ANTENNA ARRAYS

THE DIRECTIVITY OF A VERTICAL ANTENNA IN THE VERTICAL PLANE DEPENDS ON ITS HEIGHT WHICH CAN BE ALTERED BY ITS HEIGHT ABOVE GROUND (THE HEIGHT FUNCTION - LECTURE 7)

HOWEVER TO OBTAIN DIRECTIVITY IN THE HORIZONTAL PLANE, TWO OR MORE ANTENNAS MUST BE USED TO FORM AN ARRAY. THE RESULTANT FIELD PATTERN CAN BE OBTAINED BY CONSIDERING THE ANTENNAS AS POINT SOURCE ISOTROPIC RADIATORS.

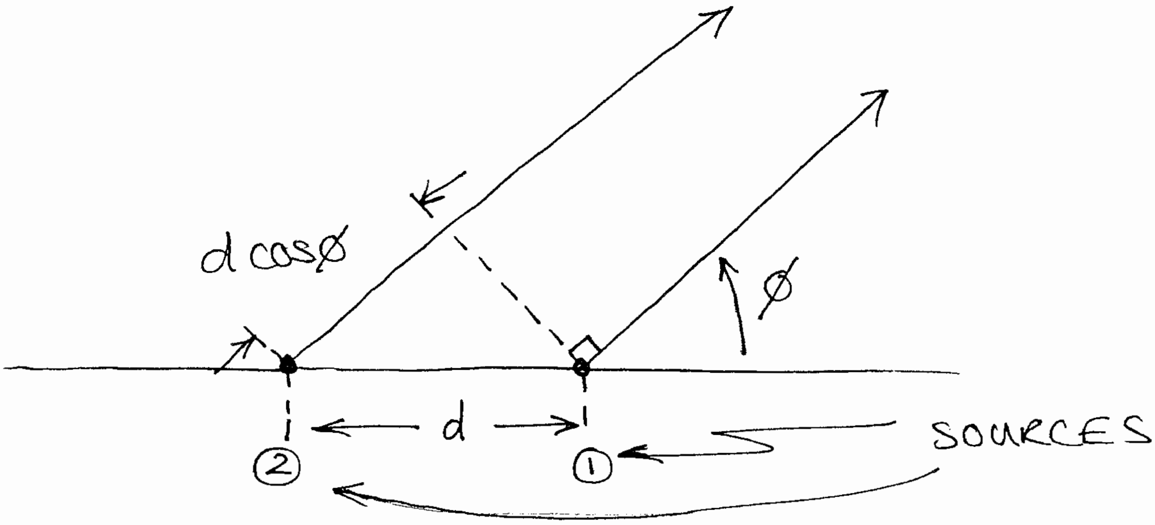
BY CONTROLLING THE SPACING, RELATIVE PHASE AND RELATIVE AMPLITUDES OF THE SOURCES, WE CAN CONTROL SOME ASPECTS OF THE RADIATION PATTERN OF THE RESULTING ARRAY.

ARRAY ANTENNAS ARE VERY USEFUL FOR CREATING HIGH RADIATED POWERS FROM MANY LOW-POWER SOURCES AND FOR ELECTRONIC BEAMFORMING AND STEERING

TWO POINT SOURCES

CONSIDER TWO POINT SOURCES SPACED AT A DISTANCE d IN THE HORIZONTAL PLANE ENERGISED BY CURRENTS IN THE SAME PHASE. AT A DISTANT POINT P THE FIELD STRENGTHS OF EACH ARE APPROXIMATELY EQUAL, BUT THERE IS A PHASE DIFFERENCE DUE TO A PATH DIFFERENCE;

POINT 'P'



