Radio Science, Volume 12, Number 6, pages 1031-1034, November-December 1977

Diagnosis of the turbulent state of ionospheric plasma by propagation methods

K. C. Yeh and C. H. Liu

Department of Electrical Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

(Received March 22, 1977.)

An electromagnetic signal randomized by scattering from plasma irregularities and by subsequent diffraction in the nonrandom part of the medium is known to bear statistical information about the plasma turbulent irregularities. For example, if the turbulent spectrum is of the form κ^{-p} in its wavenumber dependence, the received amplitude of the electromagnetic signal will fluctuate with a high frequency spectral asymptote of the form ν^{1-p}. However, a power-law turbulent spectrum of the form κ^{-p} has certain nonphysical difficulties if cutoffs at both the inner scale and the outer scale are not introduced. In addition, in this paper it is shown that the mean arrival time and the mean square pulse width are rather sensitive to values of the outer and inner scales of ionospheric turbulence. Information concerning the values of these scales is rather meager at present. It is therefore suggested that pulsed experiments be made so that the inner scale and the outer scale can be determined. Possibilities of making similar measurements on a laboratory plasma are also implied.

1. INTRODUCTION

The ionospheric plasma is known to go turbulent at various geographic locations for a substantial amount of time. That is, the ionospheric electron density $N(\vec{r})$ may have a part $\Delta N(\vec{r})$ that fluctuates randomly from the mean value $\langle N(\vec{r})\rangle$. When this happens, the scattering of electromagnetic energy can be very substantial, resulting in what is known as the spread-F echoes. To characterize this turbulent process completely requires the knowledge of a multidimensional probability density function which is impossible to measure experimentally and extremely difficult to deal with mathematically. As is usually the case, one is then forced to make correlation measurements and to work with correlation theories in which only the first two moments are needed. In this case the first moment is just the mean value and the second moment is related to the power spectrum of $N(\vec{r})$.

Even in the realm of correlation theory, a complete determination of the turbulent process requires an extremely large program to cover the three-dimensional space of interest, so much so that it has never been done. Only through accumulation of many, though incomplete, measurements which are checked internally for consistency and checked against theoretical predictions, does a coherent and average picture emerge. For example, the root mean square value of $\Delta N/\langle N\rangle$ is generally very small at temperate latitudes, being less than 1%, but it may rise to high values of above 10% in the auroral zone and even to 50% or higher near the dip equator [*Dyson*, 1969; *Sagalyn et al.*, 1974; *Basu et al.*, 1976]. The one-dimensional power spectrum of the process $\Delta N/\langle N\rangle$ follows a power law of the form $1/\kappa_x^m$ where m is roughly equal to 2 in the wavenumber range 10^{-3} m^{-1} (scale size 7 km) to 10^{-1} m^{-1} (scale size 70 m) [*Dyson et al.*, 1974; *Phelps and Sagalyn*, 1976]. With the assumption of isotropic process for $\Delta N/N$, this implies a three-dimensional spectrum of the form $1/\kappa^p$ where p is roughly equal to 4. Actually, this power-law behavior was implied in propagation measurements of a few years earlier [*Elkins and Papagiannis*, 1969; *Rufenach*, 1972]. In the following the various techniques are summarized that can be used to characterize the process $\Delta N/N$ within the realm of correlation theory.

(a) The mean square fluctuation σ_N^2 of $\Delta N/N$: In-situ measurements can give σ_N^2 readily but only along the path of the satellite or the rocket. Scintillation measurements can be related to σ_N^2 through model computations if other parameters are measured or assumed and the scintillation is weak [*Umeki et al.*, 1977].

(b) The spectral index p: In-situ measurements have been used to compute the one-dimensional spectrum but only over a two- to three-decade range [*Dyson et al.*, 1974; *Phelps and Sagalyn*, 1976]. In case of weak scintillation, the propagation theory [*Jokippi and Hollweg*, 1970; *Singleton*, 1974] predicts a high frequency asymptote of the form ν^{1-p} for both amplitude and phase. This has been used by various authors to get the spectral index p [*Rufenach*, 1972; *Crane*, 1977] but again only over a two- to three-decade range in wavenumber space.

(c) The height and the thickness of the irregularity region: Radar observations can yield both values [*McClure and Woodman*, 1972]; they can also be measured by scintillation observations at spaced stations [*Liu*, 1965; *Paul et al.*, 1970].

The above statistical properties are very important in our search for a basic understanding of the turbulent process itself. Additionally, for the purpose of computing their effects on waves propagating through these turbulent irregularities, one must first know the statistical properties of $\Delta N/N$. Hence, they are absolutely essential in model computations of the kind carried out by *Basu et al.* [1976].

