-\Gamma)} Dw_1(\tfrac{1}{2}) T_1(\tfrac{1}{2}),$$

$$X_4 = \left\langle w_1 T_1 \right\rangle, \qquad X_5 = \left\langle w_1 D\Phi_1 \right\rangle, \qquad X_6 = \left\langle (DT_1)^2 \right\rangle + a^2 \left\langle T_1^2 \right\rangle,$$

$$X_7 = \left\langle T_1^2 \right\rangle, \qquad X_8 = \left\langle \Phi_1 DT_1 \right\rangle - \Phi_1(\tfrac{1}{2}) T_1(\tfrac{1}{2}),$$

$$X_9 = \left\langle (D\Phi_1)^2 \right\rangle + M_3 a^2 \left\langle \Phi_1^2 \right\rangle + \frac{a}{1+\chi_m} \left[\left\{ \Phi_1\left(\tfrac{1}{2}\right) \right\}^2 + \left\{ \Phi_1\left(\tfrac{-1}{2}\right) \right\}^2 \right],$$

$$N_1 = \frac{1 + \Gamma_V^2 \eta \omega^2}{1 + \Gamma_V^2 \omega^2}, \qquad N_2 = \frac{\Gamma_V(1-\eta)}{1 + \Gamma_V^2 \omega^2},$$

$$N_3 = X_1 X_7 N_1 - X_1 X_6 N_2 + \frac{X_2 X_6}{Pr} - Ma X_3 X_4 N_2,$$

and $\langle uv \rangle = \int\limits_{-1/2}^{1/2} uv \, dz$. Since R is a real quantity, the imaginary part of Eq. (6.3.2)

has to vanish. This gives us two possibilities:

(i) $\omega \neq 0$, $N_3 = 0$ (oscillatory instability),

(ii) $\omega = 0$, $N_3 \neq 0$ (stationary instability).

We note that, unlike the problems discussed in Chapters III – V, the condition $N_3 = 0$ leads to an expression for the square of the frequency of oscillations

$$\omega^2 = \frac{Pr\left[\Gamma_V (1-\eta)\{X_1 X_6 + Ma X_3 X_4\} - X_1 X_7\right] - X_2 X_6}{\Gamma_V{}^2\left[X_2 X_6 + Pr\,\eta\, X_1 X_7\right]}. \qquad (6.3.3)$$

The above expression for ω^2 is in turn substituted in the real part of the expression for R given in Eq. (6.3.2) thereby yielding the oscillatory thermal Rayleigh number R^O.
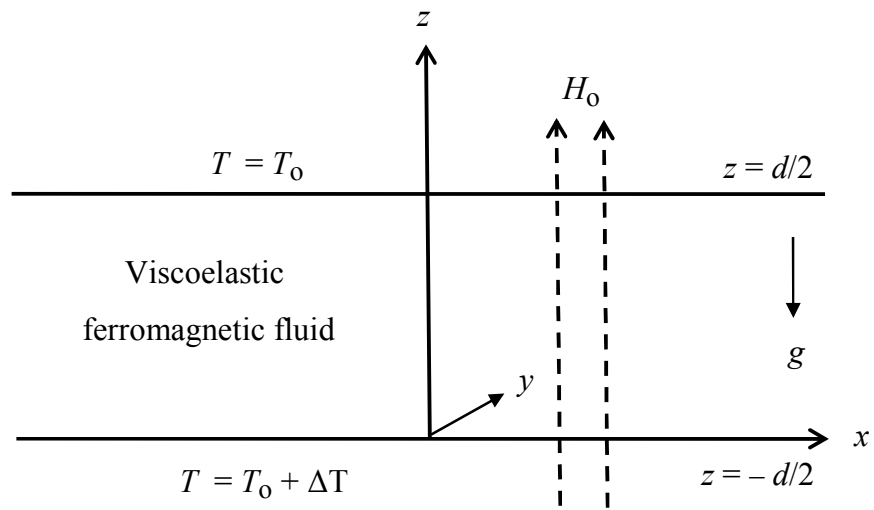
Figure 6.1: Configuration of the problem.
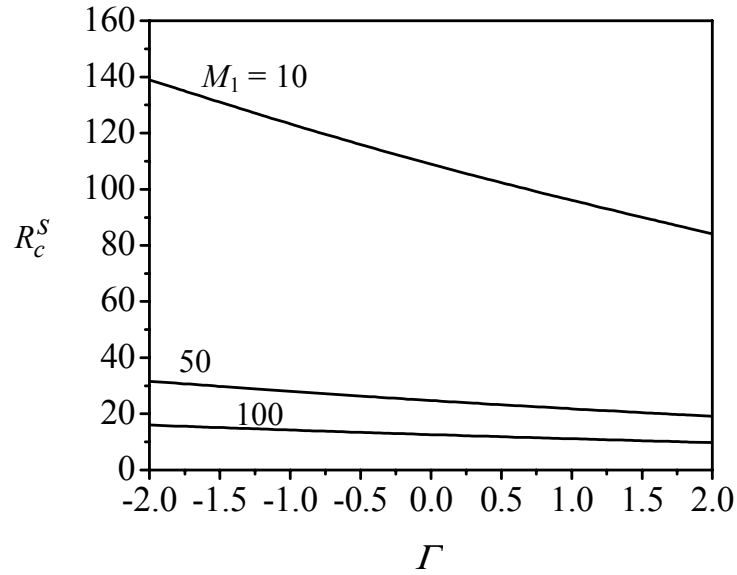
Figure 6.2: Plot of critical thermal Rayleigh number R_c^S versus effective viscosity parameter Γ for $M_3 = 1$, $\chi_m = 1$ and for different values of buoyancy-magnetization parameter M_1.
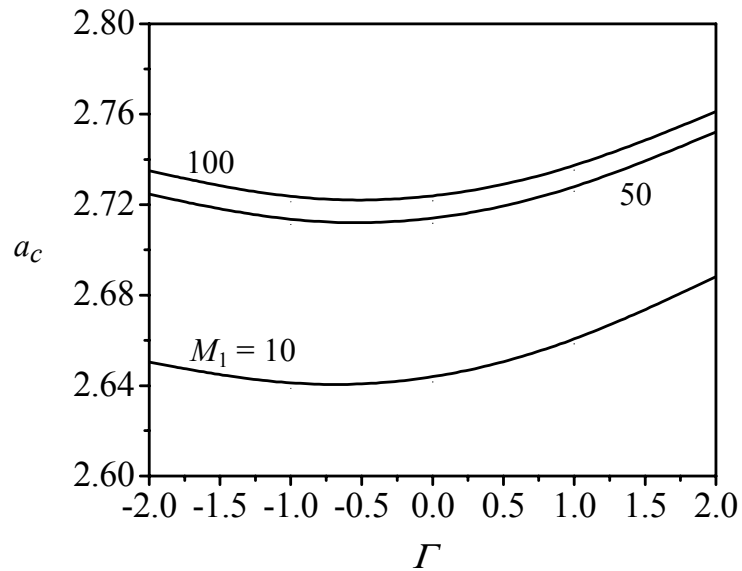
Figure 6.3: Plot of critical wavenumber a_c versus Γ relating to stationary instability for $M_3 = 1$, $\chi_m = 1$ and for different values of M_1.
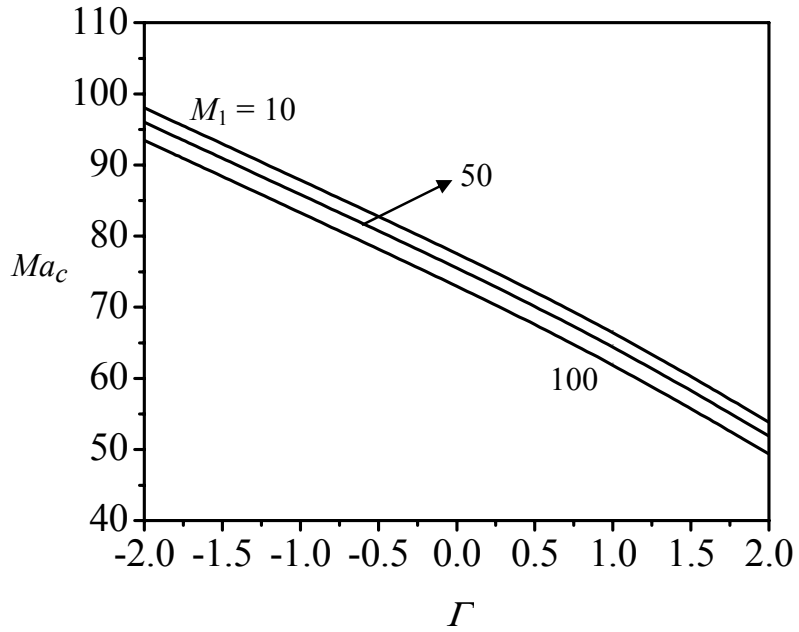
Figure 6.4: Plot of critical thermal Marangoni number Ma_c versus Γ relating to stationary instability for $M_3=1$, $\chi_m=1$ and for different values of M_1.
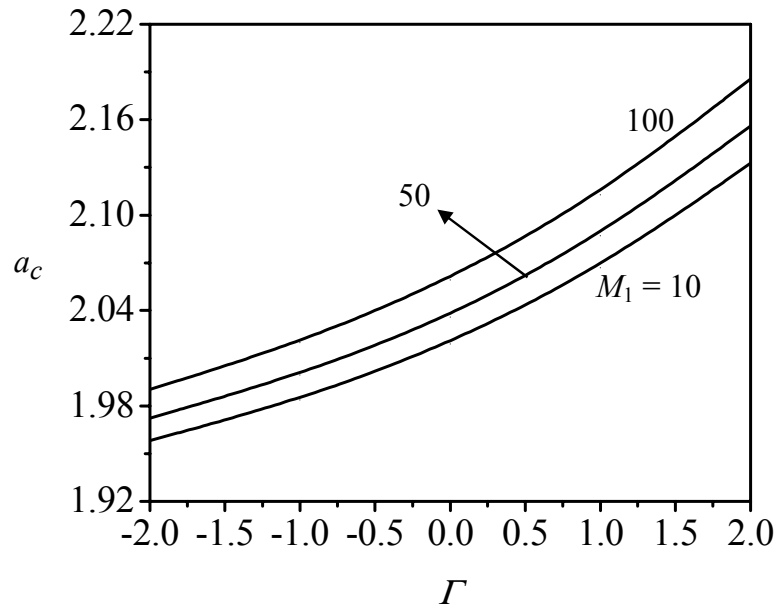
Figure 6.5: Plot of a_c versus Γ relating to stationary instability for $M_3=1$, $\chi_m=1$ and for different values of M_1.
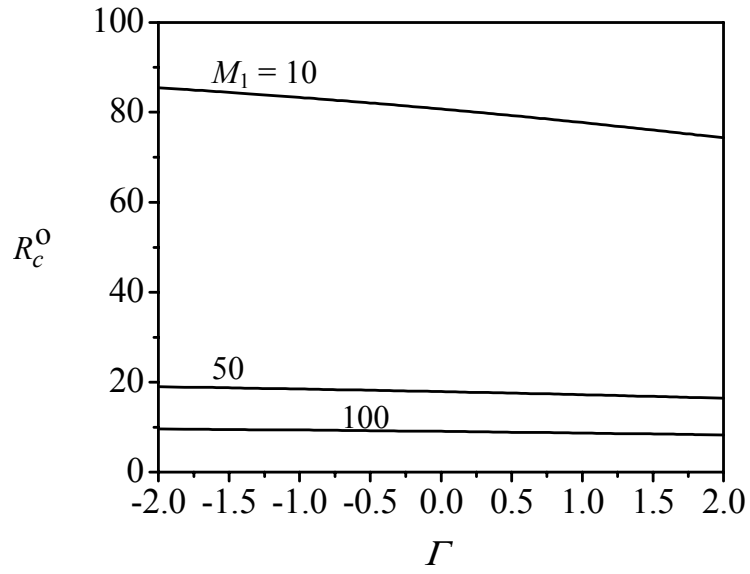
Figure 6.6: Plot of R_c^O versus Γ for $M_3=1$, $\chi_m = 1$ and for different values of M_1.
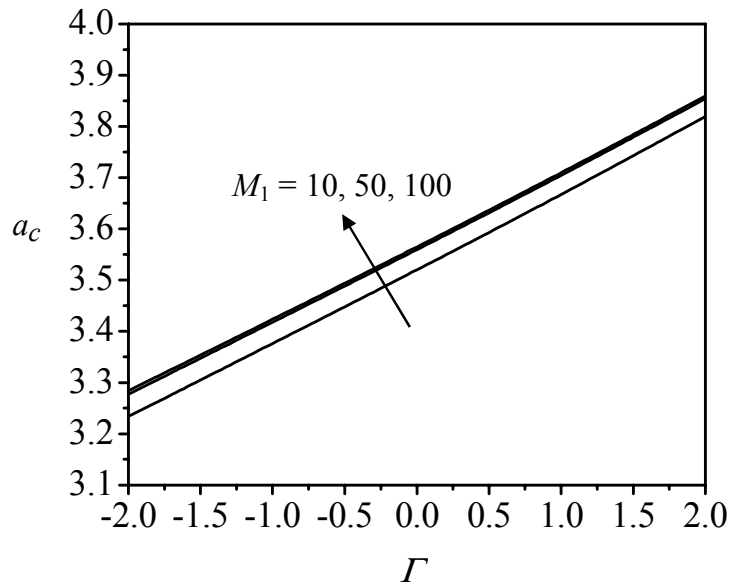
Figure 6.7: Plot of a_c versus Γ relating to oscillatory instability for $M_3=1$, $\chi_m = 1$ and for different values of M_1.
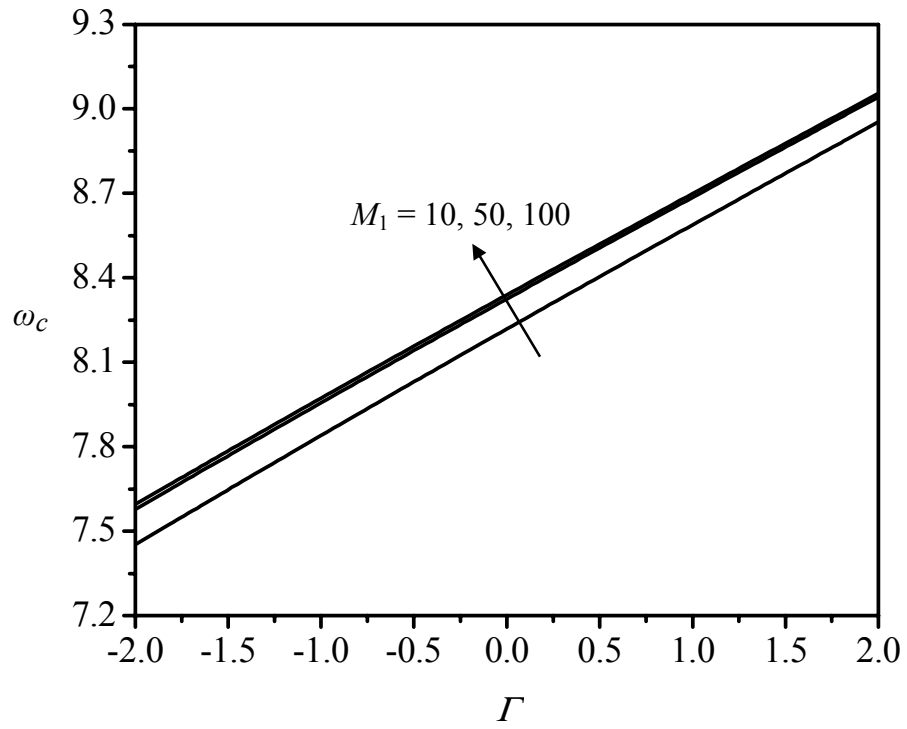
Figure 6.8: Plot of critical frequency of oscillations ω_c versus Γ for $M_3 = 1$, $\chi_m = 1$ and for different values of M_1.