THE DIFFERENCE IN PATH LENGTH IS $d\cos\phi$, WHICH PRODUCES A PHASE DIFFERENCE BETWEEN THE TWO FIELDS PRODUCED BY EACH SOURCE

$$\frac{\theta}{2\pi} = \frac{d\cos(\phi)}{\lambda}$$

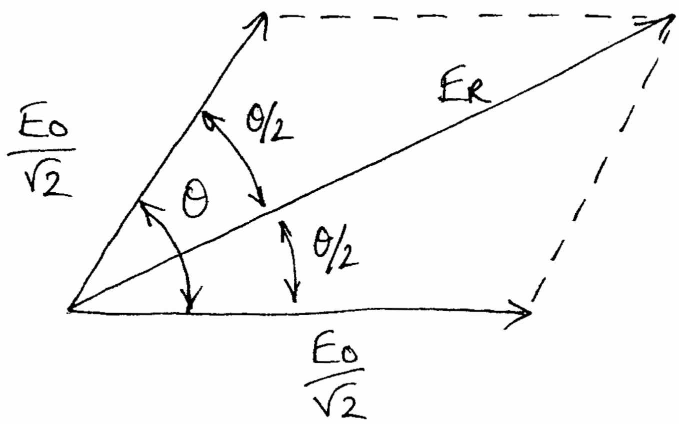
WAVE NUMBER

$$\theta = 2\pi\frac{d\cos(\phi)}{\lambda} = k_0\, d\cos(\phi)$$

IN ORDER TO MAKE A FAIR COMPARISON OF GAIN WE WILL ASSUME THAT THE POWER FED TO A SINGLE ANTENNA IS THE SAME AS THAT FED TO THE ARRAY.

HENCE, IF THE FIELD STRENGTH DUE TO A SINGLE SOURCE IS E_0 WHEN ENERGISED BY A POWER P, THEN THE FIELD STRENGTH OF EACH ELEMENT IN AN ARRAY OF TWO IS $E_0/\sqrt{2}$ AS EACH ELEMENT IS ENERGISED BY A POWER $P/2$ (THAT IS, THE FIELD STRENGTH OF EACH ELEMENT IS REDUCED BY $\sqrt{2}$.

TO FIND THE RESULTANT FIELD STRENGTH;



$$E_R = \frac{E_0}{\sqrt{2}} \cos(\theta/2) + \frac{E_0}{\sqrt{2}} \cos(\theta/2)$$

HENCE

$$E_R = \sqrt{2}\, E_0 \cos(\theta/2)$$

$$= \sqrt{2}\, E_0 \cos\left[\frac{\pi d \cos\phi}{\lambda}\right]$$

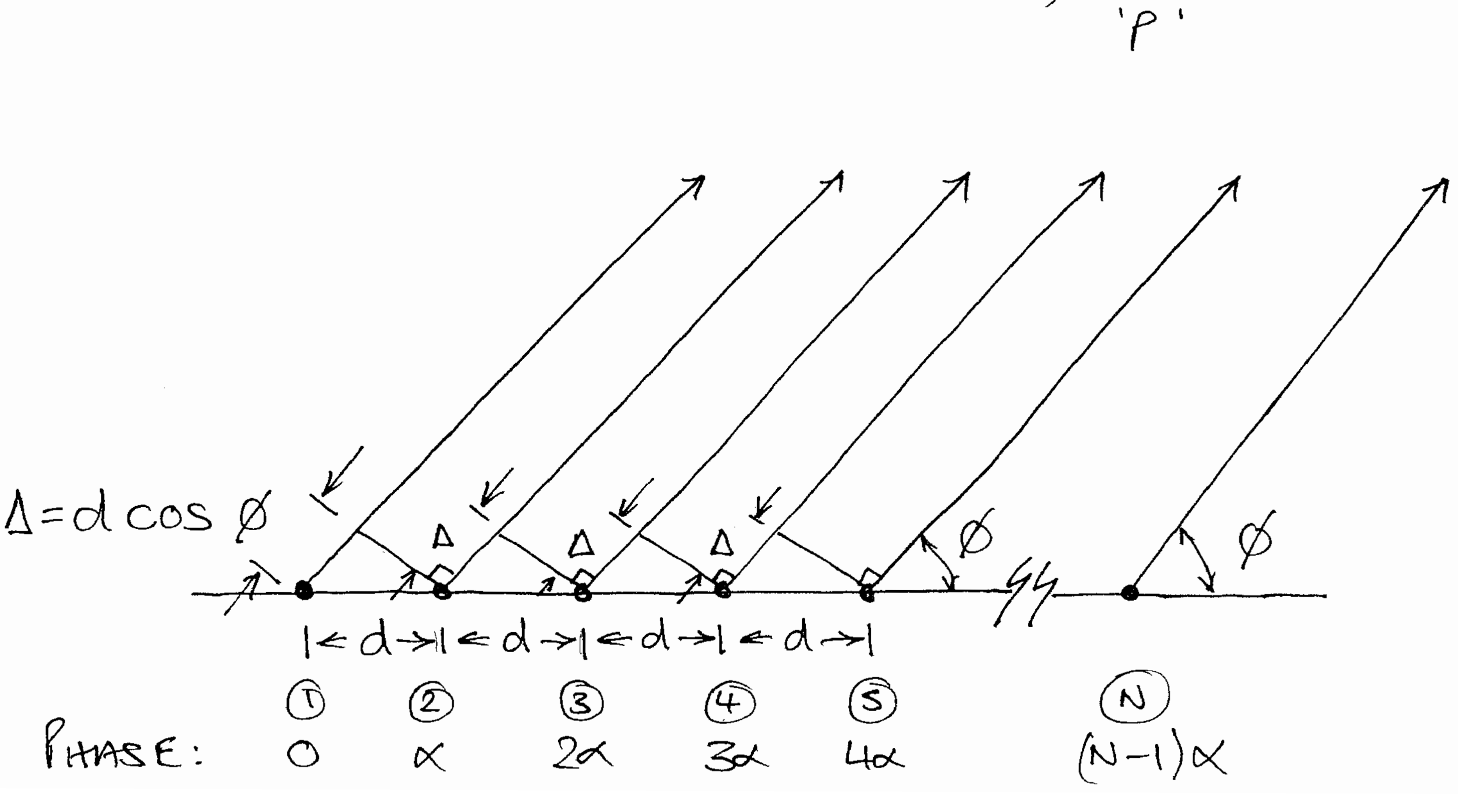
IN GENERAL, IF THE DRIVE CURRENTS LAG OR LEAD EACH OTHER BY A PHASE ANGLE α THEN