2. NEED OF INNER AND OUTER SCALES

A power spectrum of the form $1/\kappa^p$ for all values of κ has several difficulties. For example, for $p > 2$ its associated correlation function will not exist. Also, for any finite value of p, the spectral moments will fail to exist above a certain order. To partially remedy this situation an outer scale l_0 was introduced by *Tatarskii* [1971]. However, the Tatarskii spectrum still had problems with large κ and a gaussian-like cutoff near the inner scale was introduced by *Lee and Jokipii* [1976]. While such a procedure eliminated all the difficulties with the moments, the mathematical manipulation of the resulting function became very difficult. *Shkarofsky* [1968] has reviewed this problem and recommends a spectrum expressed in terms of Bessel functions with imaginary arguments. This Bessel spectrum has an inner scale r_0 and an outer scale l_0. For wavenumbers in the range $l_0^{-1} < \kappa < r_0^{-1}$ the spectrum reduces to the desired power-law spectrum. For $\kappa \ll l_0^{-1}$, the spectrum is flat and for $\kappa \gg r_0^{-1}$ the spectrum decays exponentially. Its moments of all orders can be computed analytically and are finite. Its three-dimensional, two-dimensional, and one-dimensional correlation functions can easily be derived and are also given by Bessel functions of imaginary argument but of different orders. Mathematically, they are very convenient to use and have been used in connection with the scintillation theory [*Yeh and Liu*, 1977]. Some of their formulas will be adapted in this paper.

Experimentally, information on the inner scale and outer scale is almost nonexistent. Scintillation measurements of either amplitude or phase are not expected to be of much help in locating the inner scale because the presence of noise (receiver calibration error, scaling error, quantization effects in a digital system, etc.) will prevent an accurate determination of that part of the spectrum affected by small irregularities. In-situ measurements give the power spectrum for scales not smaller than 70 m [*Dyson et al.*, 1974; *Phelps and Sagalyn*, 1976]. This has been extended downward to 3 m by using the 40 MHz radar data [*Yeh et al.*, 1975]. However, since Arecibo radar operating at 430 MHz does not seem to see spread-F irregularities of size 0.35 m, we may well surmise that the inner scale must be larger than 0.35 m, placing the inner scale in the range 0.35 to 3 m. The matter of the outer scale is probably not in much better position either. In-situ measurements give a power spectrum only up to 7 km. Amplitude scintillation measurements are affected severely by the Fresnel filtering effect and are not expected to be sensitive to scales larger than the Fresnel zone [*Wernik and Liu*, 1974] which is of the order of 1 km at 100 MHz. Large-scale perturbations in the ionosphere (e.g., traveling ionospheric disturbances which have horizontal wavelengths of the order 50 km and up) do exist and they contaminate the data because the process responsible for their existence is completely different from that for the turbulent ionosphere. Current popular values for the outer scale are in the range of tens of kilometers. For example, in their model study *Basu et al.* [1976] picked $l_0 = 20$ km.

3. DIAGNOSIS OF INNER AND OUTER SCALES BY PULSED SIGNALS

Recently *Yeh and Liu* [1977] investigated the behavior of temporal moments of signals randomized by scattering from irregularities. These temporal moments are defined in the following way. Let $A(z,t)$ be the complex amplitude of a narrow band signal propagating along the z axis. The nth moment is defined as

$$\langle\langle t^n(z)\rangle\rangle \equiv \int_{-\infty}^{\infty} t^n \langle |A(z,t)|^2 \rangle \, dt \tag{1}$$

where the angle brackets denote ensemble average. When properly normalized (i.e., $\langle\langle t^0(z)\rangle\rangle = 1$), and for symmetric modulation envelope, the first moment with $n = 1$ in (1) is just the mean arrival time t_a and the mean square pulse width τ^2 is related to the second moment through

$$\tau^2 = \langle\langle t^2(z)\rangle\rangle - t_a^2 \tag{2}$$

The signal is assumed to be impressed a distance z from the receiver on top of a turbulent ionosphere and propagates through the turbulent slab of thickness L. The phase of the signal is then mixed through diffraction below the slab. For carrier frequencies much larger than the plasma frequency, the mean arrival time t_a is found to be