Table 6.1: Limiting cases of the present study for a constant-viscosity liquid.

Nature of liquid → / Type of liquid ↓	*Ferromagnetic liquid*	*Dielectric liquid* [Replace RM_1 by R_E and M_3 by 1 in Eqs. (6.2.9)-(6.2.11)]	*Ordinary viscous liquid* [Set M_1 and M_3 equal to 0 and 1 respectively in Eqs. (6.2.9)-(6.2.11)]
Newtonian	$G_1 = 1$ in Eqs. (6.2.9)-(6.2.11) (Finlayson, 1970)	$G_1 = 1$ in Eqs. (6.2.9)-(6.2.11) (Stiles *et al.,* 1993)	$G_1 = 1$ in Eqs. (6.2.9)-(6.2.11) (Chandrasekhar, 1961)
Maxwell	$\eta = 0$ in Eqs. (6.2.9)-(6.2.11) (Siddheshwar, 1998; 2002b)	$\eta = 0$ in Eqs. (6.2.9)-(6.2.11) (Siddheshwar, 2002b)	$\eta = 0$ in Eqs. (6.2.9)-(6.2.11) (Vest and Arpaci, 1969)
Jeffrey	Eqs. (6.2.9)-(6.2.11) as they are (Siddheshwar, 2002b)	Eqs. (6.2.9)-(6.2.11) as they are (Takashima and Ghosh, 1979)	Eqs. (6.2.9)-(6.2.11) as they are (Sokolov and Tanner, 1972)
Rivlin - Ericksen	$\Gamma_V \to 0$, $\eta \to \infty$, $(\Gamma_V \eta = Q / Pr)$ in Eqs. (6.2.9)-(6.2.11) (Siddheshwar 1999; 2002b)	$\Gamma_V \to 0$, $\eta \to \infty$, $(\Gamma_V \eta = Q / Pr)$ in Eqs. (6.2.9)-(6.2.11) (Siddheshwar, 2002b)	$\Gamma_V \to 0$, $\eta \to \infty$, $(\Gamma_V \eta = Q / Pr)$ in Eqs. (6.2.9)-(6.2.11) (Siddheshwar and Srikrishna, 2002)

Table 6.2 : Critical values for the oscillatory Rayleigh-Bénard convection in a viscoelastic ferromagnetic liquid with variable viscosity and $M_1 = 10$, $M_3 = 1$, $\chi_m = 1$.

Γ_V	η	Pr	$\Gamma = -1$			$\Gamma = 0$			$\Gamma = 1$		
			R_c^o	a_c	ω_c	R_c^o	a_c	ω_c	R_c^o	a_c	ω_c
0.3			83.39	3.376	7.842	80.83	3.519	8.219	77.81	3.666	8.589
0.4	0.3	10	73.82	3.251	6.962	71.93	3.395	7.275	69.58	3.544	7.585
0.5			67.95	3.161	6.295	66.46	3.307	6.568	64.54	3.457	6.840
	0.3		83.39	3.376	7.842	80.83	3.519	8.219	77.81	3.666	8.589
0.3	0.4	10	99.48	3.251	5.776	96.95	3.396	6.117	93.82	3.544	6.452
	0.5		114.96	3.163	4.167	112.47	3.308	4.484	109.256	3.457	4.796
		10	83.39	3.376	7.842	80.83	3.519	8.219	77.81	3.666	8.589
0.3	0.3	50	83.73	3.361	8.258	81.21	3.506	8.691	78.21	3.653	9.127
		100	83.79	3.359	8.313	81.28	3.503	8.753	78.29	3.651	9.199

CHAPTER VII

RESULTS, DISCUSSIONS AND GENERAL CONCLUSIONS

In the thesis we have studied Rayleigh-Bénard and Marangoni instability problems of Newtonian/viscoelastic ferromagnetic/dielectric liquids. Particular attention has been given to study the thermorheological and magnetorheological/electrorheological effects, the effect of internal heat source (sink), the effect of thermal radiation and viscoelastic effect. Keeping in mind various application situations, the following types of boundaries have been considered.

   (i) free-free

  (ii) free-rigid and

 (iii) rigid-rigid.

These boundaries may either be isothermal or adiabatic.

In what follows we discuss the results of Chapter III – VI one after the other followed by important conclusions drawn from the study.

Chapter III: Linear and nonlinear ferro- and electro-convection

In the chapter linear and nonlinear Rayleigh-Bénard ferroconvection has been investigated using the method of normal modes and a truncated representation of double Fourier series respectively. The linear theory predicts the condition for the onset of convection, while the nonlinear theory helps in quantifying heat transfer and in understanding the transition from periodic oscillations to a behaviour that is apparently chaotic, *i.e.*, solutions are aperiodic and exhibit dependence on the initial conditions.

Before embarking on the results of this chapter, we first turn our attention to the possible values of magnetic parameters arising in the study. Table 2.3 documents these quantities for two types of ferro-liquids: (i) hydrocarbon-based ferro-liquid, (ii) water-based ferro-liquid. A few general observations on these liquids are in order. The values of heat capacity, thermal conductivity, viscosity and pyromagnetic coefficient are higher for water-based liquids compared to hydrocarbon-based ones. As a result the Prandtl number Pr is comparatively higher for hydrocarbon-based liquids. This also reiterates our contention earlier that oscillatory convection can be discounted. It should be noted that when the thickness of the fluid layer $d \propto 1\text{mm}$, a temperature gradient of about 10-100 $\text{K}\,\text{mm}^{-1}$ is necessary to drive Rayleigh-Bénard convection (Weilepp and Brand, 1996). In view of this and the data provided in Table 2.3, we have chosen the range 10-1000 for the buoyancy-magnetization parameter M_1. The parameter M_1 is the ratio of magnetic force to gravitational force. The chosen values of the non-buoyancy-magnetization parameter M_3 are 1, 5, 10 and 25 (Finlayson, 1970). The parameter M_3 represents the departure of magnetic equation of state from linearity. The range of values of magnetic susceptibility, χ_m, is 1 to 5 for most ferromagnetic liquids (Finlayson, 1970). In arriving at the value of M_1, as reported in the Table 2.3, we have assumed $\chi_m = 1$. A higher value of χ_m means a smaller value of M_1.