$$E_R = \sqrt{2}\, E_0 \cos\left[\frac{\theta \pm \alpha}{2}\right]$$

$\alpha > 0$ CURRENT LAG, $\alpha < 0$ CURRENT LEAD

'N' POINT SOURCES

CONSIDER NOW 'N' POINT SOURCES (UNIFORMLY SPACED & ISOTROPIC)



AS WITH THE PREVIOUS CASE OF TWO SOURCES, WE WILL ASSUME THAT 'P' IS FAR REMOVED FROM THE ARRAY SUCH THAT ALL THE PATHS ARE (ALMOST) PARALLEL, AND THAT 'P' IS IN THE FAR-FIELD

AS BEFORE, THERE IS A PATH DIFFERENCE BETWEEN ADJACENT ELEMENTS OF

$$\Delta = d \cos(\phi),$$

WHICH GIVES RISE TO A PHASE DIFFERENCE,

$$\theta = k_0 d \cos(\phi)$$

THIS TIME WE WILL INTRODUCE A PHASE SHIFT IN THE DRIVE CURRENTS, SUCH THAT AT 'P' THE PHASE DIFFERENCE BETWEEN TWO SIGNALS IS;

$$\theta = k_0 d \cos(\phi) + \alpha$$

WE CAN DETERMINE THE TOTAL FIELD AT 'P' BY SUPERPOSITION OF THE FIELDS FROM THE INDIVIDUAL SOURCES.

USING SOURCE #1 AS A REFERENCE WE HAVE;

$$E_R = E_1 + E_2 \exp(j\theta) + E_3 \exp(j2\theta) + \ldots$$
$$\ldots + E_N \exp(j(N-1)\theta).$$

THIS IS THE GENERAL EXPRESSION FOR THE RADIATION FROM A SET OF SOURCES WITH ARBITRARY AMPLITUDES.

IF WE MAKE THE SIMPLIFYING ASSUMPTION THAT ALL SOURCES ARE OF THE SAME AMPLITUDE $E_0 = E_1 = E_2 = E_3 = \ldots = E_N$, WE CAN ANALYSE THE ARRAY ANALYTICALLY;

$$E_R = E_0\Big[1 + \exp(j\theta) + \exp(j2\theta) + \ldots$$
$$\ldots + \exp(j[N-1]\theta)\Big] \quad - ①$$

IF WE MULTIPLY ① BY $\exp(j\theta)$ WE GET

$$E_R \exp(j\theta) = E_0\Big[\exp(j\theta) + \exp(j2\theta) + \ldots$$
$$\ldots + \exp(jN\theta)\Big] \quad - ②$$

IF WE NOW SUBTRACT ② FROM ① WE GET...