$$t_a = t_1 + t_2 + t_3 \tag{2}$$

where

$$t_1 = z/c[1 - (\omega_p^2/\omega^2)]^{1/2} \tag{3}$$

$$t_2 = 3\omega_p^2\overline{\Omega^2}z/2\omega^4 c \tag{4}$$

$$t_3 = \omega_p^4\sigma_N^2 L(2z - L)\ln(l_0/r_0)/4l_0\omega^4 c \tag{5}$$

The time t_1 is just the transit time required for a signal propagating a distance z with group velocity $c(1 - \omega_p^2/\omega^2)^{1/2}$. The time t_2 is a correction due to higher-order dispersion since the signal may have a finite bandwidth $(\overline{\Omega^2})^{1/2}/2\pi$. (For precise definition of bandwidth, see *Yeh and Liu* [1977]). The time t_3 is caused by scattering and diffraction, and its dependence on the inner scale r_0 is rather weak but its dependence on the outer scale l_0 is fairly strong. For the purpose of making numerical estimates the following parameter values are taken:

$$r_0 = 3\text{ m},\quad l_0 = 10^4\text{ m},\quad L = 300\text{ km},\quad z = 600\text{ km},$$
$$f_p = 10\text{ MHz},\quad (\overline{\Omega^2})^{1/2}/2\pi = 1\text{ MHz},\quad \sigma_N = 0.1,$$
$$f = 100\text{ MHz} \tag{6}$$

The turbulent spectrum is assumed to be given by the Bessel spectrum [*Shkarofsky*, 1968] which reduces to a power-law spectrum of the form $1/\kappa^4$ within the inner and outer scales. The assumed inner scale is nearly equal to the ionic gyroradius. The computed numerical values are

$$t_1 = 2 \times 10^{-3} + 10^{-5}\text{ sec}$$
$$t_2 = 3 \times 10^{-9}\text{ sec}$$
$$t_3 = 1.8 \times 10^{-7}\text{ sec} \tag{7}$$

In the expression for t_1, the first numerical value is just z/c and it has the largest value. The second value in t_1 is a correction which is proportional to the integrated electron density or electron content. Experimentally its contribution to time delay can be measured by monitoring the modulation phase of harmonically related frequencies as is done in radio beacon experiments. A comparison of these numerical values shows that the second-order dispersive effects given by t_2 are very small and can be ignored. The scattering term given by t_3 is also small but it may rise to a magnitude comparable to the second value of t_1 which is proportional to the electron content value in a strong turbulence. The fact that t_3 has a fairly strong dependence on the outer scale l_0 suggests that the measurement of t_3 can be used to determine the outer scale l_0.

The mean square pulse width defined by (2) can also be computed. It is given by

$$\tau^2 = \tau_0^2 + \tau_1^2 + \tau_2^2 + \tau_3^2 + \tau_4^2 + \tau_5^2 \tag{8}$$

where

$$\tau_0^2 = \langle\langle t^2(0)\rangle\rangle = 6.3 \times 10^{-15}\text{ s}^2$$
$$\tau_1^2 = z^2\omega_p^4\overline{\Omega^2}/\omega^6c^2 = 4 \times 10^{-14}\text{ s}^2$$
$$\tau_2^2 = \omega_p^4\sigma_N^2 L l_0/2\omega^4c^2 = 1.7 \times 10^{-14}\text{ s}^2$$
$$\tau_3^2 = \omega_p^4\sigma_N^2 L(L^2 - 2Lz + 3z^2)/3l_0r_0^2\omega^6 = 2.3 \times 10^{-12}\text{ s}^2$$
$$\tau_4^2 = \omega_p^8\sigma_N^4 L^2(12z^2 - 16zL + 6L^2)(\ln l_0/r_0)^2/48l_0^2\omega^8c^2 = 2.7 \times 10^{-20}\text{ s}^2$$
$$\tau_5^2 = \omega_p^8\sigma_N^2 zL(2z - L)\overline{\Omega^2}\ln(l_0/r_0)/2l_0\omega^{10}c^2 = 7.3 \times 10^{-18}\text{ s}^2 \tag{9}$$

The quantity τ_0^2 is just the mean square pulse width of the impressed signal at $z = 0$. The quantity τ_1^2 is the contribution toward lengthening of the pulse width from dispersive effects. The quantities τ_2^2 through τ_5^2 are contributions from scattering; each of these terms can be related to the value of the second derivative or the fourth derivative of the correlation function of $\Delta N/N$ at the origin. The nonlinear term is given by τ_4^2. For the numerical values given by (6) and a gaussian signal spectrum, the various values for τ_0^2 through τ_5^2 can be calculated easily and are given numerically in (9). It is seen that the original mean square pulse width is 6.3×10^{-15} s^2 and after propagating a distance 600 km in a medium described by (6), the mean square pulse width is almost a factor of 400 larger. The dominant contribution comes from τ_3^2, which is related to the fourth derivative of the correlation function at the origin and consequently is very sensitive to the choice of the inner scale. The inner scale of 3 m chosen in (6) is probably the upper bound. If a smaller inner scale were used, the value of τ_3^2 would be even larger than that given in (9). Consequently, the exact value of the inner scale is very important.