We have substantiated in Chapter III, in the case of linear theory, that stationary mode of convection is preferred to oscillatory mode. The critical values relating to the stationary instability for different values of magnetic parameters M_1 and M_3 are listed in Table 3.1. We see from Table 3.1 that the critical Rayleigh number R_c^s decreases with increase in both M_1 and M_3. Hence both the parameters M_1 and M_3 have a destabilizing effect on the system. We recall at this juncture that this particular result has been proved analytically in Chapter III with the help of the parametric perturbation method. We further notice from Table 3.1 that, as the magnetic equation of state becomes more nonlinear (M_3 large), the fluid layer is

destabilized slightly. We also observe that the cell size decreases with increasing M_1 and remains unaltered for large values of M_1. The opposite is true for the parameter M_3.

The streamlines are plotted in Figures 3.2 and 3.3 for different values of M_1 and M_3 respectively using the first of Eq. (3.4.1) with A of the steady state given by Eq. (3.4.21). These figures reiterate the remarks on the wavelength λ in Table 3.1.

The realm of nonlinear convection warrants the quantification of heat transfer. This is depicted in the Nusselt-Rayleigh numbers plane in Figures 3.4 and 3.5. We observe from these figures that the Nusselt number increases with increasing M_1 and M_3. This behaviour is consistent with the remarks made in respect of Table 3.1. Further, computations reveal that finite amplitude sub-critical instability is ruled out.

The truncated representation of Fourier series used for a weakly nonlinear stability analysis of ferroconvection yielded a nonlinear autonomous system (generalized Lorenz model, Sparrow, 1981) of differential equations, *viz.*, Eqs. (3.4.6) – (3.4.8). The conditions under which a saddle point, node or spiral may be obtained for the linear autonomous system as has been discussed in Chapter III. The nature of the critical points obtained from the linear system reveals information about the trajectories in the phase-plane. The nature of these trajectories is retained by the nonlinear system but with distortions dictated by the nonlinear terms. Further, we analyze finite amplitudes for a possible chaotic motion. Malkus and Veronis (1958) established that, for values of R up to ten times the critical value R_c^S, a steady cellular convection exists in the Boussinesq-nonmagnetic fluid. Further, they observed that at $R > 10\,R_c^S$, a new instability occurs in the fluid producing disordered aperiodic motions, quasi-cellular in appearance. This has been interpreted as the onset of some type of turbulence or chaotic motion. It is

well known that chaotic motion has been observed prominently in fluids subject to temperature gradient, differential rotation, vibrations and other forms of energy (Baker and Gollub, 1990). Here, we consider the temperature gradient and the magnetic energy to study the chaotic motion. To analyze the periodic or aperiodic nonlinear convection, we have taken the value of R to be 658 in our calculations which is appreciably greater than the critical value R_c^S for any of the combination of magnetic parameters (see Table 3.1 for these critical values).

As the system of equations (3.4.6)–(3.4.8) is dissipative, there exists a global attractor and as all the solutions approach the attractor, the solutions exhibit an almost random behaviour (Swinney, 1983). To understand the qualitative effects of magnetic parameters M_1 and M_3, and the Prandtl number Pr on the onset of nonlinear convection, we have solved the nonlinear autonomous system of equations representing the convective process numerically using Mathematica 4.0. In what follows we discuss the two key issues, namely, aperiodic solutions and sensitive dependence on initial data.

Figure 3.6 is the plot of Nusselt number, Nu, versus time for different values of M_1. When $M_1 = 10$, there exists no transient behaviour and the solution begins to converge almost at once. When $M_1 = 100$, the oscillations bifurcate into periodic doublings after a brief transient period. It may be noted that the periodic doubling is one route to chaos (Ott, 1993). As M_1 is further increased to the value of 1000, the periodic doubling reaches the accumulation point thereby giving way to a chaotic solution. This succession of period doubling and the onset of chaos at an accumulation value of a driving parameter is known as the *Feigenbaum route* and has been observed in many experiments (Khayat, 1995b).

Figure 3.7 illustrates the variations in the Nusselt numbet, Nu, versus time for different values of M_3. The periodic doubling bifurcation after a brief transient behaviour is obvious for $M_3 = 1$, 5 and 25, hence indicating the destabilizing effect of M_3. We notice that the periodic doubling behaviour continues to exist even for

higher values of M_3 and does not lead to a chaotic solution. This particular aspect should be looked at in conjunction with the result of linear stability theory, *viz.*, the fluid layer is destabilized slightly for higher values of M_3.

Figures 3.8 and 3.9 delineate the variations of the amplitude $B(t)$ versus time for different values of M_1 and M_3 respectively. These figures reiterate the remarks made in respect of Figures 3.6 and 3.7.

The effect of Prandtl number Pr on the variations of the amplitude $B(t)$ is shown in Figure 3.10. It is clear that increase in the value of Pr gives way to simple periodic motion of oscillations which are independent of the transient conditions. Thus the effect of increasing Pr is to stabilize the system (Lorenz, 1963).

To study the sensitive dependence on the initial condition, we consider the plots of variations in the Nusselt number, Nu, versus time and the amplitude $B(t)$ versus time for a fixed value of the parameters arising in the study. The chosen initial conditions are $A(0) = 0, \ B(0) = 1.0001, C(0) = 0$, a slight departure from the earlier chosen initial conditions of $(A, B, C) = (0, 1, 0)$. From Figures 3.11 and 3.12, we observe that the behaviour of the two solutions is quite different. The attracting set in this case, although of zero volume, has a rather complex structure and is called a strange attractor. As the system is sensitive to the initial conditions, the time evolution eventually leads to a chaotic motion.

A phase-space analysis can be carried out on the generalized Lorenz system (3.4.6) − (3.4.8) and one can consider its projection on the three phase-planes as done by many. This requires the scaling of the equation based on the information provided by the critical points. Due to the nonlinear term BC in Eq. (3.4.6) this turned out to be prohibitive. In view of the above we resorted to the time-series plots of Figures 3.6 − 3.12 as discussed above.

The chapter reports an analogy between linear and nonlinear Rayleigh-Bénard ferroconvection with a *dc* magnetic field and linear and nonlinear Rayleigh-Bénard

164

electroconvection with an *ac* electric field. This is an important statement on the problem of linear and nonlinear electroconvection.

In what follows we discuss the effect of variable viscosity and uniform internal heat source (sink) on the onset of Rayleigh-Bénard/Marangoni convection in Newtonian ferromagnetic/dielectric liquids. This leads to differential equations with space varying coefficients and complicated boundary conditions. This precludes, unlike in Chapter III, the possibility of obtaining a closed form solution.

Chapter IV: Thermorheological and magnetorheological effects on ferroconvection with internal heat source

External regulation of rheological properties and thereby the control of instability arising from buoyancy/surface-tension forces in a variable-viscosity ferromagnetic liquid in the presence of a vertical, uniform *dc* magnetic field with internal heat generation is studied. The principle of exchange of stabilities is shown to be valid for both *RBC* and *MC*. The critical values pertaining to stationary convection have been obtained by using the higher order Rayleigh-Ritz technique (*HORT*).

Before discussing the important results of the problem, we call attention to the values of certain needed physical quantities pertaining to ferromagnetic liquids. We have discussed about the range of values of the parameters M_1, M_3, χ_m and *Pr* in the results of Chapter IV. As to the Rayleigh-Bénard instability, we have chosen the values of buoyancy-magnetization parameter, M_1, to be 10, 50 and 100. Higher values of M_1 will not be considered here as their results are quite close to those for $M_1 = 100$. We now discuss the range of values of other parameters relevant to this chapter.

In classical Marangoni convection problems wherein $g = o(10^{-6})$, the thermal Rayleigh number R is negligibly small. Table 2.3 clearly illustrates to us that this is true in Marangoni-ferroconvection problems also. It is also obvious from the table

that, in the case of *MC*, increase in ΔT and d will also not render the situation conducive to Rayleigh-Bénard ferroconvection. At this juncture, another important parameter that remains to be discussed is the magnetic Rayleigh number R_M. Clearly, from Table 2.3, we find that R_M is comparatively greater for water-based liquids than hydrocarbon-based ones. Table 2.3 shows that ΔT and d can be manipulated upon to diminish or enhance the effect of other factors in R_M. For $\Delta T = 1\,\text{K}$ and $d = 1\text{mm}$ (Weilepp and Brand, 1996) the range of values of R_M for the liquids considered is 0.431 to 1.409. If the temperature difference ΔT or depth of the liquid d is increased, then R_M attains higher values. In view of this we have chosen the representative values of R_M to be 10, 50 and 100. We note here that R varies as ΔT and d^3 whereas R_M varies as $(\Delta T)^2$ and d^2. To keep R small in the Marangoni instability problem, it is imperative that we need to prefer variation of ΔT to variation of d.

The parameter N_S is the ratio of strength of the internal heat source to external heating. The chosen range of values for the parameter N_S is –4 to 4. As previously mentioned in Chapter II, positive values of N_S signify a heat source and the negative values a heat sink. The values of $|\Gamma|$ have to be small on reasons explained in Chapter IV. The condition $\Gamma < 0$ characterizes the dominance of magnetic field-dependent viscosity, while $\Gamma < 0$ signifies the dominance of temperature-dependent viscosity.

In order to understand better the results arrived at in the problem we analyze the nonlinear basic state temperature distribution, which throws some light on the observed effect of heat source (sink) on the stability. We consider a scaled dimensionless temperature distribution arrived at from the second of Eq. (4.2.9) in the form

$$\theta = \frac{T_b - T_a}{\Delta T} = \frac{N_S}{8} - z - \frac{N_S}{2}z^2 .$$

As in the earlier sections we have dropped the asterisks in the above equation.

Figure 4.2 is a plot of z versus θ. We note from the computations and the figure, in the $\theta - z$ plane, that the curves are asymmetrical about the lines $\theta = 0$ and $z = 0$ for $N_S \neq 0$. The asymmetry is obviously due to N_S. We discuss more on this figure by considering the cases of heat source ($N_S > 0$) and heat sink ($N_S < 0$) separately. We note that $d\theta/dz = -(1 + N_S z)$ and $d^2\theta/dz^2 = -N_S$ and hence the point $z = -1/N_S$ is a point of extrema. The extremal point $z = -1/N_S$ and the vertical range $z \in [-1/2, \ 1/2]$ suggest that $N_S \notin (-2, 2)$ for the analysis of the $\theta(z)$ profile. It is important to make a statement at this juncture on the range $|N_S| < 2$. In this range, the $\theta(z)$ profile of $N_S \neq 0$ is similar to that of $N_S = 0$, *i.e.*, there is no switch over from monotonically increasing to decreasing, or vice-versa, at any point $z \in [-1/2, \ 1/2]$. Hence a discussion involving point of extrema is inappropriate for the range $|N_S| < 2$. With the above remark let us consider first the case in which $N_S \geq 2$. From Figure 4.2 it is clear that when $N_S = 2$, the highest temperature in the liquid layer occurs at the lower bounding surface, *i.e.*, $z = -1/2$. As N_S increases beyond the value of 2, temperatures in excess of that at $z = -1/2$ occur within the liquid and further increases in N_S result in corresponding increases in liquid temperature and the location of the point of extrema approaches closer to $z = 0$. Thus, when $N_S \geq 2$, the point of extrema is always in the lower half of the layer, *i.e.*, $-1/2 \leq z < 0$. In view of the fact that temperature decreases with height in the upper half of the layer (the most important zone in so far as Marangoni instability is concerned) surface-tension effect becomes strong. It is therefore clear that the effect of increasing N_S is to hasten instability for both *RBC* and *MC*.