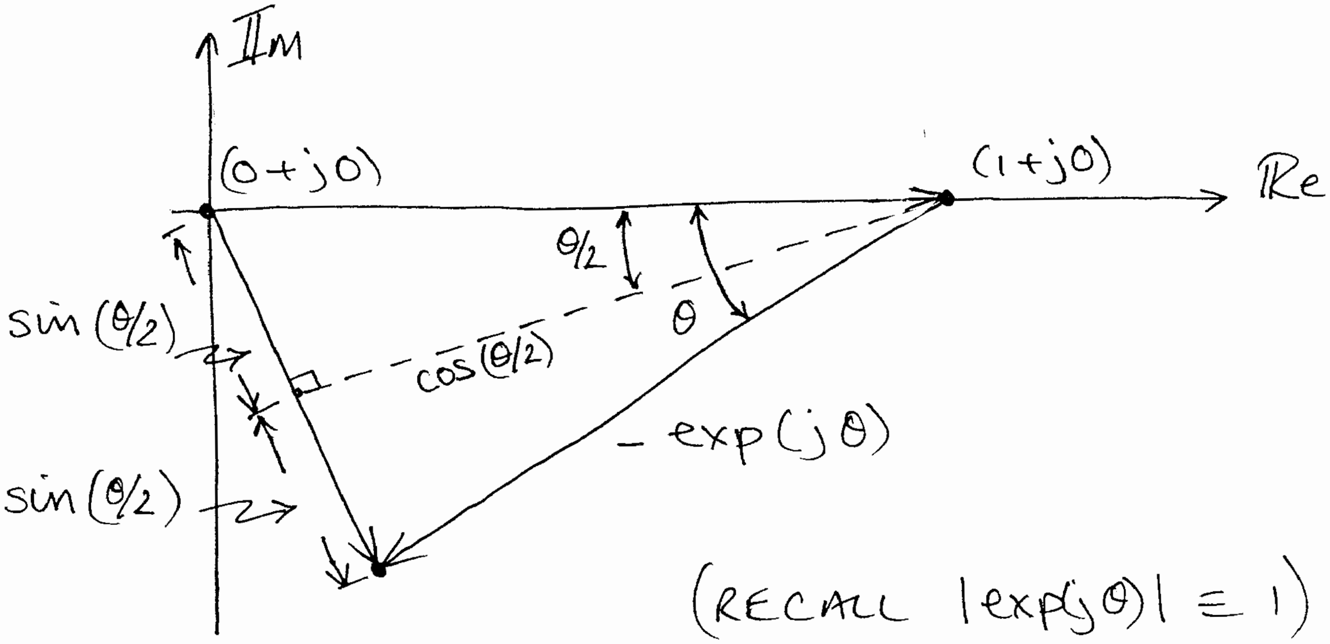
$$E_R\,(1 - \exp(j\theta)) = E_0\,(1 - \exp(jN\theta))$$

WHICH WE CAN WRITE AS;

$$E_R = E_0 \left[\frac{1 - \exp(jN\theta)}{1 - \exp(j\theta)} \right]$$

WE GENERALLY ARE INTERESTED IN THE MAGNITUDE AND PHASE OF THE RADIATED FIELD (THAT IS THE MODULUS AND ARGUMENT OF E_R)

IF WE CONSIDER $1 - \exp(j\theta)$ ON AN ARGAND DIAGRAM;



(RECALL $|\exp(j\theta)| \equiv 1$)

FROM THIS WE FIND THAT:

$$|1 - \exp(j\theta)| = \left| 2 \sin(\theta/2) \right|$$

AND

$$\arg\left[1 - \exp(j\theta)\right] = -\left[\pi/2 - \theta/2\right]$$

BY SUBSTITUTING Nθ FOR θ WE CAN GET A SIMILAR EXPRESSION FOR THE NUMERATOR.