4. DISCUSSION

We have given formulas for the mean arrival time in (2) and the mean square pulse width in (8). For numerical values applicable to ionospheric conditions it is found that the mean arrival time is rather sensitive to the outer scale while the mean square pulse width is extremely sensitive to the inner scale. It is suggested that this sensitivity can be exploited experimentally together with existing techniques in order to obtain more accurate values for the inner and outer scales which so far are very uncertain. Furthermore, the formulas given should be applicable to laboratory plasmas even though the numerical values given by (6) may not be applicable. It is entirely possible that a combination of parameters can be found so that the measurement of arrival time and pulse width can be developed as a technique for measuring the inner scale and outer scale for a laboratory plasma.

Acknowledgments. This research was supported by grant ATM 75-21755 from the Atmospheric Sciences Section of the National Science Foundation.

REFERENCES

Basu, S., S. Basu, and B. K. Khan (1976), Model of equatorial scintillations from in-situ measurements, *Radio Sci.*, *11*, 821-832.

Crane, R. K. (1977), Ionospheric scintillation, *Proc. IEEE*, *65*, 180-199.

Dyson, P. L. (1969), Direct measurements of the size and amplitude of irregularities in the topside ionosphere, *J. Geophys. Res.*, *74*, 6291-6303.

Dyson, P. L., J. P. McClure, and W. B. Hanson (1974), In-situ measurements of the spectral characteristics of *F*-region ionospheric irregularities, *J. Geophys. Res.*, *79*, 1495-1502.

Elkins, T. J., and M. D. Pagagiannis (1969), Measurement and interpretation of power law spectrums of ionospheric scintillation at a subauroral location, *J. Geophys. Res.*, *74*, 4105-4115.

Jokippi, J. R., and J. V. Hollweg (1970), Interplanetary scintillation and the structure of solar-wind fluctuations, *Astrophys. J.*, *160*, 745-753.

Lee, L. C., and J. R. Jokippi (1976), The irregularity spectrum in interstellar space, *Astrophys. J.*, *206*, 735-743.

Liu, C. H. (1965), Cross-correlation function of spherical waves propagating through a slab containing anisotropic irregularities, *J. Atmos. Terr. Phys.*, *28*, 385-395.

McClure, J. P., and R. F. Woodman (1972), Radar observations of equatorial spread-*F* in a region of electrostatic turbulence, *J. Geophys. Res.*, *77*, 5617-5621.

Paul, L. M., K. C. Yeh, and B. J. Flaherty (1970), Measurement of irregularity heights by the space receiver technique, *Radio Sci.*, *5*, 967-973.

Phelps, A. D. R., and R. C. Sagalyn (1976), Plasma density irregularities in the high-latitude topside ionosphere, *J. Geophys. Res.*, *81*, 515-523.

Rufenach, C. L. (1972), Power-law wavenumber spectrum deduced from ionospheric scintillation observations, *J. Geophys. Res.*, *77*, 4761-4772.

Sagalyn, R. C., M. Smiddy, and M. Ahmed (1974), High latitude irregularities in the topside ionosphere based on ISIS 1 thermal ion probe data, *J. Geophys. Res.*, *79*, 4252-4261.

Shkarofsky, I. P. (1968), Generalized turbulence space-correlation and wave-number spectrum-function pairs, *Can. J. Phys.*, *46*, 2133-2153.

Singleton, D. G. (1974), Power spectra of ionospheric scintillations, *J. Atmos. Terr. Phys.*, *36*, 113-134.

Tatarskii, V. I. (1971), *The Effects of the Turbulent Atmosphere on Wave Propagation*, National Technical Information Service, Springfield, Va.

Umeki, R., C. H. Liu, and K. C. Yeh (1977), Multifrequency studies of ionospheric scintillations, *Radio Sci.*, *12*, 311-317.

Wernik, A. W., and C. H. Liu (1974), Ionospheric irregularities causing scintillation GHz frequency radio signals, *J. Atmos. Terr. Phys.*, *36*, 871-879.

Yeh, K. C., and C. H. Liu (1977), An investigation of temporal moments of stochastic waves, *Radio Sci.*, *12*, 671-680.

Yeh, K. C., C. H. Liu, and M. Y. Youakim (1975), A theoretical study of the ionospheric scintillation behavior caused by multiple scattering, *Radio Sci.*, *10*, 97-106.