We next consider the case in which $N_S \leq -2$. In this case the point of extrema is always in the upper half of the layer, *i.e.*, $0 < z \leq 1/2$. Further, when $N_S < -2$, temperatures within the liquid fall below that of the upper surface, with the result

that temperature increases with height in the upper half of the layer and surface-tension effect is weakened leading to stability. A thought comes to mind here as to why both heat source and heat sink have identical effect (destabilizing effect) on *RBC* and not on *MC* as discussed earlier in Chapter IV. This can be answered by realizing that in the case of *MC*, as discussed above, the upper half temperature distribution is important due to surface-tension being the cause for convection. In the case of *RBC* it can be either the lower half temperature distribution or that of the upper half, that decides on the stability or otherwise. In the case of *RBC*, as temperature increases with height in the lower half and temperature decreases with height in the upper half for $N_S > 2$, liquid near the center of the layer becomes lighter than the liquid at the top leading to instability, while for $N_S < -2$, it is the lower half of the layer that facilitates instability. Thus, for both $N_S > 2$ and $N_S < -2$, the effect of N_S is to destabilize the system in the case of *RBC*, unlike the case of *MC*.

Tables 4.1 – 4.4 reiterate the N_S-effect on stability discussed in the context of Figure 4.2. In the case of *RBC* with free-rigid boundaries, the destabilizing influence of heat sink for $N_S < -2$ can be seen from Table 4.2. Thus we conclude that, in the case of *RBC*, the effect of heat source and heat sink does not depend on whether or not the boundaries are symmetric. It is clear from Tables 4.1 – 4.4 that the qualitative effect of the magnetization parameter M_3 and the magnetic susceptibility χ_m on the onset of convection is akin to that in a constant viscosity ferromagnetic liquid (Finlayson, 1970). We also find that

$$R_c^{RR} > R_c^{FR} > R_c^{FF},$$

$$a_c^{RR} > a_c^{FR} > a_c^{FF},$$

where the superscripts represent the three different velocity boundary combinations. It can be proved that the above qualitative results are true for both isothermal and adiabatic boundaries.

From Table 4.4 we also notice that the dominance of magnetic field dependence of viscosity enhances the destabilizing effect of M_3 on Ma_c, noticed in the constant viscosity case. Qualitatively, the effect of N_S on Ma_c is more significant compared to the almost insignificant effect of χ_m on Ma_c. This is true for all Γ. From physically comprehensible reasons we can convince ourselves that this is due to magnetic field strength and temperature being antagonistic in their influence on viscosity. In what follows we discuss first about convergence of the solution and then discuss about the inadequacy of a linear viscosity-temperature-magnetic field relationship. This is followed by a discussion on the effects of M_1, R_M and Γ on the onset of ferroconvection.

In arriving at the documented values in Table 4.1 – 4.4, we have made use of a five-term Rayleigh-Ritz technique. Tables 4.5 and 4.6 respectively present a comparison between the results of *RBC* and *MC* problems and those of other standard works for the limiting case of a constant-viscosity, nonmagnetic fluid without internal heating. Table 4.7 compares the critical values pertaining to the *MC* problem with those obtained by others for the limiting case of a constant viscosity, nonmagnetic liquid with internal heating. Tables 4.5 – 4.7 highlight the need to use *HORT* in the present problem. It can be seen that the numerical values agree quite well with the standard results and a fifth order Rayleigh-Ritz technique assures us of the desired accuracy. We have ascertained by actual computation that this is true for the results obtained in the present problem also. The results documented in Tables 4.5 – 4.6 quite obviously spell out the fact that *HORT* gives accurate values. In fact, the critical values given in Table 4.5 agree well with those obtained by Chandrasekhar (1961). Even for $N_S \neq 0$ the results compare very well with existing results of limiting cases (see Table 4.7).

Before we proceed further on the other results of the problem, we first settle the issue of the inadequacy of a linear viscosity variation. To this end, we restrict our attention to the case of temperature-dependent viscosity and $N_S = 0$. The critical values marking the onset of Rayleigh-Bénard convection in a nonmagnetic fluid

with linear dependence of viscosity on temperature are listed in Table 4.8. The relation

$$\mu(T) = \mu_1 \left[1 - \delta_L (T - T_a)\right], \tag{7.1}$$

with $\delta_L = -(1/\mu_1)(\partial\mu/\partial T)_{T_a}$ is used in place of the quadratic law given by Eq. (4.2.8) to obtain the values in Table 4.8 and $\Gamma_L = \delta_L \Delta T$ is the dimensionless effective viscosity parameter corresponding to the linear variation. We find that the effect of temperature-dependent viscosity is stabilizing for free-free boundaries and destabilizing for free-rigid and rigid-rigid boundaries. We, however, note that the effect of quadratic variation is always destabilizing for the all the boundary combinations. On the other hand, the critical values listed in Table 4.9 for $N_S = 0$ enlighten us as to why a quadratic variation in the effective viscosity must be taken and not a linear one in the case of MC. Cloot and Lebon (1985) studied the effect of temperature-dependent viscosity on Marangoni instability in a Newtonian nonmagnetic liquid with a deformable free surface. Lam and Bayazitoglu (1987) extended the above study to include the effect of internal heat generation. Both these studies show that linear viscosity-temperature relationship is good enough to demonstrate that the system is less stable compared to a constant viscosity one. This result was incidentally obtained because of the classical Pearson boundary condition used by them, *viz.*, $w = D^2 w + a^2 Ma T = DT = 0$ relating to the upper non-deformable free surface, which is a particular case of the two aforementioned studies. This condition needed to be modified to include variable viscosity effect (Selak and Lebon, 1997). On incorporation of the linear variation in the temperature-dependent viscosity given in Eq. (7.1), the aforementioned classical Pearson boundary condition gets modified into

$$w = (1 + \Gamma_L) D^2 w + a^2 Ma T = DT = 0. \tag{7.2}$$

Evaluations using this modified boundary condition at the upper non-deformable free surface reveal that the effect of temperature-dependent viscosity is stabilizing!

Table 4.9 documents this observation. The above proceeding points to the fact that the viscosity-temperature relationship cannot be linear and must, at least, be quadratic for *RBC* problem with free-free boundaries and for the *MC* problem. Straughan (2004) documents this aspect for the viscosity-temperature relationship. The same reasoning applies to the viscosity-magnetic field relationship. We now discuss the effect of other parameters of the problem on stability.

Figures 4.3 – 4.5 depict the variation of R_c with N_S for different values of Γ and M_1. The destabilizing effect of magnetic mechanism is obvious from the figures for $\Gamma \neq 0$ as well as for $\Gamma = 0$, *i.e.*, when the thermorheological and magnetorheological effects are absent. It is of interest to note that large values of M_1 reduce the destabilizing effect of N_S and the mutually antagonistic effect of Γ. Of particular interest is the effect of N_S, M_1 and Γ on the critical wavenumber a_c. This is depicted in Figures 4.6 – 4.8. We see that a_c increases with increasing $|N_S|$ and M_1. This result is true for all boundary combinations. However, the effect of Γ on a_c needs special attention. We observe that, for free-free boundaries, a_c increases with increasing the effect of temperature-dependent viscosity and decreases with the effect of magnetic field-dependent viscosity. The above result is also true for free-rigid and rigid-rigid boundaries as long as $N_S > 2$. When $N_S < 2$, the opposite behaviour is observed for free-free and rigid-rigid boundaries. It is worth mentioning that, when $N_S = 0$ and when the viscosity is temperature-dependent, the effect of Γ on a_c for all the three boundary combinations is in keeping with what was reported by Stengel *et al*. (1982).

Figure 4.9 shows the variation of Ma_c with N_S for different values of Γ and R_M. The destabilizing effect of magnetic mechanism is obvious from the figures for $\Gamma \neq 0$ as well as for $\Gamma = 0$. A striking result from the figure is that mutually antagonistic influence of Γ is more pronounced for a uniform heat sink than for a uniform heat source. Now coming to Figure 4.10, we see the individual variation of

171

the critical wavenumber a_c with all the three parameters, *viz.*, N_S, R_M and Γ is in line with that applicable to the *RBC* problem with free-free boundaries. In what follows we discuss an interesting aspect concerning the general boundary conditions on the magnetic potential Φ.

Qin and Kaloni (1994) examined the stability of a constant viscosity ferromagnetic liquid layer with surface-tension effect but with no internal heating. Shivakumara *et al*. (2002) investigated the stability of a constant viscosity ferromagnetic liquid layer with surface-tension and buoyancy mechanisms operating together and with no internal heating. In the present paper we have corrected a technical mistake appearing in the above two works. We note that the above investigators used a boundary condition on the magnetic potential Φ pertaining to the upper free surface that did not involve a temperature term. A meticulous derivation of boundary conditions on Φ reveals that, for adiabatic boundaries, a temperature term appears in the boundary condition (see Appendix B). So we cannot compare our results with those of the above investigators.

The chapter further gives an account of an analogy for a variable-viscosity fluid with internal heat source between ferroconvection with a *dc* magnetic field and electroconvection with an *ac* electric field. This essentially means that the study of the latter is redundant.

In what follows we discuss the effect of thermal radiation on the onset of Rayleigh-Bénard/Marangoni convection in Newtonian ferromagnetic/dielectric liquids. This leads to differential equations with variable coefficients. This precludes, as in Chapter IV, the possibility of obtaining an analytical solution.

Chapter V: Thermal radiation effects on ferroconvection

The effect of radiation on the onset of Rayleigh-Bénard/Marangoni convection in an absorbing and emitting variable-viscosity ferromagnetic fluid layer in the presence of a vertical, uniform *dc* magnetic field is studied. The presumption, *viz.*,

the boundaries are black bodies, seems to be a fairly good approximation inasmuch as most ferromagnetic fluids are black. The optical properties of the ferromagnetic fluid are considered to be independent of the wavelength of radiation. This assumption pertaining to a gray medium allows us to consider two asymptotic cases:

    (i) Optically thin fluid medium   (transparent medium) and

    (ii) Optically thick fluid medium (opaque medium).