THEREFORE, THE TOTAL FIELD CAN BE WRITTEN AS;

$$E_R = E_0 \left| \frac{\sin\left(N\theta/2\right)}{\sin\left(\theta/2\right)} \right| \exp\left[j\left(\frac{N-1}{2}\right)\theta\right]$$

THE ARGUMENT OF THE RESULTANT FIELD E_R IS THE PHASE CENTRE;

PHASE CENTRE : $\left(\frac{N-1}{2}\right)\theta$

THE MODULUS TERM OF THE RESULTANT FIELD E_R IS THE <u>ARRAY FACTOR</u>

ARRAY FACTOR : $E_0 \left| \frac{\sin\left(\frac{N\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \right|$

WHERE : $\theta = k_0 d \cos(\phi) + \alpha$

THE ARRAY FACTOR DESCRIBES THE VARIATION OF THE RADIATED FIELD AS A FUNCTION OF:

* ANGLE: ϕ
* ELEMENT SPACING: d
* NUMBER OF ELEMENTS: N
* DRIVE CURRENT PHASE: α
* FREQUENCY : (VIA $k_0 = 2\pi/\lambda$)

THE ARRAY FACTOR DETERMINES:

BEAM NULLS: NUMERATOR IS ZERO

MAJOR LOBES: DENOMINATOR IS ZERO

MINOR LOBES: WHEN $\frac{\partial E_R}{\partial \phi} = 0$.

MAJOR LOBES

THESE OCCUR WHEN THE DENOMINATOR OF THE ARRAY FACTOR IS ZERO, THAT IS WHEN:

$\sin(\theta/2) = 0$, WHICH IS WHEN
$\theta = 0, 2\pi, 4\pi, \ldots \quad = 2m\pi$

HOWEVER AT THESE POINTS THE NUMERATOR IS ALSO ZERO. IN ORDER TO BE ABLE TO EVALUATE THE RADIATED FIELD, WE MUST USE L'HÔPITAL'S RULE;

FOR A FUNCTION WHICH CAN BE WRITTEN AS

$\frac{f(x)}{g(x)}$, IF AS $x \to a$, $g(a) = f(a) = 0$

THEN $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$

WHERE PRIME DENOTES $\partial/\partial x$

SO,

$$\partial/\partial\theta\left(\sin(N\theta/2)\right) = N/2 \cos(N\theta/2)$$

$$\partial/\partial\theta\left(\sin(\theta/2)\right) = 1/2 \cos(N\theta/2)$$

SETTING $\theta = m2\pi$ WE HAVE

$$\lim_{\theta \to m2\pi} E_0 \left| \frac{\sin(N\theta/2)}{\sin(\theta/2)} \right| = E_0 \left| \frac{N/2 \cos(Nm\pi)}{1/2 \cos(m\pi)} \right|$$

$$= NE_0 \quad \text{FOR ALL } m$$

THEREFORE AT A MAJOR BEAM (LOBE) THE TOTAL FIELD IS SIMPLY THE SUM OF THE AMPLITUDES OF THE INDIVIDUAL SOURCES.

THIS MEANS THAT A HIGH POWER LOBE CAN BE GENERATED FROM MANY LOW POWER SOURCES.

THE MAJOR LOBES OCCUR AT AN ANGLE ϕ_b DETERMINED BY;

$$\theta = 2m\pi = k_0 d \cos(\phi_b) - \alpha$$

REARRANGING WE OBTAIN

$$\cos(\phi_b) = \frac{2\pi m + \alpha}{k_0 d}$$

$$= \frac{m\lambda}{d} + \frac{\alpha}{k_0 d}$$

THE PRIMARY BEAM OCCURS (FOR $m=0$), AT

$$\cos(\phi_0) = \frac{\alpha}{k_0 d}$$

CLEARLY THE PRIMARY BEAM ANGLE IS CONTROLLED BY THE RELATIVE DRIVE PHASE ANGLE α.

IF THE ELEMENTS OF THE ARRAY ARE ALL DRIVEN IN PHASE ($\alpha = 0°$), THE PRIMARY LOBE OCCURS AT $\phi_0 = 90°$, AND THE LOBE IS CALLED A BROADSIDE LOBE

IF THE PHASE SHIFT IS SET TO $\alpha = k_0 d$ THEN $\phi_0 = 0°$. IN THIS CASE THE LOBE IS CALLED AN ENDFIRE LOBE

BY VARYING THE PHASE, INTERMEDIATE ANGLES CAN BE GENERATED.