The latter approximation is appropriate for shallow layers in the laboratory. The principle of exchange of stabilities is shown to be valid for both *RBC* and *MC*. The critical values pertaining to stationary instability are obtained by using the more accurate higher order Rayleigh-Ritz technique. Before discussing the important results of the problem, we turn our attention to the range of values of different parameters arising in the study.

The chosen values of the parameters M_1, Γ, R_M, M_3 and χ_m are as considered in Chapter IV. As to the radiative parameters, we note that large radiative effects are more likely if a gas rather than a liquid is used as a fluid (Howell and Menguc, 1998). In view of this, we assume the range of values of the conduction-radiation parameter χ to be 10^{-1}–10^5 and that of absorptivity parameter τ to be 10^{-1}–10^3.

In order to understand better the results arrived at in the problem, we analyze the nonlinear basic state temperature distribution which throws some light on the effect of radiative heat transfer on the stability of the fluid layer. Figures 5.2 and 5.3 are plots of z versus $\beta/\bar{\beta}$ for different values of χ and τ respectively. It is clear that $\beta/\bar{\beta} \to 1$ as either χ or $\tau \to 0$. We further notice that as χ or τ increases, the basic state temperature profile is exponential as is evident from Eq. (5.2.19). Computations reveal that the basic state temperature profile remains unaffected when τ is large and the opposite is true for the parameter χ. We also

note that the effect of radiative heat transfer is prominent in the boundary layer regions at the two bounding surfaces. It is also clear from Eq. (5.2.19) that $\beta/\bar{\beta}$ is an even function of z. This feature renders the basic state temperature distribution symmetric about the lines $\beta/\bar{\beta} = 0$ and $z = 0$, which is in contrast to the heat source (sink) problem where the basic state temperature distribution is asymmetric about the lines $\theta = 0$ and $z = 0$ (see Figure 4.2). The symmetric property is largely responsible for the stabilizing effect of χ and τ in the radiative heat transfer problem. This feature has not been addressed by the earlier investigators.

In what follows we first discuss the results pertaining to Rayleigh-Bénard instability followed by those to Marangoni instability. Free-free boundaries are considered to study the Rayleigh-Bénard instability as the results corresponding to the free-rigid and rigid-rigid boundaries show a similar trend as that for free-free case.

The results relating to the transparent approximation are shown in Figures 5.4 – 5.7. Figure 5.4 is a plot of R_c versus τ for different values of χ and Γ for fixed values of M_1, M_3 and χ_m. The parameter τ is the characteristic of absorption coefficient and distance between the horizontal planes. The parameter χ is indicative of the temperature in the equilibrium state. Figure 5.4 reiterates the behaviour of Γ observed in the results of Chapter IV. We see that the mutually antagonistic influence (*MAI*) of thermo- and magneto-rheological effects is not affected by the radiative heat transfer for large values of χ and τ. The stabilizing influence of χ and τ is obvious from Figure 5.4. This results from the fact that radiative transfer tends to damp out any motions which may arise due to the heat transfer from hotter to colder parts of the ferromagnetic fluid and that the radiative damping increases as the layer depth increases. The effect of τ, χ and Γ on a_c is depicted in Figure 5.5. The effect of Γ on a_c is in accordance with what has been reported in Chapter IV. The effect of τ and χ on a_c is in agreement with the existing results relating to a nonmagnetic fluid (Murgai and Khosla, 1962).

174

Figure 5.6 delineates the effect of τ, χ and M_1 on R_c. The destabilizing influence of magnetic mechanism is obvious. It is interesting to note that increasing the temperature in the equilibrium state enhances the destabilizing behaviour of M_1. This is quite reasonable as the fluid magnetization is temperature sensitive. Further, we notice that for large values of both χ and M_1, the stabilizing effect of τ is nullified. The effect of M_1 on a_c is shown in Figure 5.7, which is as expected.

The results of opaque medium pertaining to the Rayleigh-Bénard instability are shown in Figures 5.8 – 5.11. The effects of Γ, M_1 and χ are qualitatively similar to those corresponding to the transparent medium. It is worthwhile noting from Figures 5.4 and 5.6, and 5.8 and 5.10 that the critical thermal Rayleigh number R_c corresponding to the opaque case is much higher than that of the transparent one if χ is large; meaning maximum stabilization is achieved when the ferromagnetic fluid is optically thick. The effect of τ on both R_c and a_c, however, needs special attention. As previously mentioned, increasing τ contributes much to radiative damping if the fluid layer is optically thin. On the other hand, increasing τ leads to a more unstable basic temperature gradient in the fluid layer thereby increasing the interior temperature gradients if the fluid medium is opaque. This feature can be seen in Figures 5.8 and 5.10. These figures also indicate that radiation begins to act as conduction process for large values of τ in the case of opaque medium. The earlier works (Goody, 1956; Murgai and Khosla, 1962) substantiated for a nonmagnetic fluid that the radiative heat transfer has no effect on the cell size in the case of opaque medium if both τ and χ are large. This is true for the present study also as can be seen from Figures 5.9 and 5.11.

The critical values marking the onset of *RBC* relating to the transparent and opaque media for different values of χ_m, M_3, Γ are documented respectively in Tables 5.1 and 5.2. These tables reiterate the remarks made earlier in the context of *RBC* in Chapter IV. As mentioned earlier, the above results applicable to free-free

boundaries are also true for the other boundary combinations, *viz.*, free-rigid and rigid-rigid boundaries.

The values of R_c and a_c for the limiting case relating to a Newtonian, constant viscosity nonmagnetic fluid with vanishing radiation effects are given in Table 4.5. We find that the critical values in Table 4.5 agree well with those obtained by Chandrasekhar (1961).

In what follows we discuss the influence of radiative effects on the onset of Marangoni instability in a variable-viscosity ferromagnetic fluid. Figures 5.12 and 5.13 show respectively the effect of χ on Ma_c and a_c for different values of Γ. We note that these figures show a similar trend as in the transparent case of *RBC*. Of particular interest is the influence of Biot number *Bi* on Ma_c and a_c. This is depicted in Figures 5.14 and 5.15. Biot number, *Bi*, is the ratio of conductive resistance within the fluid layer to convective resistance at the free surface of the fluid layer. When *Bi* = 0, an insulated surface retains more energy within the fluid layer and thus the system is less stable. Once the heat release to the gas is allowed, *i.e.*, $Bi \neq 0$, the fluid becomes more stable. The stabilizing influence of *Bi* is quite evident from the Figure 5.14. It is of interest to observe that the stabilizing influence of *Bi* gets enhanced when the temperature in the equilibrium state is increased. We see from Figure 5.15 that a_c increases with increasing *Bi*. This means that the cell size gets reduced with increasing *Bi*. This is true for nonmagnetic fluid also (Char and Chiang, 1994).

The results of Figures 5.16 and 5.17 are in accordance with the remarks made in the context of *RBC*. However, it is interesting to note that the stabilizing effect of *Bi* is nullified when the fluid layer depth is increased. This feature is explained in Figure 5.18. Figure 5.19 indicates that the cell size at the onset of convection is reduced with increasing τ and *Bi*.

The results corresponding to the opaque ferromagnetic fluid layer are given in Figures 5.20 – 5.27. The effect of χ, Bi and Γ on Ma_c are qualitatively the same for both transparent and opaque media. The effect of τ on Ma_c is in conformity with the remarks made in the case of opaque medium of RBC. Figures 5.21 and 5.23 suggest that the cell size is not affected by the radiation if the fluid layer is opaque. We recall that this result is true for RBC as well.

Figures 5.24 and 5.26 show that the mutually antagonistic influence of thermo- and magneto-rheological effects and the stabilizing effect of Bi can be diminished by increasing the fluid layer depth. The latter result is true for the transparent case also (see Figure 5.18), while the former one is true only for opaque case (see Figure 5.16).

Figures 5.25 and 5.27 show that a_c increases with increasing τ for moderate values τ and decreases for large values τ. It seems that the reversing effect of τ after a particular value may be owing to the fact that the linear theory applies only till that point. This observation is, however, speculative and needs to be examined better through experimental ratification.

The critical values for the transparent and opaque media corresponding to the Marangoni instability in a variable-viscosity ferromagnetic fluid are listed in Tables 5.3 – 5.5. These tables essentially suggest that the radiation and variable-viscosity effects are more significant in a thin layer of ferromagnetic fluid (no matter the fluid is transparent or opaque) when compared with the almost negligible effect of M_1, M_3 and χ_m on the stability.

It is also our prime concern to validate the results obtained. It should be noted that, unlike the Rayleigh-Bénard convection wherein the boundary conditions are the same as in Chapter IV, the boundary conditions for the radiation-affected Marangoni instability are different from those considered in Chapter IV due to the introduction of the Biot number Bi (see Eq. 5.5.14). A comparison is therefore made between the results of the present study and those of the standard works for

the pure Marangoni convection (see Table 5.6). It is quite clear that the critical values agree well with the existing results and a fourth order Rayleigh-Ritz technique assures us of the desired accuracy.

The chapter further reports an analogy for a variable-viscosity fluid with thermal radiation between ferroconvection with a *dc* magnetic field and electroconvection with an *ac* electric field.

In what follows we discuss the variable-viscosity effect on the onset of Rayleigh-Bénard/Marangoni convection in viscoelastic ferromagnetic/dielectric liquids. This leads to differential equations with variable coefficients. This precludes, as in Chapters IV and V, the possibility of obtaining a closed form solution.

Chapter VI: Ferroconvection in viscoelastic liquids

Thermorheological and magnetorheological effects on Rayleigh-Bénard and Marangoni instabilities in viscoelastic ferromagnetic liquids are addressed. The present study emphasizes on the need to resort to the use of general equations that encompass within its realm both Newtonian and viscoelastic descriptions as limiting cases. In deed, the study unveils the fact that the results of Rayleigh-Bénard and Marangoni type of instabilities in Jeffrey ferromagnetic liquids leads to those of Maxwell/Rivlin-Ericksen/Newtonian ferromagnetic liquids with recourse to suitable limiting process discussed in Sections (6.2.3)-(6.2.5). The study also makes plain an analogy between viscoelastic ferromagnetic and viscoelastic dielectric liquids. The demonstrated convergence renders the study of rheological effects on *RBC* and *MC* in Newtonian/viscoelastic dielectric liquids with *ac* electric field redundant as the same can be drawn from the corresponding study in ferromagnetic liquids with *dc* magnetic field. The aforementioned results are summarized in Table 6.1.

In view of the results documented in Table 6.1, we mainly discuss the results of stationary and oscillatory convection for both *RBC* and *MC* in Jeffrey ferromagnetic liquids with the understanding that the results can be extended to other models discussed in the study. Higher and first order Rayleigh-Ritz methods are used to compute the eigenvalues for the stationary and oscillatory modes of convection.

Computations reveal that oscillatory Rayleigh-Bénard convection is possible provided that $\eta < 1$. In other words, the principle of exchange of stabilities (*PES*) is valid for viscoelastic ferromagnetic/dielectric liquids only if $\eta > 1$. The condition on η for the oscillatory Rayleigh-Bénard convection is also true for the corresponding problem in which the rheological effects are absent (Takashima and Ghosh, 1979; Siddheshwar, 2002b). Thus to open up the possibility of the conditional oscillatory Rayleigh-Bénard convection, we take values of η to be less than 1. Furthermore, from Eq. (6.3.3), we find that the square of the frequency of oscillations, ω^2, depends explicitly on the effective viscosity parameter Γ but depends on the magnetic parameters M_1 and M_3 implicitly via the wavenumber a. Surprisingly, on the other hand, exhaustive computations reveal that the frequency of oscillations corresponding to the Marangoni instability becomes imaginary for any combination of the parameters arising in the study. This essentially suggests to us that oscillatory Marangoni convection in viscoelastic ferromagnetic/dielectric liquids does not occur. We also note that stationary convection is devoid of viscoelastic effects. We thus conclude that, under microgravity conditions, surface-tension driven convection in a viscoelastic ferromagnetic/dielectric liquid is well-nigh impossible. Odenbach (1999) found experimentally that the surface-tension effect is unimportant in viscoelastic ferromagnetic liquids under microgravity conditions. We find that the results of the present study relating to *MC* do agree with that obtained by Odenbach (1999). This is an important finding of the problem under consideration.

The results pertaining to stationary convection are depicted in Figures 6.2 – 6.5 and those relating to oscillatory convection are in Figures 6.6 – 6.8. The mutually antagonistic influence (*MAI*) of thermo- and magneto-rheological effects on Rayleigh-Bénard and Marangoni instabilities can be seen from Figures 6.2 – 6.8. These figures reiterate the results obtained in Chapter IV and V. It is worth noting that the observed *MAI* is more pronounced in the case of Marangoni instability rather than the Rayleigh-Benard instability in so far as the Newtonian ferromagnetic fluid is concerned. This particular result, taking into account the magnetic field dependency of effective viscosity, can be seen to be compliant with the finding of Odenbach (1999), *viz.*, microgravity conditions can amplify the viscoelastic effects in ferromagnetic fluids.

Figures 6.2 – 6.8 also reveal that the magnetic mechanism has a destabilizing influence on *RBC* and *MC* as in the earlier chapters. From Figure 6.8 we observe that the frequency of oscillations is dependent on the effective viscosity parameter; another important result of the study at hand.

The effect of stress relaxation parameter, Γ_V, the viscoelastic parameter, η, and the Prandtl number *Pr* on the onset of oscillatory Rayleigh-Bénard convection in a viscoelastic ferromagnetic liquid can be seen from Table 6.2. It is clear that the viscoelastic parameters Γ_V and η have opposing influence on the stability of the system. In fact, Γ_V destabilizes, while η stabilizes. This result is also found to be valid for viscoelastic nonmagnetic fluids (Siddheshwar, 1998; 2002b). Computations reveal that the following relationships hold good for the three types of viscoelastic liquids discussed in the problem:

$$R_c^{\text{Maxwell}} < R_c^{\text{Jeffrey}} < R_c^{\text{Rivlin-Ericksen}},$$

$$a_c^{\text{Rivlin-Ericksen}} < a_c^{\text{Jeffrey}} < a_c^{\text{Maxwell}},$$

$$\omega_c^{\text{Rivlin-Ericksen}} < \omega_c^{\text{Jeffrey}} < \omega_c^{\text{Maxwell}}.$$

Conclusions

The important conclusions drawn from the study in the Chapters III – VI are:

1. Principle of exchange of stabilities holds good for a Newtonian ferromagnetic liquid. This result holds good whether or not variable-viscosity, internal heat source (sink) and thermal radiation are present.

2. Finite amplitude sub-critical instability is ruled out for a Newtonian ferromagnetic liquid.

3. In the case of finite amplitude nonlinear convection, increase in the magnetic force results in chaotic motion.

4. In the presence of non-Boussinesq effects the magnetization parameters have a diminished or enhanced influence on the onset of Marangoni convection compared to that in a constant viscosity fluid layer with no heat source (sink), depending on whether the viscosity-temperature dependence is dominating or the viscosity-magnetic field strength dependence is dominating.

5. Viscosity variation plays a more significant role for the heat sink than for the heat source in the case of Marangoni convection.

6. Further, unlike the Rayleigh-Bénard instability wherein the heat source and heat sink have identical effect, the Marangoni convection problem with heat source must be treated separately from the problem with heat sink. This is, perhaps, recorded for the first time in the literature with appropriate physical reasoning.

7. Through our numerical experiments, it is clear that the viscosity-temperature and viscosity-magnetic field relationships have to be nonlinear for the Rayleigh-Bénard instability with free-free boundaries and for the Marangoni instability.

8. The mutually antagonistic influence (*MAI*) of thermo- and magneto-rheological effects is more pronounced in Newtonian ferromagnetic fluids if the instability is due to surface-tension rather than buoyancy.

9. For adiabatic boundaries, a temperature term appears in the boundary conditions on the magnetic potential Φ.

10. Thermal radiation inhibits the onset of convection in both the transparent and opaque media.

11. The opaque medium is shown to release heat for convection more slowly than the transparent medium.

12. Radiation affects the cell size at the onset of convection in the case of transparent medium and the opposite is true for opaque medium.

13. The stabilizing effect of Biot number *Bi* is more pronounced for large values of the conduction-radiation parameter χ. This is true for both transparent and opaque media.

14. The stabilizing effect of Biot number *Bi* vanishes as the fluid layer depth increases. This is true for both transparent and opaque media.

15. The mutually antagonistic influence of temperature and magnetic field strength dependent viscosity vanishes as the fluid layer depth increases if the fluid medium is opaque.

16. The results of the linear stability analysis of *RBC* and *MC* in Newtonian ferromagnetic liquids are valid provided that the temperature as well as the strength of magnetic field is not very large. Very strong strengths of magnetic field will induce non-Newtonian characteristics in the liquid Large temperatures, in addition to a stronger magnetic field, will render the local and convective accelerations important giving scope for manifestation of oscillatory or even sub-critical motions.

17. Oscillatory mode of instability is preferred to the stationary instability for the Rayleigh-Bénard convection in a viscoelastic ferromagnetic liquid provided $\eta < 1$.

18. The results pertaining to the Jeffrey ferromagnetic liquids lead to those of Maxwell, Rivlin-Ericksen and Newtonian ferromagnetic liquids by a suitable limiting process.

19. Surface-tension effect does not have a say on the onset of convection in a viscoelastic ferromagnetic liquid layer.

20. The viscoelastic parameters Γ_V and η have opposing influence on the onset of convection.

21. The results pertaining to Newtonian/viscoelastic ferromagnetic liquids with a *dc* magnetic field under a suitable limiting process lead to the results of Newtonian/viscoelastic dielectric liquids with an *ac* electric field. The analogy holds good whether or not variable viscosity, heat source (sink) and thermal radiation are present. This analogy suggests that one can discount an isolated study of the problem involving dielectric liquids.

APPENDIX – A

A1. PREPARATION OF MAGNETIC FLUIDS

Bitter (1932) prepared a colloidal solution of magnetite which was stable under zero gravity but became unstable in the presence of a magnetic field. This phenomenon, known as Bitter's technique, is used to study domain boundaries of the surface of the ferromagnetic materials. Although the dream and challenge of producing a liquid that possessed strong magnetic properties was put forward simultaneously by several investigators, it was Papell (1963) who realized the preparation of a ferrofluid. Papell was interested in synthesizing magnetic fluids which could be mixed with rocket fuels so that fuels could be pumped under zero gravity condition by means of external magnetic fields. Using a ball milling technique, Papell dispersed magnetite in kerosene with the use of an oleic acid as a surfactant. It so happened that a suspension of coarse magnetic particles had been in use since the 1940's in magnetic clutches, but Papell's ferrofluid resembled a clutch fluid only superficially, the size of the particles of a ferrofluid being 1000 times smaller in linear dimensions than that of a clutch fluid. Under the influence of a magnetic field a clutch fluid congeals into a solid mass; hence when it is magnetically active it is not a fluid and vice versa, whereas a ferrofluid does not congeal when it is subjected to a magnetic field. It becomes magnetized, but remains a liquid. Papell's ferrofluid having kerosene as a base with high evaporation rate is unsuitable for many industrial applications.

Around the same time, Rosensweig and his collaborators (Neuringer and Rosensweig, 1964), on the other hand, were interested in the formation of magnetic fluids that had implications for the energy conversion devices. They prepared magnetic fluids which were magnetically about 10 times as strong as Papell's original fluid. The systematic study with various aspects involving these fluids resulted in a new branch of hydrodynamics known as *Ferrohydrodynamics* (Rosensweig, 1986).

A2. THREE COMPONENTS OF MAGNETIC FLUIDS

(*i*) *Magnetic particles*

The magnetic properties of the magnetic fluid are decided by the magnetic moment related to the dispersed magnetic particles. The size of the colloidal particles is restricted in such a way that each particle acts as a single domain. Ferrofluids based on small particles tend to have a narrow size distribution compared to large particles. The number density of these particles is roughly $10^{23}/m^3$ (Rosensweig, 1986). The size of the magnetic particles must be sufficiently small because the stability of a magnetic fluid as a colloidal suspension is ensured by thermal motion of the particles, preventing agglomeration and precipitation. This motion increases with decreasing particle size. On the other hand, the magnetic particles must not be very small for at sizes less than 1–2 nm, their magnetic properties disappear (Yaacob *et al*., 1995). The commonly used magnetic materials in ferrofluid are magnetite (Fe_3O_4), maghemite (Fe_2O_3) and other mixed ferrites (Rosensweig, 1986). Depending on the application, the magnetic particles can be chosen. In character recognition ink-jet inks, for example, where a high remanence is required, ferrites such as cobalt ferrite and barium hexaferrite are preferred. In situations where enhanced viscosity is to be avoided as in rotating shaft seals, the particles with low magneocrystalline anisotropy are used.

(*ii*) *Carrier liquid*

At present magnetic fluids are available in any carrier liquid, for instance, in water, hydrocarbon, fluorocarbon, diester and silicon oils. The choice of this dominating component depends on the type of application of the fluid. In particular, when a rapid evaporation of liquid is desired such as in domain observation, ink-jet printing, catalysis and synthesis of polymer beads, aqueous or organic based magnetic fluids are used. In loudspeakers, seals, bearings, stepper motors and gauges, magnetic fluids must stay in working gaps for several years (often under high temperature conditions). In these applications, high molecular synthetic lubricating oils are of great use. Ferrofluids having ester as their carrier liquid can

operate over a wide temperature range and have a high viscosity index and are more oxidative stable than the hydrocarbon ones. For harsh chemical environments, perfluoropoly ethers are the best carrier liquids and under nuclear radiation polyphenyl ethers are the most durable. Vegetable oils offer the best choice when biodegradibility and toxicity are of concern; however, they suffer from poor thermal stability and their pour points are high. Silicones have the highest viscosity index of all the carriers; they exhibit low volatility and can be used over a wide operating temperature range, but the magnetization values are limited to about 100 G. Ferrofluids which are aqueous based are environmental friendly with a high volatility an a restricted temperature range of 10-80 ^{o} C. More information about carrier fluids can be seen from Table A2.1.