OF COURSE IT IS POSSIBLE TO PRODUCE OTHER LOBES BY CHOOSING OTHER VALUES OF m. FOR THE m^{th} LOBE WE HAVE

$$\cos(\phi_m) = \frac{m\lambda}{d} + \frac{\alpha}{k_0 d}$$

OR

$$\cos(\phi_m) = \frac{m\lambda}{d} + \cos(\phi_0)$$

THESE LOBES ARE GENERALLY UNWANTED AND ARE CALLED GRATING LOBES

TO AVOID THE m^{th} GRATING LOBE WE NEED TO SATISFY THE CONDITION:

$$d < \frac{m\lambda}{1 - \cos(\phi_0)}$$

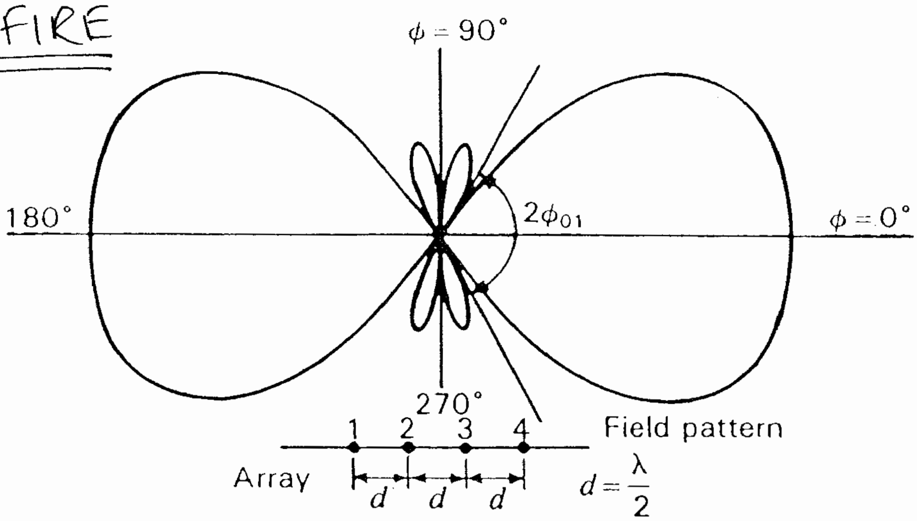
IF THIS IS MET THE GRATING LOBES CAN NOT POSSIBLY EXIST IN REAL SPACE AS IT WOULD IMPLY THAT $|\cos(\phi_m)| > 1$, WHICH CANNOT BE!

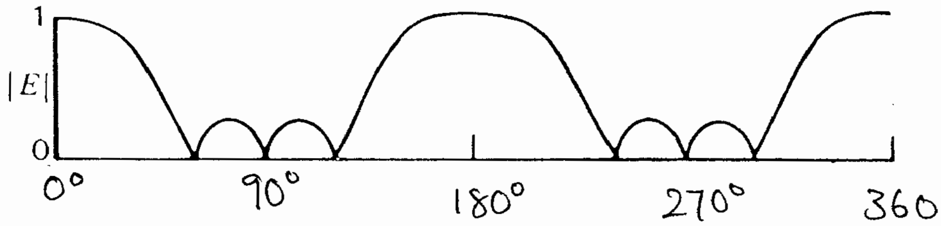
FOR A BROADSIDE BEAM ($\phi_0 = 90°$) WE NEED $d < m\lambda$, SO IF THE FIRST GRATING LOBE IS TO BE AVOIDED WE NEED $d < \lambda$.

GENERALLY A SPACING OF $d = \lambda/2$ IS MOST OFTEN USED, HOWEVER THE EQUATIONS THAT WE HAVE DEVELOPED ARE FOR ISOTROPIC SOURCES, PRACTICAL ELEMENTS (DIPOLES FOR EXAMPLE) ARE DESIGNED TO RADIATE WELL AT $\phi = 90°$, AND POORLY AT $\phi = 0, 180°$. HENCE PRACTICAL ELEMENTS DO A REASONABLE JOB AT SUPRESSING GRATING LOBES.

IT IS FOR THIS REASON THAT $d = \lambda$ IS SOMETIMES ACCEPTABLE FOR SOME PRACTICAL ANTENNAS.

END-FIRE





BROADSIDE