Inorganic solvent **:** aqueous   Organic solvents **:** heptane, xylene, toluene, MEK   Oils **:** esters   Hydrocarbons **:** mineral and synthetic   Silahydrocarbons   Perfluoropolyethers   Polyphenylethers   Glycols   Silicones   Vegetable **:** sunflower, canola, castor

Table A2.1 : Liquid carriers used in the synthesis of ferrofluids.

Most of the commercially available magnetic fluids are electrically non-conducting but it is desirable to use conducting ferrofluids in several applications (Shizawa and Tanahashi, 1986; Qi *et al.*, 2001).

(*iii*) *Surfactant*

The role of the surfactant is to maintain a certain minimum distance namely 1–2 nm between the particles. Since the number density of magnetic particles in the magnetic fluid is very high ($\sim 10^{23}/m^3$), the particles can be close to each other and they are capable of experiencing a short range van der Waal's force as well as magnetic attraction force. If these two forces dominate the thermal energy, then the

particles tend to aggregate and sediment out from the dispersion. To prevent this, a repulsive interaction is created between the particles. This can be done in two ways.

(*a*) *Entropic / steric repulsion (Scholten, 1978)*

This can be created by coating fatty acids on the surface of each magnetic particle. This type of repulsion is present in surfacted magnetic fluid.

(*b*) *Electrostatic repulsion (Scholten, 1978)*

This type of repulsion is present in ionic magnetic fluids. This is done by creating Coulombic repulsion between the like charges on each individual magnetic particle. A molecule of a typical surfactant consists of a linear chain of hydrocarbon atoms at one end and anchor polar group of atom at the other end. The chain is called "tail" and the anchor group is called "head". When a molecule of a surfactant is adsorbed on the surface of the particle, the head remains attached on the surface while the tail is free to perform thermal movements in the carrier liquid. When a second particle approaches closely, the chains have to bend aside and their motion is restricted severely. This causes steric or entropic repulsion. It should be remarked that an excess of stabilizer in a solution leads to an increase in the viscosity.

A3. PROPERTIES OF MAGNETIC FLUIDS

Hydrodynamic, magnetic and rheological properties of a magnetic fluid determine its applicability in technological devices. The variation of fluid properties depends on the fluid composition, ratio of its components and on its preparation techniques to some extent. On the other hand, several of its physical properties are modified under the influence of an external magnetic field.

(*i*) *Hydrodynamical properties*

When a magnetic fluid is exposed to a magnetic field, a body force is developed within the fluid which can change its response to the magnetic field, while retaining

the essential fluid characteristics. The magnetic force $\left(\vec{M}.\nabla\right)\vec{H}$ produces an additional term in the Bernoulli's equation (Neuringer and Rosensweig, 1964) given by

$$p + \frac{1}{2}\rho q^2 + \rho g h'' - \mu_o \int_0^H M\,dH = \text{constant}\,, \tag{A3.1}$$

where p and q are pressure and velocity respectively, ρ is the fluid density, g is the gravitational constant, h'' is the height relative to some reference, μ_o is the permeability constant, M is the magnetization and H is the field intensity.
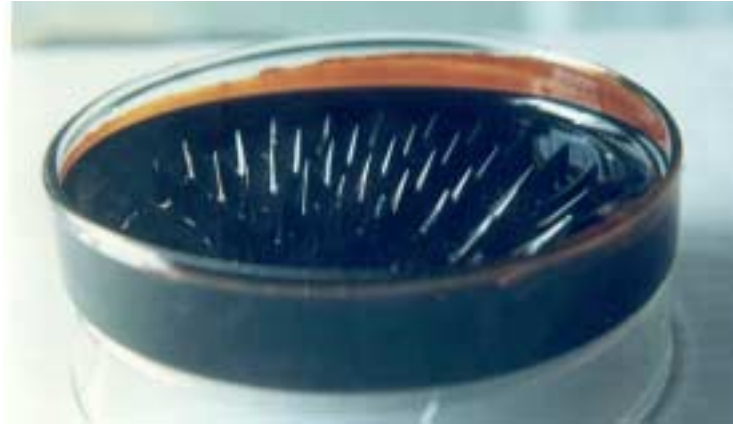
Figure A3.1 : Spike formation in magnetic fluid.
(From Upadhyay, 2000)

Several new phenomena arise because of this new body force; the fluid can be suspended in space by applied magnetic field, spontaneous formation of stable liquid spikes in the presence of a perpendicular magnetic field (Figure A3.1), ability to flow and conduct magnetic flux, stable levitation of nonmagnetic as well as magnetized materials (Figure A3.2) and generation of fluid motion by thermal and magnetic means without any external moving parts (Curtis, 1971).

Figure A3.2 : Levitation of a nonmagnetic object.
(From Upadhyay, 2000)

(*ii*) *Magnetic properties*

The magnetic response of a magnetic fluid is due to the coupling of individual magnetic particles with a substantial volume of the surrounding carrier liquid. Thus, when a magnetic field is applied, each particle experiences a force in the direction of the magnetic gradient and this force is also transmitted to the volume of the associated liquid phase. For a magnetic fluid, magnetization (magnetic moment per unit volume) is an important property. The way in which this varies with the external magnetic field and temperature determines its suitability for a particular application.

Under normal conditions a magnetic fluid consists of millions of tiny magnets moving randomly in the carrier liquid. Under the influence of an external magnetic field, particles try to align in the direction of the external magnetic field, while Brownian motion destroys this alignment. When magnetic fluids move in the presence of a magnetic field or are exposed to unsteady magnetic fields, one has to consider magnetization relaxation towards its equilibrium value. The main mechanisms responsible for the relaxation processes are discussed below.

189

(*a*) *Neel relaxation mechanism*

This mechanism is characterized by the properties of ferromagnetic particles. This mechanism corresponds to reversal of the magnetic moment within the grain to align along the direction of the field without mechanical rotation of the particles. The concept of "*blocking*" is associated with this mechanism.

(*b*) *Brownian relaxation mechanism*

In this mechanism, the magnetic moment of the particles can rotate by mechanical rotation of the particle with respect to the carrier liquid. Here, the magnetization direction is frozen with the particle and it will change its direction only if the particles rotate. At normal temperatures, the Brownian rotation generally predominates in magnetic fluids.

A4. RHEOLOGICAL PROPERTIES

The accompanying magnetic and fluid properties of a magnetic fluid make it all the more attractive. In deed, the magnetic fluid retains its flowability in the presence of a magnetic field. It has been reported that a magnetic fluid may show Newtonian or non-Newtonian behaviour depending on the particle number density, the property of the carrier fluid and the method of preparation (Rosensweig, 1969). In many technological applications, the rheological properties of magnetic fluids play a vital role and one must determine how these properties are modified by an external magnetic field. The rheology of a magnetic fluid is closely related to its viscosity. Many workers have studied the effect of viscosity on the potential applications, such as, fluid magnetic levitation, shaft seals, dampers etc. of magnetic fluids. A comprehensive discussion on the variable nature of viscosity with respect to temperature and magnetic field/electric field has been given in Chapter II.

A5. OPTICAL PROPERTIES

Magnetic fluids are, by and large, black and practically opaque. However, a transparent one can be made out of a thin film of a magnetic fluid and its optical properties can be studied. In the absence of a magnetic field the optical properties of a ferrofluid are isotropic. If a field is applied then it exhibits properties similar to a uniaxial crystal. The direction of the applied field becomes the optical axis. Thus, the fluid exhibits optical anisotropy which increases with magnetic field and reaches saturation. Magnetic birefringence, dichroism and anisotropy in light scattering are some of the consequences of this induced anisotropy. Magneto-optical effects in ferrofluids are utilized in the development of light shutters, field sensors and spatial filters (Trivedi *et al*., 2004).

The other physical properties like dielectric constant, ultrasonic propagation, thermal conductivity, surface tension etc. are modified by the application of a magnetic field. The study of all these properties as a function of magnetic field and temperature can lead to the development of many useful applications (Berkovsky *et al*., 1993).

APPENDIX – B

B1. SURFACE-TENSION BOUNDARY CONDITIONS FOR A VARIABLE-VISCOSITY FERROMAGNETIC / DIELECTRIC LIQUID WITH INTERNAL HEAT SOURCE

We assume that the surface-tension σ_s depends on the temperature as well as the magnetic field strength. The free surface is assumed to be non-deformable for simplicity. The linearized surface-tension boundary conditions in the perturbed state for a variable-viscosity ferromagnetic fluid with internal heat source can be obtained by equating the shear stresses at the surface to the variations of surface-tension, *i.e.*,

$$\tau_{xz} = \mu_b(z)\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right) = \frac{\partial \sigma_s{}'}{\partial x} \tag{B1.1}$$

and

$$\tau_{yz} = \mu_b(z)\left(\frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y}\right) = \frac{\partial \sigma_s{}'}{\partial y}, \tag{B1.2}$$

where $\mu_b(z) = \mu_1\left[1 - \delta\{f(z)\}^2\right]$ (see Eq. 4.2.9). Using Eq. (2.2.3), differentiating Eq. (B1.1) with respect to 'x' and Eq. (B1.2) with respect to 'y' and adding the resulting equations, we obtain

$$\mu_b(z)\frac{\partial}{\partial z}\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = \nabla_1^2 \sigma_s{}'. \tag{B1.3}$$

In arriving at Eq. (B1.3) we have used the fact that w does not vary with x and y at the boundaries.

Eq. (B1.3), on using the continuity equation (2.2.6), reduces to

$$-\mu_b(z)\frac{\partial^2 w'}{\partial z^2} = \nabla_1^2 \sigma_s{}'. \tag{B1.4}$$

From Eqs. (2.1.6) and (B1.4), we arrive at

$$\mu_b(z) \frac{\partial^2 w'}{\partial z^2} = \sigma_T \nabla_1^2 T' - \sigma_H \nabla_1^2 H_3'.$$
(B1.5)

Substituting the expression for $\mu_b(z)$, introducing the perturbed magnetic potential Φ' and using the normal mode technique, we obtain

$$\mu_1 \left[1 - \delta \{ f(z) \}^2 \right] D^2 w + \sigma_T k^2 T - \sigma_H k^2 D\Phi = 0.$$
(B1.6)

Non-dimensionalizing the above equation using the scaling given in Chapter II, we obtain

$$\left[1 - \Gamma \{ g(z) \}^2 \right] D^2 w + a^2 Ma\, T - a^2 Ma_H\, D\Phi = 0,$$
(B1.7)

where $g(z) = (N_S z^2/2) + z - (N_S/8)$ (see Eq. (4.2.20). Thus the boundary conditions for a free surface of a variable-viscosity magnetic fluid in the presence of surface-tension, magnetic field and internal heat source are

$$w = 0 \text{ and } \left[1 - \Gamma \{ g(z) \}^2 \right] D^2 w + a^2 Ma\, T - a^2 Ma_H\, D\Phi = 0.$$
(B1.8)

Similarly, using Eq. (2.1.15), the boundary conditions for a free surface of a variable-viscosity dielectric fluid in the presence of surface-tension and electric field with internal heat source are

$$w = 0 \text{ and } \left[1 - \Gamma \{ g(z) \}^2 \right] D^2 w + a^2 Ma\, T - a^2 Ma_E\, D\Phi = 0.$$
(B1.9)

We note that in the absence of internal heat source, *i.e.*, when $N_S = 0$, we arrive at the boundary conditions for the free upper surface (that is, at $z = 1/2$) of a variable-viscosity magnetic and dielectric fluid given in Eqs. (5.4.14) and (5.5.14) from Eqs. (B1.8) and (B1.9) respectively. The boundary conditions for a free surface of a constant-viscosity magnetic and dielectric liquid can be recovered respectively from Eqs. (B1.8) and (B1.9) by taking $\Gamma = 0$. It is worth mentioning

193

that the boundary conditions in Eqs. (B1.8) and (B1.9) are influenced by the viscosity and magnetic field but unaffected by the internal heat source. This observation can be understood if we replace z by 1/2 in the expression for $g(z)$.

B2. GENERAL BOUNDARY CONDITIONS ON THE MAGNETIC POTENTIAL

The magnetic boundary conditions are that the normal component of the magnetic induction and tangential components of the magnetic field are continuous across the boundary. Thus the matching conditions at the boundaries are

$$[H_1'] = [H_2'] = [B_3'] = 0 \quad \text{at} \quad z = \pm \frac{d}{2}, \tag{B2.1}$$

where the square bracket denotes the difference between the values of bracketed quantity at both sides of the boundary.

Eq. (2.1.7b), in the perturbed state, becomes

$$\vec{H}' = \nabla \Phi', \tag{B2.2}$$

where Φ' is the perturbed magnetic scalar potential. The boundary conditions on Φ' can be obtained using the fact that the periodic nature of Φ' within the fluid layer induces a periodic magnetic potential ζ_m' outside the layer. Thus outside the layer

$$\vec{H}' = \nabla \zeta_m'. \tag{B2.3}$$

From Eqs. (2.1.7a), (2.1.8) and (B2.3), we arrive at

$$\nabla^2 \zeta_m' = 0 \quad \text{(outside the layer).} \tag{B2.4}$$

The third condition in Eq. (B2.1), using Eq. (2.1.8), leads to

$$\zeta_m' = \Phi', \quad D\zeta_m' = H_3' + M_3' \quad \text{at} \quad z = \pm \frac{d}{2}. \tag{B2.5}$$

194

The above equation, on using the second of Eq. (4.2.12) and normal mode analysis, leads to

$$\zeta_m = \Phi, \quad D\zeta_m = (1 + \chi_m)D\Phi - K_1 T \quad \text{at} \quad z = \pm\frac{d}{2}. \tag{B2.6}$$

Equation (B2.4), using the normal mode analysis, becomes

$$(D^2 - k^2)\zeta_m = 0 \quad \text{(outside the layer).} \tag{B2.7}$$

From Eqs. (B2.6) and (B2.7), we obtain

$$\left.\begin{array}{l} (1 + \chi_m)D\Phi + k\,\Phi - K_1 T = 0 \quad \text{at} \quad z = \dfrac{1}{2}, \\[2mm] (1 + \chi_m)D\Phi - k\,\Phi - K_1 T = 0 \quad \text{at} \quad z = -\dfrac{1}{2}. \end{array}\right\} \tag{B2.8}$$

The dimensionless form of Eq. (B2.8) is

$$\left.\begin{array}{l} D\Phi + \dfrac{a\,\Phi}{1 + \chi_m} - T = 0 \qquad \text{at} \quad z = \dfrac{1}{2}, \\[3mm] D\Phi - \dfrac{a\,\Phi}{1 + \chi_m} - T = 0 \qquad \text{at} \quad z = -\dfrac{1}{2}. \end{array}\right\} \tag{B2.9}$$

The boundary conditions given by Eq. (B2.9) are the general boundary conditions on the magnetic potential Φ and are derived for the first time in the literature. In particular, when both boundaries are isothermal, Eq. (B2.9) reduces to

$$\left.\begin{array}{l} D\Phi + \dfrac{a\,\Phi}{1 + \chi_m} = 0 \quad \text{at} \quad z = \dfrac{1}{2}, \\[3mm] D\Phi - \dfrac{a\,\Phi}{1 + \chi_m} = 0 \quad \text{at} \quad z = -\dfrac{1}{2}. \end{array}\right\} \tag{B2.10}$$

The conditions in Eq. (B2.10) have been used by Finlayson (1970) while studying the *RBC* in a ferromagnetic fluid using rigid-rigid, isothermal boundaries.

B3. GENERAL BOUNDARY CONDITIONS ON THE ELECTRIC POTENTIAL

The electric boundary conditions are that the normal component of dielectric field $\vec{D}$ and tangential components of electric field $\vec{E}$ are continuous across the boundaries. Thus the matching conditions at the boundaries are

$$[E_1'] = [E_2'] = [D_3'] = 0 \quad \text{at} \quad z = \pm\frac{d}{2}, \tag{B3.1}$$

where the square bracket denotes the difference between the values of bracketed quantity at both sides of the boundary.

Eq. (2.1.16b), in the perturbed state, becomes

$$\vec{E}' = \nabla \Phi', \tag{B3.2}$$

where Φ' is the perturbed electric scalar potential. The boundary conditions on Φ' can be obtained using the fact that the periodic nature of Φ' within the fluid layer induces a periodic electric potential ζ_e' outside the layer. Thus outside the layer

$$\vec{E}' = \nabla \zeta_e'. \tag{B3.3}$$

From Eqs. (2.1.16a), (2.1.17a) and (B3.3), we arrive at

$$\nabla^2 \zeta_e' = 0 \quad \text{(outside the layer).} \tag{B3.4}$$

The third condition in Eq. (B3.1), using Eqs. (2.1.17a) and (2.1.17b), leads to

$$\zeta_e' = \Phi', \quad \varepsilon_o D\zeta_e' = \varepsilon_o E_3' + P_3' \quad \text{at} \quad z = \pm\frac{d}{2}. \tag{B3.5}$$

The above equation, on using the second of Eq. (4.4.10) and normal mode analysis, leads to

$$\zeta_e = \Phi, \quad D\zeta_e = (1+\chi_e)D\Phi - eE_oT \quad \text{at} \quad z = \pm\frac{d}{2}. \tag{B3.6}$$

Equation (B3.4), using the normal mode analysis, becomes

$$(D^2 - k^2)\zeta_e = 0 \quad \text{(outside the layer).} \tag{B3.7}$$

From Eqs. (B3.6) and (B3.7), we obtain

$$\left.\begin{aligned}
(1+\chi_e)D\Phi + k\,\Phi - eE_oT = 0 \quad &\text{at} \quad z = \frac{1}{2}, \\
(1+\chi_e)D\Phi - k\,\Phi - eE_oT = 0 \quad &\text{at} \quad z = -\frac{1}{2}.
\end{aligned}\right\} \tag{B3.8}$$

The dimensionless form of Eq. (B3.8) is

$$\left.\begin{aligned}
D\Phi + \frac{a\Phi}{1+\chi_e} - T = 0 \quad &\text{at} \quad z = \frac{1}{2}, \\
D\Phi - \frac{a\Phi}{1+\chi_e} - T = 0 \quad &\text{at} \quad z = -\frac{1}{2}.
\end{aligned}\right\} \tag{B3.9}$$

The boundary conditions given by Eq. (B3.9) are the general boundary conditions on the electric potential Φ and are derived for the first time in the literature. In particular, when both boundaries are isothermal, Eq. (B3.9) reduces to

$$\left.\begin{aligned}
D\Phi + \frac{a\Phi}{1+\chi_e} = 0 \quad &\text{at} \quad z = \frac{1}{2}, \\
D\Phi - \frac{a\Phi}{1+\chi_e} = 0 \quad &\text{at} \quad z = -\frac{1}{2}.
\end{aligned}\right\} \tag{B3.10}$$

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CONFERENCES ATTENDED / PAPERS PRESENTED

1. Attended **"Third National Conference on Magnetic Fluids"** organized by Department of Physics, Pondicherry Engineering College, Pondicherry during March 29-31, 2000.

2. Attended "**National Conference on Recent Trends in Applied Mathematics**" organized by Kanchi Mamunivar Centre for Post Graduate Studies, Pondicherry during February 1-2, 2001 and presented a paper entitled "Onset of convection in a variable viscosity ferromagnetic fluid ".

3. Attended a series of lectures delivered by Prof. J. L. Bona on **"Non - linear Analysis and its applications to problems in wave propagation"**, organized by Department of Mathematics, Indian Institute of Science and Tata Institute of Fundamental Research, Bangalore during July 23-August 10, 2001.

4. Attended "**46th Congress of Indian Society of Theoretical and Applied Mechanics"** held at Regional Engineering College, Hamirpur, Himachal Pradesh, India during December 19-22, 2001 and presented a paper entitled "Effect of radiation on Rayleigh-Bénard convection in ferromagnetic fluids ".

5. Attended "**National Seminar on Recent Advances in Fluid Mechanics**" organized by Department of Mathematics, Gulbarga University during September 11-12, 2002 and presented a paper entitled "Onset of convection in a variable viscosity ferromagnetic fluid under μg conditions".

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7. Attended "**90th Session of Indian Science Congress"** held at Bangalore University, Bangalore, Karnataka during January 03 – 07, 2003 and presented a paper entitled "Effect of radiation on electroconvective instability in dielectric fluids**".**

8. Attended "**International workshop on Recent Advances in Nanotechnology of Magnetic Fluids**" organized by National Physical Laboratory, New Delhi, India during January 22-24 and presented a paper entitled "Effect of internal heat generation on the onset of convection in a variable viscosity ferromagnetic fluid layer under μg conditions".

9. Attended "**Symposium on Advances in Fluid Mechanics**" organized by Jawaharlal Nehru Centre for Advanced Scientific research and Department of Aerospace Engineering, I.I.Sc., Bangalore during July 24-25, 2003 and presented a paper entitled "Effect of radiation on non-gravity ferroconvection".

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PUBLICATIONS

1. "Effect of radiation on Rayleigh-Bénard convection in ferromagnetic fluids", *International Journal of Applied Mechanics and Engineering*, **8**(3), pp. 449–459, 2003.

2. "Analogy between instabilities in two smart liquids", *Proc of Symposium on Modelling of Nano and Smart Materials* (Ed.: N. Rudraiah)**,** Bangalore, India, pp. 311–320, 2003 (with P. G. Siddheshwar).

3. Thermorheological and magnetorheological effects on Marangoni-ferroconvection with internal heat generation and analogy with electroconvection", *Fluid Dynamics Research* (resubmitted) 2005 (with P. G. Siddheshwar).